香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2021年香港中學文憑考試
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2021

數學　　必修部分　　試卷一
MATHEMATICS　COMPULSORY PART　PAPER 1

評卷參考
MARKING SCHEME

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Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $(\alpha\beta^3)(\alpha^{-2}\beta^4)^5$		
$=(\alpha\beta^3)(\alpha^{-10}\beta^{20})$	1M	for $(a^h)^k=a^{hk}$ or $(ab)^l=a^lb^l$
$=\alpha^{-9}\beta^{23}$	1M	for $c^pc^q=c^{p+q}$ or $d^{-r}=\dfrac{1}{d^r}$
$=\dfrac{\beta^{23}}{\alpha^9}$	1A	
	----------(3)	
2. $\dfrac{4-3a}{b}=5$		
$4-3a=5b$	1M	
$-3a=5b-4$	1M	
$a=\dfrac{4-5b}{3}$	1A	or equivalent
$\dfrac{4-3a}{b}=5$		
$\dfrac{4}{b}-\dfrac{3a}{b}=5$		
$-3a=b\left(5-\dfrac{4}{b}\right)$	1M+1M	
$a=\dfrac{-b}{3}\left(5-\dfrac{4}{b}\right)$		
$a=\dfrac{4}{3}-\dfrac{5b}{3}$	1A	or equivalent
	----------(3)	
3. (a) $6x^2+xy-2y^2$		
$=(2x-y)(3x+2y)$	1A	or equivalent
(b) $8x-4y-6x^2-xy+2y^2$		
$=8x-4y-(2x-y)(3x+2y)$	1M	for using the result of (a)
$=4(2x-y)-(2x-y)(3x+2y)$		
$=(2x-y)(4-3x-2y)$	1A	or equivalent
	----------(3)	
4. (a) $\dfrac{7(x-2)}{5}+11>3(x-1)$		
$7x-14>15x-70$		
$-8x>-56$	1M	for putting x on one side
$x<7$	1A	
$x+4\geq 0$		
$x\geq -4$		
Thus, the required range is $-4\leq x<7$.	1A	
(b) 6	1A	
	----------(4)	

Solution	Marks	Remarks
5. Let x be the number of stickers owned by the girl.		
Then, the number of stickers owned by the boy is $3x$.	1A	
$2(3x-20)=x+20$	1M+1A	
$6x-40=x+20$		
$x=12$		
Thus, the total number of stickers owned by the boy and the girl is 48 .	1A	
Alternative: Let x and y be the number of stickers owned by the girl and the number of stickers owned by the boy respectively.		
Then, we have $3x=y$ and $2(y-20)=x+20$.	1A+1A	
$2(3x-20)=x+20$	1M	
$6x-40=x+20$		
$x=12$		
$y=36$		
Thus, the total number of stickers owned by the boy and the girl is 48 .	1A	
Alternative: Let n be the total number of stickers owned by the boy and the girl.		
Then, we have $2\left(\frac{3}{4}n-20\right)=\frac{1}{4}n+20$.	1M+1A+1A	
$\frac{5}{4}n=60$		
$n=48$	1A	
Thus, the total number of stickers owned by the boy and the girl is 48 .		
	----------(4)	
6. Let $\$x$ be the marked price of the shirt.		
The cost of the shirt $=\$(x-80)$	1M	
The selling price of the shirt $=(90\%)x$ $=\$0.9x$	1M	
$0.9x=(x-80)(1+30\%)$	1M	
$0.9x=1.3x-104$		
$x=260$	1A	
Thus, the marked price of the shirt is $\$260$.		
Alternative: Let $\$c$ be the cost of the shirt.		
The marked price of the shirt $=\$(c+80)$	1M	
The selling price of the shirt $=(c+80)(90\%)$ $=\$(0.9c+72)$	1M	
$0.9c+72=(1+30\%)c$	1M	
$0.9c+72=1.3c$		
$c=180$		
Thus, the marked price of the shirt is $\$260$.	1A	
	----------(4)	

