# Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

## General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

| | |
|---|---|
| 'M' marks | awarded for correct methods being used; |
| 'A' marks | awarded for the accuracy of the answers; |
| Marks without 'M' or 'A' | awarded for correctly completing a proof or arriving at an answer given in a question. |

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

5. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

**Paper 1**

| Solution | Marks | Remarks |
|---|---|---|
| 1. $9(h+6k)=7h+8$ | | |
| [redacted] | 1M | |
| | 1M | for putting $h$ on one side |
| $2h=8-54k$ | | |
| $h=4-27k$ | 1A | or equivalent |
| $9(h+6k)=7h+8$ | | |
| [redacted] | 1M | |
| | 1M | for putting $h$ on one side |
| $\frac{2h}{9}=\frac{8-54k}{9}$ | | |
| $2h=8-54k$ | | |
| $h=4-27k$ | 1A | or equivalent |
| | ----------(3) | |
| 2. $\frac{3}{7x-6}-\frac{2}{5x-4}$ | | |
| [redacted] | 1M | |
| | 1M | |
| $=\frac{x}{(7x-6)(5x-4)}$ | 1A | or equivalent |
| | ----------(3) | |
| 3. $24^2+(13+r)^2=(17-3r)^2$ | 1M | |
| $576+169+26r+r^2=289-102r+9r^2$ | | |
| $8r^2-128r-456=0$ | 1M | for $ar^2+br+c=0$ |
| $r^2-16r-57=0$ | | |
| $(r+3)(r-19)=0$ | | |
| $r=-3$ [redacted] | 1A | |
| Thus, we have $r=-3$ . | | |
| | ----------(3) | |
| 4. (a) $4m^2-9$ | | |
| $=(2m+3)(2m-3)$ | 1A | or equivalent |
| (b) $2m^2n+7mn-15n$ | | |
| $=n(2m^2+7m-15)$ | | |
| $=n(2m-3)(m+5)$ | 1A | or equivalent |
| (c) $4m^2-9-2m^2n-7mn+15n$ | | |
| $=4m^2-9-(2m^2n+7mn-15n)$ | | |
| [redacted] | 1M | for using the results of (a) and (b) |
| $=(2m-3)(2m-mn-5n+3)$ | 1A | or equivalent |
| | ----------(4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 5. (a) Let $\$m$ be the marked price of the wallet. | | |
| $(1-25\%)m=690$ | 1M | |
| $m=\dfrac{690}{0.75}$ | | |
| $m=920$ | 1A | |
| Thus, the marked price of the wallet is $\$920$ . | | |
| (b) Let $\$c$ be the cost of the wallet. | | |
| $(1+15\%)c=690$ | 1M | |
| $c=\dfrac{690}{1.15}$ | | |
| $c=600$ | 1A | |
| Thus, the cost of the wallet is $\$600$ . | | |
| | ----------(4) | |
| 6. (a) $\dfrac{7x+26}{4}\le 2(3x-1)$ | | |
| $7x+26\le 24x-8$ | | |
| $7x-24x\le -8-26$ | 1M | for putting $x$ on one side |
| $-17x\le -34$ | | |
| $x\ge 2$ | 1A | |
| (b) $45-5x\ge 0$ | | |
| $x\le 9$ | 1A | |
| By (a), we have $2\le x\le 9$ . | | |
| Thus, the required number is 8 . | 1A | |
| | ----------(4) | |
| 7. Let $13k$ and $6k$ be the original number of adults and the original number of children in the playground respectively [redacted]. | 1A | can be absorbed |
| $\dfrac{13k+9}{6k+24}=\dfrac{8}{7}$ | 1M+1A | |
| $91k-48k=192-63$ | | |
| $k=3$ | | |
| Thus, the original number of adults in the playground is 39 . | 1A | |
| Let $x$ and $y$ be the original number of adults and the original number of children in the playground respectively. | | |
| $\begin{cases}\dfrac{x}{y}=\dfrac{13}{6}\\ \dfrac{x+9}{y+24}=\dfrac{8}{7}\end{cases}$ | 1A+1A | |
| $\begin{cases}6x=13y\\ 7x-8y=129\end{cases}$ | | |
| So, we have $7x-8\left(\dfrac{6x}{13}\right)=129$ . | 1M | for getting a linear equation in $x$ or $y$ only |
| Solving, we have $x=39$ . | 1A | |
| Thus, the original number of adults in the playground is 39 . | | |
| | ----------(4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 8. (a) 2 | 1A | |
| (b) Note that $360° - 54° - 90° - 144° = 72°$ . | | |
| The mean of the distribution $= \frac{2(144) + 3(54) + 5(72) + 7(90)}{360}$ | 1M | |
| $= 4$ | 1A | |
| (c) The required probability $= \frac{72 + 90}{360}$ | 1M | |
| $= \frac{9}{20}$ | 1A | 0.45 |
| The required probability $= \frac{360 - 54 - 144}{360}$ | 1M | |
| $= \frac{9}{20}$ | 1A | 0.45 |
| | ----------(5) | |
| 9. (a) Note that the ratio of the radius of the larger sphere to the radius of the smaller sphere is $2:1$ . So, the ratio of the volume of the larger sphere to the volume of the smaller sphere is $8:1$ . | 1M | |
| The volume of the larger sphere $= 324\pi\left(\frac{8}{1+8}\right)$ | | |
| $= 288\pi \text{ cm}^3$ | 1A | |
| (b) Let $R$ cm be the radius of the larger sphere. | | |
| $\frac{4}{3}\pi R^3 = 288\pi$ | 1M | |
| $R = 6$ So, the radius of the smaller sphere is 3 cm . | | |
| The sum of the surface areas of the two spheres $= 4\pi(6^2) + 4\pi(3^2)$ | 1M | |
| $= 180\pi \text{ cm}^2$ | 1A | |
| | ----------(5) | |

