2010-CE
MATH
PAPER 1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2010

Please stick the barcode label here.

Candidate Number							

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.

3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

4. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

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SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Simplify $a^{14}\left(\frac{b^3}{a^2}\right)^5$ and express your answer with positive indices. (3 marks)

2. (a) Solve the inequality $\frac{29x-22}{7} \leq 3x$.

(b) Write down the greatest integer satisfying the inequality in (a). (3 marks)

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3. Factorize

(a) $m^2 + 12mn + 36n^2$,

(b) $m^2 + 12mn + 36n^2 - 25k^2$.

(3 marks)

4. For each positive integer n, the nth term of a sequence is $\tan\frac{180°}{n+2}$.

(a) Find the 2nd term of the sequence.

(b) Write down, in surd form, two different terms of the sequence such that the product of these two terms is equal to the 2nd term of the sequence.

(3 marks)

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5. Consider the formula $3(2c+5d+4)=39d$.

(a) Make c the subject of the above formula.

(b) If the value of d is decreased by 1 , how will the value of c be changed?

(4 marks)

6. The cost of a bottle of orange juice is the same as the cost of 2 bottles of milk. The total cost of 3 bottles of orange juice and 5 bottles of milk is $\$66$. Find the cost of a bottle of milk.

(4 marks)

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7. Mary has 50 badges. The number of badges owned by Tom is 30% less than that owned by Mary.

(a) How many badges does Tom have?

(b) If Mary gives a certain number of her badges to Tom, will they have the same number of badges? Explain your answer.

(4 marks)

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8. Three students, Peter, John and Henry have $16.8, $24.3 and $32.5 respectively.

(a) By rounding down the amount owned by each student to the nearest dollar, estimate the total amount they have.

(b) If the three students want to buy a football of price $70, will they have enough money to buy the football? Use the result of (a) to explain your answer.

(4 marks)

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9. In Figure 1, $AB = CD$, $AE // CD$, $\angle BAE = 108°$ and $\angle BCD = 126°$.

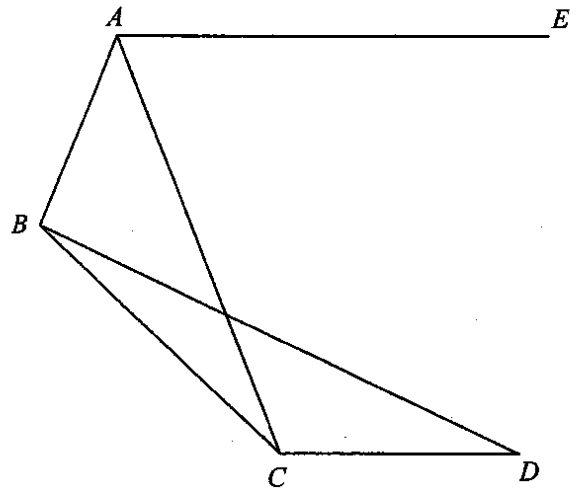

Figure 1

(a) Find $\angle ABC$.

(b) Prove that $\Delta ABC \cong \Delta DCB$.

(5 marks)

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SECTION A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. The cost of a tablecloth of perimeter x metres is $\$C$. It is given that C is the sum of two parts, one part varies as x and the other part varies as x^2. When $x=4$, $C=96$ and when $x=5$, $C=145$.

(a) Express C in terms of x. (4 marks)

(b) If the cost of a tablecloth is $\$288$, find its perimeter. (3 marks)

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11. The stem-and-leaf diagram below shows the ages of the players of a football team:

Stem (tens)	Leaf (units)
1	8 9 9
2	0 1 1 1 3 3 5 6 6 7 7 8 8 8 8
3	0 0 1 1

(a) Find the mean, the median and the range of the ages of the players of the football team. (3 marks)

(b) As the two oldest players leave the team, three new players join the football team. After the three players join the football team, the manager of the team finds that the mean age of the players of the football team is the same as the mean found in (a).

(i) Find the mean age of the three new players.

(ii) Furthermore, the manager finds that the median and the range of the ages of the players of the football team are the same as the median and the range found in (a) respectively. Write down two sets of possible ages of the three new players.

(5 marks)

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12. In Figure 2, the straight line passing through A and B is perpendicular to the straight line passing through A and C, where C is a point lying on the x-axis.

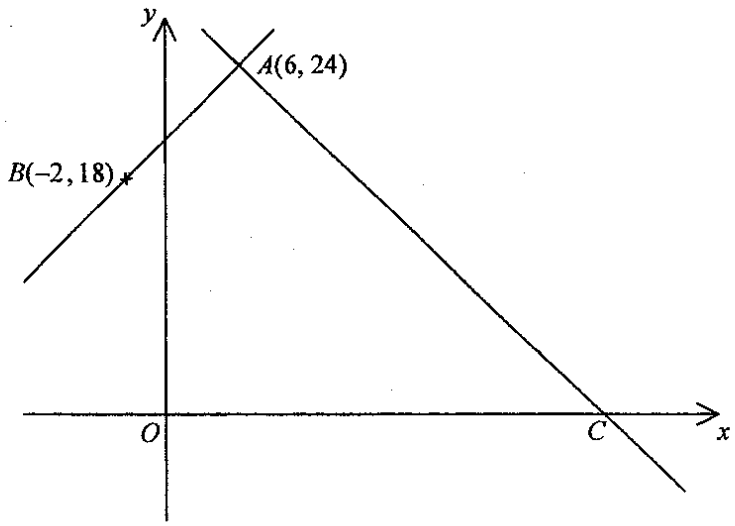

Figure 2

(a) Find the equation of the straight line passing through A and B. (2 marks)

(b) Find the coordinates of C. (3 marks)

(c) Find the area of ΔABC. (2 marks)

(d) A straight line passing through A cuts the line segment BC at D such that the area of ΔABD is 90 square units. Let $BD : DC = r : 1$. Find the value of r. (2 marks)

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13. In Figure 3(a), $ABCDEF$ is a wooden block in the form of a right prism. It is given that $AB = AC = 17$ cm , $BC = 16$ cm and $CD = 20$ cm .