Solution	Marks	Remarks
7. (a) $\angle POQ$		
$= 140° - 80°$		
$= 60°$	1A	
(b) Since ΔOPQ is an equilateral triangle, we have $r = 21$.	1A	
(c) The perimeter of ΔOPQ		
$= 3(21)$	1M	
$= 63$	1A	
	----------(4)	
8. (a) $\angle CAE = \angle BDE$ (given)		[已知]
$\angle AEC = \angle DEB$ (common $\angle$)		[公共角]
$\angle ACE = \angle DBE$ ($\angle$ sum of Δ)		[Δ內角和]
$\Delta ACE \sim \Delta DBE$ (AAA)		(AA) (equiangular) [等角]
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(b) (i) $AC^2 + AE^2$		
$= 25^2 + 60^2$		
$= 4\,225$		
$= 65^2$		
$= CE^2$		
Thus, ΔACE is a right-angled triangle.	1A	f.t.
(ii) $\frac{DE}{AE} = \frac{BD}{AC}$	1M	
$\frac{DE}{60} = \frac{15}{25}$		
$DE = 36$ cm		
Note that $\angle BDE = 90°$.		
The area of ΔBDE		
$= \frac{15(36)}{2}$		
$= 270 \text{ cm}^2$	1A	
	----------(5)	
9. (a) $\frac{12+k+16}{12+k+16+9+11+4} = \frac{7}{10}$	1M	
$k = 28$	1A	
(b) The range		
$= 5$	1A	
The inter-quartile range		
$= 2$	1A	
The standard deviation		
≈ 1.43	1A	r.t. 1.43
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $f(x)=m(x+4)^2+n$, where m and n are non-zero constants.	1M	
Since $f(-3)=0$ and $f(2)=105$, we have $m+n=0$ and $36m+n=105$.	1M	for either substitution
Solving, we have $m=3$ and $n=-3$.		
Thus, we have $f(0)=45$.	1A	
	----------(3)	
(b) (i) 48	1M	$(a)+3$
(ii) For $f(x)+3=0$, we have		
$3(x+4)^2=0$	1M	
$x=-4$	1A	
Thus, the x-intercept of G is -4 .		
	----------(3)	
11. (a) The mean		
$=\frac{1(15)+2(9)+3(2)+4(5)+5(4)+6(2)+7(5)}{15+9+2+5+4+2+5}$	1M	
$=\frac{126}{42}$		
$=3$	1A	
	----------(2)	
(b) The median and the mode are 2 and 1 respectively.	1M	for either one
Thus, the median and the mode of the distribution are not equal.	1A	f.t.
	----------(2)	
(c) (i) 42	1A	
(ii) 11	1A	
(iii) 10	1A	
	----------(3)	

Solution	Marks	Remarks
12. (a) Let $p(x)=(x^2+x+1)(2x^2-37)+cx+c-1$.	1M	
$p(5)=0$	1M	
$(5^2+5+1)(2(5^2)-37)+5c+c-1=0$		
$6c+402=0$		
$c=-67$	1A	
	----------(3)	
(b) $p(x)$		
$=(x^2+x+1)(2x^2-37)-67x-68$ (by (a))		
$=2x^4+2x^3-35x^2-104x-105$		
$p(-3)$		
$=2(-3)^4+2(-3)^3-35(-3)^2-104(-3)-105$		
$=0$		
Thus, $x+3$ is a factor of $p(x)$.	1	
	----------(1)	
(c) By (b), we have $p(x)=2x^4+2x^3-35x^2-104x-105$.		
Therefore, we have $p(x)=(x+3)(x-5)(2x^2+6x+7)$.	1M	$p(x)=(x+3)(x-5)(lx^2+mx+n)$
$p(x)=0$		
$(x+3)(x-5)(2x^2+6x+7)=0$		
$x=-3$, $x=5$ or $2x^2+6x+7=0$		
$6^2-4(2)(7)$	1M	
$=-20$		
<0		
So, the roots of the equation $2x^2+6x+7=0$ are not real numbers.		
Thus, the claim is not correct.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
13. (a) Note that the coordinates of G are $(6,8)$.		
OG		
$=\sqrt{(6-0)^2+(8-0)^2}$	1M	
$=10$	1A	
	----------(2)	
(b) The radius of C		
$=\frac{1}{2}\sqrt{(-12)^2+(-16)^2+4(69)}$		
$=13$		
$>OG$ (by (a))		
Thus, O lies inside C .	1A	f.t.
	----------(1)	
(c) By (b), we have $GM=13$. Let Q be the mid-point of MN . Note that Γ is the perpendicular bisector of OG . Since Q lies on Γ , Q is the mid-point of OG .		
GQ		
$=\frac{1}{2}OG$		
$=\frac{1}{2}(10)$ (by (a))	1M	for using the result of (a)
$=5$		
Also note that $\angle GQM=90°$.		
MQ		
$=\sqrt{GM^2-GQ^2}$	1M	
$=\sqrt{13^2-5^2}$		
$=12$		
Since both M and N lie on Γ , we have $OM=GM$ and $ON=GN$. Further note that $GM=GN$. So, we have $OM=GM=GN=ON$. Hence, the quadrilateral $OMGN$ is a rhombus.		
The area of the quadrilateral $OMGN$		
$=4\left(\frac{1}{2}(GQ)(MQ)\right)$	1M	
$=4\left(\frac{1}{2}(5)(12)\right)$		
$=120$	1A	
	----------(4)	