| Solution | Marks | Remarks |
|---|---|---|
| 10. (a) Let $h(x) = r + sx$ [redacted]. | 1A | |
| So, we have $r - 2s = -96$ and $r + 5s = 72$ . | 1M | for either substitution |
| Solving, we have $r = -48$ and $s = 24$ . | 1A | for both correct |
| Thus, we have $h(x) = 24x - 48$ . | | |
| | ----------(3) | |
| (b) $h(x) = 3x^2$ | | |
| $3x^2 - 24x + 48 = 0$ | 1M | |
| $x = 4$ | 1A | |
| | ----------(2) | |
| 11. (a) Let $ax + b$ be the required quotient [redacted]. | 1M | |
| Then, we have $p(x) = (ax+b)(2x^2+9x+14)$ . | | |
| Note that $p(1) = 50$ and $p(-2) = -52$ . | 1M | for either one |
| Hence, we have $(a(1)+b)(2(1)^2+9(1)+14) = 50$ and | | |
| $(a(-2)+b)(2(-2)^2+9(-2)+14) = -52$ . | | |
| So, we have $a + b = 2$ and $-2a + b = -13$ . | | |
| Solving, we have $a = 5$ and $b = -3$ . | 1A | for both correct |
| Thus, the required quotient is $5x - 3$ . | | |
| | ----------(3) | |
| (b) $p(x) = 0$ | | |
| $(5x-3)(2x^2+9x+14) = 0$ ( by (a) ) | | |
| $5x - 3 = 0$ or $2x^2 + 9x + 14 = 0$ | | |
| $9^2 - 4(2)(14)$ | 1M | |
| $= -31$ | | |
| $< 0$ | | |
| So, the quadratic equation $2x^2 + 9x + 14 = 0$ does not have real roots. | 1M | |
| Note that $\frac{3}{5}$ is a rational root of the equation $p(x) = 0$ . | | |
| Thus, the equation $p(x) = 0$ has 1 rational root. | 1A | f.t. |
| | ----------(3) | |

