2019-DSE
MATH CP
PAPER 1

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2019

Candidate Number									

MATHEMATICS Compulsory Part

PAPER 1

Question-Answer Book

8:30 am – 10:45 am (2¼ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of THREE sections, A(1), A(2) and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

(7) The diagrams in this paper are not necessarily drawn to scale.

(8) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 0 E 0 0 1 *

SECTION A(1) (35 marks)

1. Make h the subject of the formula $9(h+6k)=7h+8$. (3 marks)

2. Simplify $\dfrac{3}{7x-6}-\dfrac{2}{5x-4}$. (3 marks)

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3. The length and the breadth of a rectangle are 24 cm and $(13+r)$ cm respectively. If the length of a diagonal of the rectangle is $(17-3r)$ cm, find r. (3 marks)

4. Factorize

(a) $4m^2-9$,

(b) $2m^2n+7mn-15n$,

(c) $4m^2-9-2m^2n-7mn+15n$.

(4 marks)

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5. A wallet is sold at a discount of 25% on its marked price. The selling price of the wallet is \$690 .

(a) Find the marked price of the wallet.

(b) After selling the wallet, the percentage profit is 15% . Find the cost of the wallet.

(4 marks)

6. (a) Solve the inequality $\frac{7x+26}{4} \leq 2(3x-1)$.

(b) Find the number of integers satisfying both inequalities $\frac{7x+26}{4} \leq 2(3x-1)$ and $45-5x \geq 0$.

(4 marks)

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7. In a playground, the ratio of the number of adults to the number of children is $13:6$. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is $8:7$. Find the original number of adults in the playground. (4 marks)

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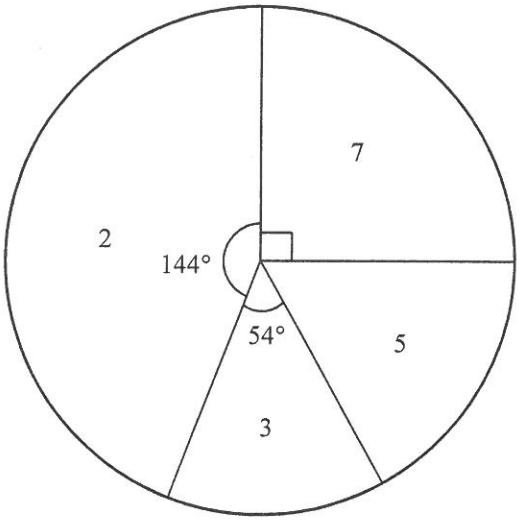
8. The pie chart below shows the distribution of the numbers of rings owned by the girls in a group.



Distribution of the numbers of rings owned by the girls in the group

(a) Write down the mode of the distribution.

(b) Find the mean of the distribution.

(c) If a girl is randomly selected from the group, find the probability that the selected girl owns more than 3 rings.

(5 marks)

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9. The sum of the volumes of two spheres is $324\pi \text{ cm}^3$. The radius of the larger sphere is equal to the diameter of the smaller sphere. Express, in terms of π,

(a) the volume of the larger sphere;

(b) the sum of the surface areas of the two spheres.

(5 marks)

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SECTION A(2) (35 marks)

10. It is given that $h(x)$ is partly constant and partly varies as x. Suppose that $h(-2) = -96$ and $h(5) = 72$.

(a) Find $h(x)$. (3 marks)

(b) Solve the equation $h(x) = 3x^2$. (2 marks)

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11. Let $p(x)$ be a cubic polynomial. When $p(x)$ is divided by $x-1$, the remainder is 50. When $p(x)$ is divided by $x+2$, the remainder is -52. It is given that $p(x)$ is divisible by $2x^2+9x+14$.

(a) Find the quotient when $p(x)$ is divided by $2x^2+9x+14$. (3 marks)

(b) How many rational roots does the equation $p(x)=0$ have? Explain your answer. (3 marks)

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12. The stem-and-leaf diagram below shows the distribution of the results (in seconds) of some boys in a 400 m race.

Stem (tens)	Leaf (units)
5	a
6	0 0 3 c c 8 9 9 9
7	0 1 1 1 2 2 5 6 9
8	b

It is given that the inter-quartile range of the distribution is 8 seconds.

(a) Find c. (2 marks)

(b) It is given that the range of the distribution exceeds 34 seconds and the mean of the distribution is 69 seconds. Find

(i) a and b,

(ii) the least possible standard deviation of the distribution.

(6 marks)

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13. In Figure 1, O is the centre of circle $ABCDE$. AC is a diameter of the circle. BD and OC intersect at the point F. It is given that $\angle AED = 115°$.

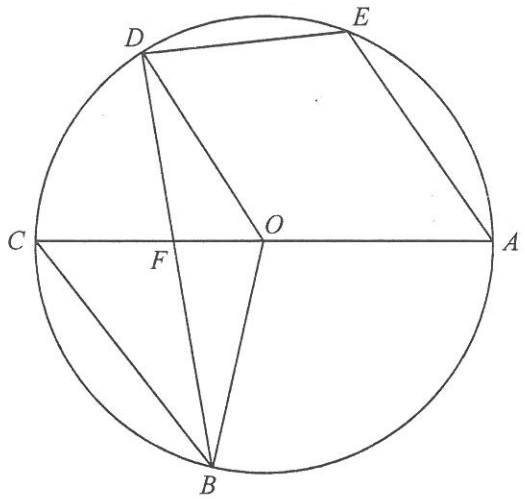

Figure 1

(a) Find $\angle CBF$. (3 marks)

(b) Suppose that $BC // OD$ and $OB = 18$ cm. Is the perimeter of the sector OBC less than 60 cm ? Explain your answer. (5 marks)

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14. In Figure 2, $ABCD$ is a square. It is given that E is a point lying on AD. BD and CE intersect at the point F. Let G be a point such that $BG // EC$ and $CG // DB$.