Solution	Marks	Remarks
14. (a) Let r cm be the base radius of Y.		
$\frac{24\pi r^2}{3}=800\pi$	1M	
$r=10$		
Thus, the base radius of Y is 10 cm .	1A	
	----------(2)	
(b) The volume of Z		
$=\pi(10^2)(20)+800\pi$	1M	
$=2\,800\pi$ cm^3		
$\left(\frac{\text{The base radius of } Y}{\text{The base radius of } Z}\right)^3=\left(\frac{1}{2}\right)^3=\frac{1}{8}$		
$\frac{\text{The volume of } Y}{\text{The volume of } Z}=\frac{800}{2\,800}=\frac{2}{7}$		
$\frac{\text{The volume of } Y}{\text{The volume of } Z}\neq\left(\frac{\text{The base radius of } Y}{\text{The base radius of } Z}\right)^3$	1M	for comparing the two ratios
Thus, Y and Z are not similar.	1A	f.t.
	----------(3)	
(c) The curved surface area of X		
$=2\pi(10)(20)$	1M	
$=400\pi$ cm^2		
The curved surface area of Y		
$=\pi(10)\left(\sqrt{10^2+24^2}\right)$	1M	either one
$=260\pi$ cm^2		
Let h cm be the height of Z.		
$\frac{\pi(20^2)(h)}{3}=2\,800\pi$		
$h=21$		
Therefore, the height of Z is 21 cm .		
The curved surface area of Z		
$=\pi(20)\left(\sqrt{20^2+21^2}\right)$		
$=580\pi$ cm^2		
The sum of the curved surface area of X and the curved surface area of Y		
$=400\pi+260\pi$		
$=660\pi$ cm^2		
$>580\pi$ cm^2		
Thus, the claim is agreed.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
15. (a) The required number		
$= P_{10}^{10}$		
$= 3\ 628\ 800$	1A	
	----------(1)	
(b) The required probability		
$= \dfrac{7!\,C_3^8\,3!}{3\ 628\ 800}$	1M+1M	1M for denominator + 1M for $7!3!$
$= \dfrac{1\ 693\ 440}{3\ 628\ 800}$		
$= \dfrac{7}{15}$	1A	r.t. 0.467
	----------(3)	
16. (a) The slope of L_1		
$= \dfrac{6-3}{2-0}$		
$= \dfrac{3}{2}$		
The equation of L_1 is		
$y-3=\dfrac{3}{2}(x-0)$	1M	either one (1M: equation of L_1 or L_2)
$3x-2y+6=0$	1A	either one (1A: equation of L_1 or L_2)
The equation of L_2 is		
$y-6=\dfrac{-2}{3}(x-2)$		
$2x+3y-22=0$		
Thus, the system of inequalities is $\begin{cases} 3x-2y+6\ge 0 \\ 2x+3y-22\le 0 \\ y\ge 0 \end{cases}$.	1A	or equivalent
	----------(3)	
(b) Note that the vertices of R are the points $(-2,0)$, $(2,6)$ and $(11,0)$.		
When $x=-2$ and $y=0$, we have $8x-5y=-16$.	1M	any one
When $x=2$ and $y=6$, we have $8x-5y=-14$.		
When $x=11$ and $y=0$, we have $8x-5y=88$.		
Thus, the least value of $8x-5y$ is -16 .	1A	
	----------(2)	