| Solution | Marks | Remarks |
|---|---|---|
| 12. (a) $72-(60+c)=8$ | 1M | |
| $c=4$ | 1A | |
| | ----------(2) | |
| (b) (i) $(80+b)-(50+a)>34$ | 1M | |
| $b-a>4$ | | |
| $\frac{50+a+60(2)+63+64(2)+68+69(3)+70+71(3)+72(2)+75+76+79+80+b}{20}=69$ | 1M | |
| Therefore, we have $a+b=7$ . | | |
| Thus, we have $\begin{cases}a=0\\b=7\end{cases}$ or $\begin{cases}a=1\\b=6\end{cases}$ . | 1A+1A | 1A for one pair + 1A for all |
| (ii) By (b)(i), there are two cases. | | |
| Case 1: $a=0$ and $b=7$<br>The standard deviation of the distribution<br>$\approx 7.582875444$ | 1M | either one |
| Case 2: $a=1$ and $b=6$<br>The standard deviation of the distribution<br>$\approx 7.341661937$ | | |
| Thus, the least possible standard deviation of the distribution is 7.34 seconds. | 1A | f.t. |
| Note that $(50-69)^2+(87-69)^2>(51-69)^2+(86-69)^2$ . | 1M | |
| When $a=1$ and $b=6$ , the standard deviation of the distribution is the least. | | |
| The standard deviation<br>$\approx 7.341661937$ | | |
| Thus, the least possible standard deviation of the distribution is 7.34 seconds. | 1A | f.t. |
| | ----------(6) | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| 13. (a) Note that $\angle ABF + \angle AED = 180°$ . | 1M | |
| So, we have $\angle ABF + 115° = 180°$ . | | |
| Hence, we have $\angle ABF = 65°$ . | | |
| Also note that $\angle ABC = 90°$ . | 1M | |
| Therefore, we have $\angle CBF + 65° = 90°$ . | | |
| Thus, we have $\angle CBF = 25°$ . | 1A | |
| $\angle AOD$ | | |
| $= 360° - 2\angle AED$ | 1M | |
| $= 360° - 2(115°)$ | | |
| $= 130°$ | | |
| $\angle COD$ | | |
| $= 180° - \angle AOD$ | 1M | |
| $= 180° - 130°$ | | |
| $= 50°$ | | |
| Since $2\angle CBF = \angle COD$ , we have $\angle CBF = 25°$ . | 1A | |
| | ----------(3) | |
| (b) $\angle ODF = \angle CBF = 25°$ | 1M | |
| $\angle OBF = \angle ODF = 25°$ | 1M | |
| $\angle DOF$ | | |
| $= 2\angle CBF$ | | |
| $= 2(25°)$ | | |
| $= 50°$ | | |
| $\angle BOC$ | | |
| $= 180° - \angle DOF - \angle OBF - \angle ODF$ | 1M | |
| $= 180° - 50° - 25° - 25°$ | | |
| $= 80°$ | | |
| The perimeter of the sector $OBC$ | | |
| $= \frac{80}{360}(2\pi(18)) + 2(18)$ | 1M | |
| $= 8\pi + 36$ | | |
| $> 8(3) + 36$ | | |
| $= 60$ | | |
| Thus, the perimeter of the sector $OBC$ is not less than 60 cm . | 1A | f.t. |
| $\angle ODF = \angle CBF = 25°$ | 1M | |
| $\angle OBF = \angle ODF = 25°$ | 1M | |
| $\angle BOC$ | | |
| $= 180° - \angle COD - \angle OBF - \angle ODF$ | 1M | |
| $= 180° - 50° - 25° - 25°$ | | |
| $= 80°$ | | |
| The perimeter of the sector $OBC$ | | |
| $= \frac{80}{360}(2\pi(18)) + 2(18)$ | 1M | |
| $= 8\pi + 36$ | | |
| $> 8(3) + 36$ | | |
| $= 60$ | | |
| Thus, the perimeter of the sector $OBC$ is not less than 60 cm . | 1A | f.t. |
| | ----------(5) | |