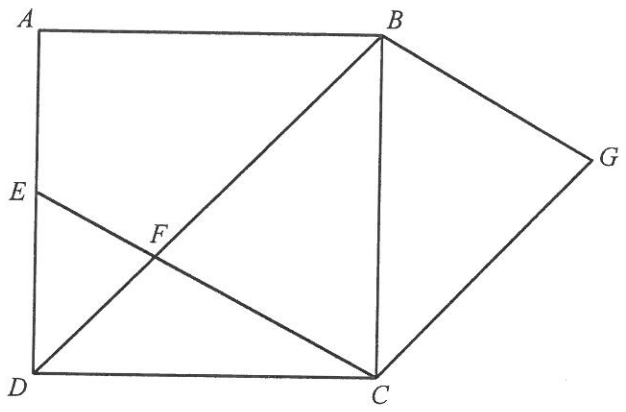

Figure 2

(a) Prove that

(i) $\Delta BCG \cong \Delta CBF$,

(ii) $\Delta BCF \sim \Delta DEF$.

(4 marks)

(b) Suppose that $\angle BCF = \angle BGC$.

(i) Let $BC = \ell$. Express DF in terms of ℓ.

(ii) Someone claims that $AE > DF$. Do you agree? Explain your answer.

(4 marks)

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SECTION B (35 marks)

15. There are 21 boys and 11 girls in a class. If 5 students are selected from the class to form a committee consisting of at least 1 boy, how many different committees can be formed? (3 marks)

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16. Let α and β be real numbers such that $\begin{cases} \beta = 5\alpha - 18 \\ \beta = \alpha^2 - 13\alpha + 63 \end{cases}$.

(a) Find α and β. (2 marks)

(b) The 1st term and the 2nd term of an arithmetic sequence are $\log\alpha$ and $\log\beta$ respectively. Find the least value of n such that the sum of the first n terms of the sequence is greater than 888. (4 marks)

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17. (a) Let a and p be the area and the perimeter of ΔCDE respectively. Denote the radius of the inscribed circle of ΔCDE by r. Prove that $pr = 2a$. (2 marks)

(b) The coordinates of the points H and K are $(9, 12)$ and $(14, 0)$ respectively. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to OH is equal to the perpendicular distance from P to HK, where O is the origin. Denote the locus of P by Γ.

(i) Describe the geometric relationship between Γ and $\angle OHK$.

(ii) Using (a), find the equation of Γ.

(5 marks)

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18. Figure 3 shows a tetrahedron $ABCD$. Let P be a point lying on AD such that BP is perpendicular to AD. A craftsman finds that $AC = AD = CD = 13$ cm, $BC = 8$ cm, $BD = 12$ cm and $\angle ABD = 72°$.

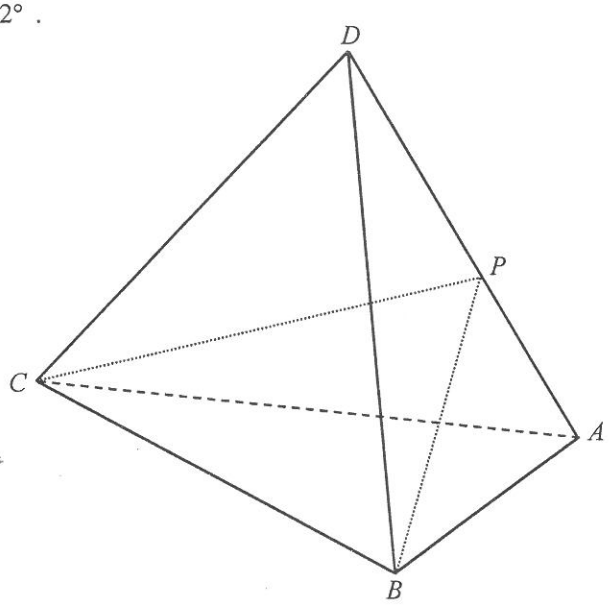

Figure 3

(a) Find

(i) $\angle BAD$,

(ii) CP.

(5 marks)

(b) The craftsman claims that $\angle BPC$ is the angle between the face ABD and the face ACD. Is the claim correct? Explain your answer. (2 marks)

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19. Let $f(x)=\frac{1}{1+k}\left(x^2+(6k-2)x+(9k+25)\right)$, where k is a positive constant. Denote the point $(4,33)$ by F.

(a) Prove that the graph of $y=f(x)$ passes through F. (1 mark)

(b) The graph of $y=g(x)$ is obtained by reflecting the graph of $y=f(x)$ with respect to the y-axis and then translating the resulting graph upwards by 4 units. Let U be the vertex of the graph of $y=g(x)$. Denote the origin by O.

(i) Using the method of completing the square, express the coordinates of U in terms of k.

(ii) Find k such that the area of the circle passing through F, O and U is the least.

(iii) For any positive constant k, the graph of $y=g(x)$ passes through the same point G. Let V be the vertex of the graph of $y=g(x)$ such that the area of the circle passing through F, O and V is the least. Are F, G, O and V concyclic? Explain your answer.

(11 marks)

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