Solution	Marks	Remarks
17. (a) Let d be the common difference of the arithmetic sequence.		
So, we have $A(1)+4d=26$ and $A(1)+11d=61$.	1M	for either one
Solving, we have $d=5$.		
Thus, we have $A(1)=6$.	1A	
	----------(2)	
(b) $\log_8\left(G(1)G(2)G(3)\cdots G(k)\right)<999$		
$\dfrac{\log_2\left(G(1)G(2)G(3)\cdots G(k)\right)}{\log_2 8}<999$	1M	
$\log_2\left(G(1)G(2)G(3)\cdots G(k)\right)<2\,997$		
$\log_2 G(1)+\log_2 G(2)+\log_2 G(3)+\cdots+\log_2 G(k)<2\,997$	1M	
$A(1)+A(2)+A(3)+\cdots+A(k)<2\,997$		
$\dfrac{k}{2}(2(6)+(k-1)(5))<2\,997$	1M	
$5k^2+7k-5\,994<0$		
$\dfrac{-7-\sqrt{7^2-4(5)(-5\,994)}}{2(5)}<k<\dfrac{-7+\sqrt{7^2-4(5)(-5\,994)}}{2(5)}$	1M	
$-35.33076667<k<33.93076667$		
Thus, the greatest value of k is 33 .	1A	
	----------(5)	

Solution	Marks	Remarks
18. (a) Let P be a point lying on AD such that $AB // PC$. By sine formula, we have		
$\frac{CD}{\sin \angle CPD}=\frac{CP}{\sin \angle CDP}$	1M	
$\frac{CD}{\sin 50^\circ}=\frac{45}{\sin 70^\circ}$		
$CD \approx 36.68433611$ cm		
$CD \approx 36.7$ cm	1A	r.t. 36.7 cm
	---------(2)	
(b) (i) $AE = AB \cos \angle BAE = 45 \cos 50^\circ \approx 28.92544244$ cm		
$DE = BC + CD \cos \angle CDE \approx 40 + 36.68433611 \cos 70^\circ \approx 52.54678189$ cm		
$AD = \sqrt{AE^2+DE^2} \approx \sqrt{(28.92544244)^2+(52.54678189)^2} \approx 59.98204321$ cm	1M	either one
Note that $\angle ABC = 90^\circ$. $AC = \sqrt{AB^2+BC^2} = \sqrt{45^2+40^2} \approx 60.20797289$ cm		
By cosine formula, we have		
$\cos \angle CAD = \frac{AC^2+AD^2-CD^2}{2(AC)(AD)}$	1M	
$\cos \angle CAD \approx \frac{(60.20797289)^2+(59.98204321)^2-(36.68433611)^2}{2(60.20797289)(59.98204321)}$		
$\angle CAD \approx 35.54210789^\circ$		
$\angle CAD \approx 35.5^\circ$	1A	r.t. 35.5°
(ii) Let Q be the foot of the perpendicular from A to CD .		
The angle between the plane ACD and the plane $BCDE$ is $\angle AQE$.	1M	
$\frac{(AQ)(CD)}{2}=\frac{(AC)(AD)\sin \angle CAD}{2}$		
$\frac{(AQ)(36.68433611)}{2} \approx \frac{(60.20797289)(59.98204321)\sin 35.54210789^\circ}{2}$		
$AQ \approx 57.22631076$ cm		
$\sin \angle AQE = \frac{AE}{AQ}$		
$\sin \angle AQE \approx \frac{28.92544244}{57.22631076}$		
$\angle AQE \approx 30.36169732^\circ$		
Since $\angle AQE > 30^\circ$, the angle between the plane ACD and the plane $BCDE$ exceeds 30° .	1A	f.t.
	---------(5)	