| Solution | Marks | Remarks |
|---|---|---|
| 14. **Marking Schemes for (a)(i) and (a)(ii) :** | | |
| Case 1 Any correct proof with correct reasons. | 2 | |
| Case 2 Any correct proof without reasons. | 1 | |
| (a) (i) $BC = BC$ (common side) | | |
| $\angle BCG = \angle CBF$ (alt. $\angle$s, $CG // DB$) | | |
| $\angle CBG = \angle BCF$ (alt. $\angle$s, $BG // EC$) | | |
| $\Delta BCG \cong \Delta CBF$ (ASA) | | |
| (ii) $\angle CBF = \angle EDF$ (alt. $\angle$s, $BC // ED$) | | |
| $\angle BFC = \angle DFE$ (vert. opp. $\angle$s) | | |
| [redacted] | | |
| $\Delta BCF \sim \Delta DEF$ (AAA) | | (AA) (equiangular) |
| | ----------(4) | |
| (b) (i) By (a)(i), we have $\angle BGC = \angle BFC$ . | | |
| Since $\angle BCF = \angle BGC$ , we have $\angle BCF = \angle BFC$ . | | |
| Therefore, we have $BF = BC = \ell$ . | 1M | |
| Since $BD\cos 45° = \ell$ , we have $BD = \sqrt{2}\ell$ . | | |
| $DF$ | | |
| $= BD - BF$ | | |
| $= \sqrt{2}\ell - \ell$ | 1A | |
| $= (\sqrt{2} - 1)\ell$ | | |
| (ii) By (b)(i), $\Delta BCF$ is an isosceles triangle with $BC = BF$ . | | |
| By (a)(ii), $\Delta DEF$ is an isosceles triangle with $DE = DF$ . | | |
| $AE$ | | |
| $= AD - DE$ | | |
| $= AD - DF$ | | |
| $= \ell - (\sqrt{2} - 1)\ell$ (by (b)(i)) | 1M | for using the result of (b)(i) |
| $= (2 - \sqrt{2})\ell$ | | |
| $> \left(2 - \dfrac{3}{2}\right)\ell$ | | |
| $= \dfrac{\ell}{2}$ | | |
| Note that $AE + DE = \ell$ . | | |
| So, we have $DE < \dfrac{\ell}{2}$ . | | |
| Since $DE = DF$ , we have $DF < \dfrac{\ell}{2}$ . | | |
| Therefore, we have $AE > DF$ . | | |
| Thus, the claim is agreed. | 1A | f.t. |
| | ----------(4) | |

| Solution | Marks | Remarks |
|---|---|---|
| 15. The required number | | |
| $= C_5^{32} - C_5^{11}$ | 1M+1M | 1M for $C_p^m - C_q^n$ <br> +1M for either one |
| $= 200\,914$ | 1A | |
| The required number | | |
| $= C_1^{21}C_4^{11} + C_2^{21}C_3^{11} + C_3^{21}C_2^{11} + C_4^{21}C_1^{11} + C_5^{21}$ | 1M+1M | 1M for considering 5 cases <br> +1M for either one |
| $= 200\,914$ | 1A | |
| | ----------(3) | |
| 16. (a) Putting $\beta = 5\alpha - 18$ in $\beta = \alpha^2 - 13\alpha + 63$ , we have | | |
| $5\alpha - 18 = \alpha^2 - 13\alpha + 63$ | 1M | |
| $\alpha^2 - 18\alpha + 81 = 0$ | | |
| Solving, we have $\alpha = 9$ and $\beta = 27$ . | 1A | for both correct |
| | ----------(2) | |
| (b) Let $T(n)$ be the $n$th term of the arithmetic sequence. | | |
| Since $T(1) = \log 9 = \log 3^2 = 2\log 3$ and $T(2) = \log 27 = \log 3^3 = 3\log 3$ , | 1M | for either one |
| the common difference of the sequence is $\log 3$ . | | |
| $T(1) + T(2) + T(3) + \cdots + T(n) > 888$ | | |
| $2\log 3 + 3\log 3 + 4\log 3 + \cdots + (n+1)\log 3 > 888$ | | |
| $\frac{n}{2}\left(2(2\log 3) + (n-1)\log 3\right) > 888$ | 1M | |
| $(\log 3)n^2 + (3\log 3)n - 1776 > 0$ | | |
| $n < -62.52928981$ or $n > 59.52928981$ | 1M | |
| Thus, the least value of $n$ is $60$ . | 1A | |
| Let $T(n)$ be the $n$th term of the arithmetic sequence. | | |
| Since $T(1) = \log 9 = \log 3^2$ and $T(2) = \log 27 = \log 3^3$ , | | |
| the common difference of the sequence is $\log 3$ . | | |
| $T(1) + T(2) + T(3) + \cdots + T(n) > 888$ | | |
| $\log 9 + \log 27 + \log 81 + \cdots + \log 3^{n+1} > 888$ | | |
| $\log 3^2 + \log 3^3 + \log 3^4 + \cdots + \log 3^{n+1} > 888$ | | |
| $\log\left(3^2 \cdot 3^3 \cdot 3^4 \cdots 3^{n+1}\right) > 888$ | 1M | |
| $\log\left(3^{2+3+4+\cdots+(n+1)}\right) > 888$ | | |
| $\log 3^{\frac{n(n+3)}{2}} > 888$ | 1M | |
| $3^{\frac{n(n+3)}{2}} > 10^{888}$ | | |
| $\frac{n(n+3)}{2} > \log_3 10^{888}$ | | |
| $n^2 + 3n - 2\log_3 10^{888} > 0$ | | |
| $n < -62.52928981$ or $n > 59.52928981$ | 1M | |
| Thus, the least value of $n$ is $60$ . | 1A | |
| | ----------(4) | |