Solution	Marks	Remarks
19. (a) $f(x)$		
$= x^2 - 12kx - 14x + 36k^2 + 89k + 53$		
$= x^2 - 2(6k+7)x + (6k+7)^2 - (6k+7)^2 + 36k^2 + 89k + 53$	1M	
$= (x-6k-7)^2 + 5k + 4$		
Thus, the coordinates of Q are $(6k+7, 5k+4)$.	1A	
	----------(2)	
(b) $(7-6k, 5k+4)$	1M	
	----------(1)	
(c) (i) The slope of the straight line passing through Q and S $= \frac{5k+4-(4-3k)}{6k+7-7} = \frac{4}{3}$		
The required equation is		
$\frac{y-(4-3k)}{x-7} = \frac{4}{3}$	1M	
$4x - 3y - 9k - 16 = 0$	1A	
(ii) Let r be the radius of C .		
Note that $QS = RS$.		
So, the coordinates of the centre of C are $(7, 5k+4-r)$.	1M	
Hence, the equation of C is $(x-7)^2 + (y-5k-4+r)^2 = r^2$.		
Putting $y = \frac{4x-16}{3} - 3k$ in $(x-7)^2 + (y-5k-4+r)^2 = r^2$,		
we have $(x-7)^2 + \left(\frac{4x-16}{3} - 3k - 5k - 4 + r\right)^2 = r^2$	1M	
$25x^2 + (24r - 192k - 350)x + 576k^2 - 144kr + 1344k - 168r + 1225 = 0$		
Since QS is a tangent to C , we have		
$(24r - 192k - 350)^2 - 4(25)(576k^2 - 144kr + 1344k - 168r + 1225) = 0$	1M	
Simplifying, we have $r^2 + 9kr - 36k^2 = 0$.		
Therefore, we have $r = 3k$ or $r = -12k$ (rejected) .		
Thus, the equation of C is		
$(x-7)^2 + (y-5k-4+3k)^2 = (3k)^2$		
$(x-7)^2 + (y-2k-4)^2 = 9k^2$	1A	$x^2 + y^2 - 14x - (4k+8)y - 5k^2 + 16k + 65 = 0$
(iii) For $ST // VU$, the slope of UV is equal to the slope of QS .	1M	
Therefore, we have $\frac{-14-(2k+4)}{-29-7} = \frac{4}{3}$.		
Solving, we have $k = 15$.	1A	
The coordinates of S and U are $(7, -41)$ and $(7, 34)$ respectively.		
The slope of SV $= \frac{-14+41}{-29-7} = \frac{-3}{4}$		
So, the product of the slope of QS and the slope of SV is -1 .		
Hence, we have $ST \perp SV$.		
Since $ST \perp TU$, we have $SV // TU$.		
When $k = 15$, we have $ST // VU$, $SV // TU$ and $ST \perp TU$.		
Thus, it is possible that $STUV$ is a rectangle.	1A	f.t.

Solution	Marks	Remarks
Note that T lies on QS and $QR = 12k = 2QT$. So, we have $QT = 6k$. Let $\left(t, \frac{4t-16}{3} - 3k\right)$ be the coordinates of T. $(t-6k-7)^2+\left(\frac{4t-16}{3}-3k-5k-4\right)^2=(6k)^2$ $25(t-7)^2-300k(t-7)+576k^2=0$ $t=\frac{12k}{5}+7$ or $t=\frac{48k}{5}+7$ (rejected) Hence, the coordinates of T are $\left(\frac{12k}{5}+7, \frac{k}{5}+4\right)$.		
For $ST = UV$, we have $\left(\frac{12k}{5}+7-7\right)^2+\left(\frac{k}{5}+4+3k-4\right)^2=(7+29)^2+(2k+4+14)^2$	1M	
Simplifying, we have $12k^2-72k-1620=0$. Solving, we have $k=15$ or $k=-9$ (rejected) .	1A	
The coordinates of S , T and U are $(7,-41)$, $(43,7)$ and $(7,34)$ respectively. $SV^2=(7+29)^2+(-14+41)^2=2025$ $TU^2=(7+29)^2+(34-7)^2=2025$ Therefore, we have $SV=TU$. Also note that $ST \perp TU$. When $k=15$, we have $ST=UV$, $SV=TU$ and $ST \perp TU$. Thus, it is possible that $STUV$ is a rectangle.	1A	f.t.
	-----------(9)	