| Solution | Marks | Remarks |
| --- | --- | --- |
| 17. (a) $\frac{r(CD)}{2}+\frac{r(DE)}{2}+\frac{r(CE)}{2}=a$ | 1M | |
| $r(CD+DE+CE)=2a$ | | |
| $pr=2a$ | 1 | |
| | ----------(2) | |
| (b) (i) $\Gamma$ is the angle bisector of $\angle OHK$ . | 1M | |
| (ii) $OH$ $=\sqrt{9^2+12^2}$ $=15$ | | |
| $HK$ $=\sqrt{(9-14)^2+12^2}$ $=13$ | | |
| Note that the area of $\Delta OHK=\frac{14(12)}{2}=84$ . | | |
| Also note that the perimeter of $\Delta OHK=13+14+15=42$ . | | |
| Let $r$ be the radius of the inscribed circle of $\Delta OHK$ . | | |
| By (a), we have $42r=2(84)$ . | 1M | for using (a) |
| So, we have $r=4$ . | | |
| Let $(h,4)$ be the coordinates of the in-centre of $\Delta OHK$ . | | |
| Hence, we have $(15-h)+(14-h)=13$ . | 1M | |
| Therefore, we have $h=8$ . | | |
| The slope of $\Gamma$ $=\frac{12-4}{9-8}$ $=8$ | | |
| The equation of $\Gamma$ is | | |
| $y-4=8(x-8)$ | 1M | |
| $8x-y-60=0$ | 1A | or equivalent |
| | ----------(5) | |

| Solution | Marks | Remarks |
|---|---|---|
| 18. (a) (i) By sine formula, we have | | |
| $\frac{\sin\angle BAD}{BD}=\frac{\sin\angle ABD}{AD}$ | 1M | |
| $\frac{\sin\angle BAD}{12}=\frac{\sin 72^\circ}{13}$ | | |
| [redacted] | | |
| Thus, we have $\angle BAD \approx 61.4^\circ$ . | 1A | r.t. $61.4^\circ$ |
| (ii) $\angle ADB \approx 180^\circ - 72^\circ - 61.38986936^\circ$ | | |
| $\angle ADB \approx 46.61013064^\circ$ | | |
| $\cos\angle ADB = \frac{AD-AP}{BD}$ | 1M | |
| $AP \approx 13 - 12\cos 46.61013064^\circ$ | | |
| $AP \approx 4.756491614$ | | |
| Note that $\angle CAP = 60^\circ$ . | | |
| By cosine formula, we have | | |
| $CP^2 = AC^2 + AP^2 - 2(AC)(AP)\cos\angle CAP$ | 1M | |
| $CP^2 \approx 13^2 + 4.756491614^2 - 2(13)(4.756491614)\cos 60^\circ$ | | |
| [redacted] | | |
| $CP \approx 11.4$ cm | 1A | r.t. 11.4 cm |
| By sine formula, we have | | |
| $\frac{AB}{\sin\angle ADB}=\frac{AD}{\sin\angle ABD}$ | | |
| $\frac{AB}{\sin(180^\circ - 72^\circ - 61.38986936^\circ)} \approx \frac{13}{\sin 72^\circ}$ | | |
| $AB \approx 9.933216094$ | | |
| $\cos\angle BAD = \frac{AP}{AB}$ | 1M | |
| $AP = AB\cos\angle BAD$ | | |
| $AP \approx 4.756491614$ | | |
| Note that $\angle CAP = 60^\circ$ . | | |
| By cosine formula, we have | | |
| $CP^2 = AC^2 + AP^2 - 2(AC)(AP)\cos\angle CAP$ | 1M | |
| $CP^2 \approx 13^2 + 4.756491614^2 - 2(13)(4.756491614)\cos 60^\circ$ | | |
| [redacted] | | |
| $CP \approx 11.4$ cm | 1A | r.t. 11.4 cm |
| | ----------(5) | |
| (b) $AP^2 + CP^2$ | | |
| $\approx 4.756491614^2 + 11.39253359^2$ | | |
| $\approx 152.4140341$ | | |
| $AC^2$ | | |
| $= 169$ | | |
| Hence, we have $AP^2 + CP^2 \neq AC^2$ . | | |
| Therefore, $\angle APC$ is not a right angle. | 1M | |
| So, $\angle BPC$ is not the angle between the face $ABD$ and the face $ACD$ . | | |
| Thus, the claim is not correct. | 1A | f.t. |
| | ----------(2) | |

| Solution | Marks | Remarks |
|---|---|---|
| 19. (a) $f(4)$ | | |
| $=\frac{1}{1+k}\left(4^2+4(6k-2)+(9k+25)\right)$ | | |
| $=\frac{1}{1+k}(33+33k)$ | | |
| $=33$ | 1 | |
| Thus, the graph of $y=f(x)$ passes through $F$. | | |
| | ----------(1) | |
| (b) (i) $g(x)$ | | |
| $=f(-x)+4$ | 1M | |
| $=\frac{1}{1+k}\left((-x)^2+(6k-2)(-x)+(9k+25)\right)+4$ | | |
| $=\frac{1}{1+k}\left(x^2-(6k-2)x+(3k-1)^2-(3k-1)^2+(9k+25)\right)+4$ | 1M | for completing the square |
| $=\frac{1}{k+1}\left((x-3k+1)^2-(k+1)(9k-24)\right)+4$ | | |
| $=\frac{1}{k+1}(x-(3k-1))^2+(28-9k)$ | 1M | |
| Thus, the coordinates of $U$ are $(3k-1, 28-9k)$ . | 1A | |
| (ii) Note that the area of the circle passing through $F$ and $O$ is the least when $FO$ is a diameter of the circle. | 1M | |
| If $U$ lies on this circle, then we have $\angle FUO=90°$ . | | |
| Under this case, we have $k\neq\frac{1}{3}$ and $k\neq\frac{5}{3}$ . | | |
| $\left(\frac{(28-9k)-0}{(3k-1)-0}\right)\left(\frac{33-(28-9k)}{4-(3k-1)}\right)=-1$ | 1M+1A | |
| $\frac{(28-9k)(5+9k)}{(3k-1)(5-3k)}=-1$ | | |
| $2k^2-5k-3=0$ | | |
| $k=3$ [redacted] | 1A | |
| Thus, the area of the circle passing through $F$, $O$ and $U$ is the least when $k=3$ . | | |
| Note that the area of the circle passing through $F$ and $O$ is the least when $FO$ is a diameter of the circle. | 1M | |
| Let $M$ be the mid-point of $FO$. | | |
| The coordinates of $M$ | | |
| $=\left(2,\frac{33}{2}\right)$ | | |
| If $U$ lies on this circle, then we have $FO=2MU$. | | |
| $\sqrt{(0-4)^2+(0-33)^2}=2\sqrt{(2-(3k-1))^2+\left(\frac{33}{2}-(28-9k)\right)^2}$ | 1M+1A | |
| $2k^2-5k-3=0$ | | |
| $k=3$ [redacted] | 1A | |
| Thus, the area of the circle passing through $F$, $O$ and $U$ is the least when $k=3$ . | | |

| Solution | Marks | Remarks |
|---|---|---|
| (iii) The coordinates of $G$ are $(-4,37)$ . | 1A | |
| The product of the slope of $FG$ and the slope of $GO$<br>$=\left(\frac{37-33}{-4-4}\right)\left(\frac{37-0}{-4-0}\right)$ | 1M | |
| $=\frac{37}{8}$<br>$\neq -1$<br>So, we have $\angle FGO \neq 90°$ .<br>Since $\angle FVO = 90°$ , $G$ does not lie on the circle passing through $F$, $O$ and $V$. | | |
| Thus, $F$, $G$, $O$ and $V$ are not concyclic. | 1A | f.t. |
| When the area of the circle passing through $F$, $O$ and $V$ is the least, $FO$ is a diameter of the circle.<br>The coordinates of $G$ are $(-4,37)$ . | 1A | |
| $FO^2$<br>$=1105$ | 1M | any one |
| $GO^2$<br>$=1385$ | | |
| $FG^2$<br>$=80$ | | |
| $FG^2+GO^2$<br>$=1465$<br>As $FG^2+GO^2 \neq FO^2$ , $\angle FGO$ is not a right angle.<br>Since $\angle FVO = 90°$ , $G$ does not lie on the circle passing through $F$, $O$ and $V$. | | |
| Thus, $F$, $G$, $O$ and $V$ are not concyclic. | 1A | f.t. |
| When the area of the circle passing through $F$, $O$ and $V$ is the least, $FO$ is a diameter of the circle.<br>The coordinates of the centre of the circle passing through $F$, $O$ and $V$<br>$=\left(2,\frac{33}{2}\right)$<br>Note that the circle passes through $(0,0)$ .<br>Let $x^2+y^2+Dx+Ey=0$ be the equation of the circle passing through $F$, $O$ and $V$. | | |
| So, we have $\frac{-D}{2}=2$ and $\frac{-E}{2}=\frac{33}{2}$ . | 1M | |
| Solving, we have $D=-4$ and $E=-33$ .<br>Therefore, the equation of the circle passing through $F$, $O$ and $V$ is $x^2+y^2-4x-33y=0$ . | | |
| Also note that the coordinates of $G$ are $(-4,37)$ . | 1A | |
| Since $(-4)^2+(37)^2-4(-4)-33(37)\neq 0$ , $G$ does not lie on the circle passing through $F$, $O$ and $V$. | | |
| Thus, $F$, $G$, $O$ and $V$ are not concyclic. | 1A | f.t. |
| | ----------(11) | |

**Paper 2**

| Question No. | Key | Question No. | Key |
|---|---|---|---|
| 1. | C (67) | 26. | D (68) |
| 2. | D (88) | 27. | B (56) |
| 3. | B (90) | 28. | C (66) |
| 4. | C (69) | 29. | B (84) |
| 5. | A (75) | 30. | C (78) |
| 6. | D (80) | 31. | B (34) |
| 7. | B (61) | 32. | D (35) |
| 8. | C (69) | 33. | A (61) |
| 9. | D (65) | 34. | D (46) |
| 10. | A (68) | 35. | C (53) |
| 11. | C (79) | 36. | C (41) |
| 12. | B (66) | 37. | A (31) |
| 13. | A (69) | 38. | A (35) |
| 14. | C (91) | 39. | B (49) |
| 15. | D (61) | 40. | C (38) |
| 16. | D (25) | 41. | B (44) |
| 17. | A (58) | 42. | C (65) |
| 18. | D (26) | 43. | D (47) |
| 19. | A (85) | 44. | B (72) |
| 20. | C (51) | 45. | A (56) |
| 21. | B (53) | | |
| 22. | B (59) | | |
| 23. | A (32) | | |
| 24. | A (65) | | |
| 25. | D (69) | | |

*Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.*