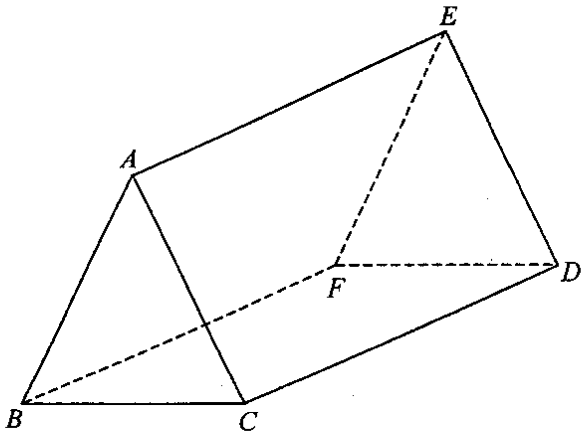

Figure 3(a)

(a) Find the area of ΔABC . (2 marks)

(b) Find the volume of the wooden block $ABCDEF$. (2 marks)

(c) The plane $PQRS$ which is parallel to the face $BCDF$ cuts the wooden block $ABCDEF$ into two blocks $APQRES$ and $BCQPSFDR$ as shown in Figure 3(b). It is given that $PQ = 4$ cm .

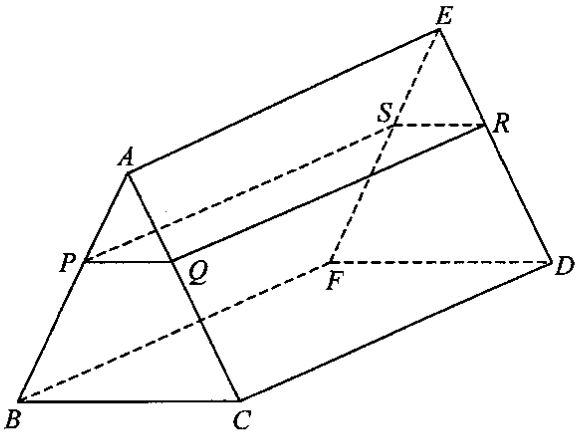

Figure 3(b)

(i) Find the volume of the wooden block $APQRES$.

(ii) Are the wooden blocks $APQRES$ and $ABCDEF$ similar? Explain your answer.

(5 marks)

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SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14. An athlete, Alice, of a school gets the following results (in seconds) in 10 practices of 1 500 m race:

279 , 280 , 264 , 267 , 283 , 281 , 281 , 266 , 284 , 265

(a) Two results are randomly selected from the above results.

(i) Find the probability that both the best two results are not selected.

(ii) Find the probability that only one of the best two results is selected.

(iii) Find the probability that at most one of the best two results is selected.

(6 marks)

(b) Another athlete, Betty, of the school gets the following results (in seconds) in 10 practices of 1 500 m race:

272 , 269 , 275 , 274 , 273 , 274 , 270 , 275 , 266 , 272

Alice and Betty will represent the school to participate in the 1 500 m race in the inter-school athletic meet.

(i) Which athlete is likely to get a better result? Explain your answer.

(ii) The best record of the 1 500 m race in the past inter-school athletic meets is 267 seconds. Which athlete has a greater chance of breaking the record? Explain your answer.

(5 marks)

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15. (a) Figure 4(a) shows a piece of paper card $ABCD$ in the form of a quadrilateral with $AB = AD$ and $BC = CD$. It is given that $BC = 24$ cm, $\angle BAD = 146°$ and $\angle ABC = 59°$. Find the length of AB.

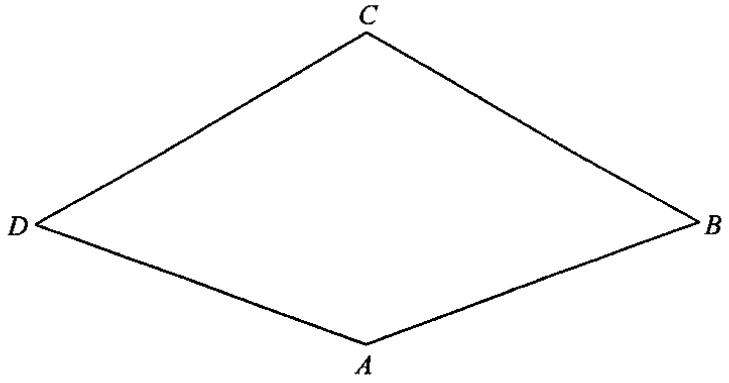

Figure 4(a)

(2 marks)

(b) The paper card described in (a) is folded along AC such that AB and AD lie on the horizontal ground as shown in Figure 4(b). It is given that $\angle BAD = 92°$.

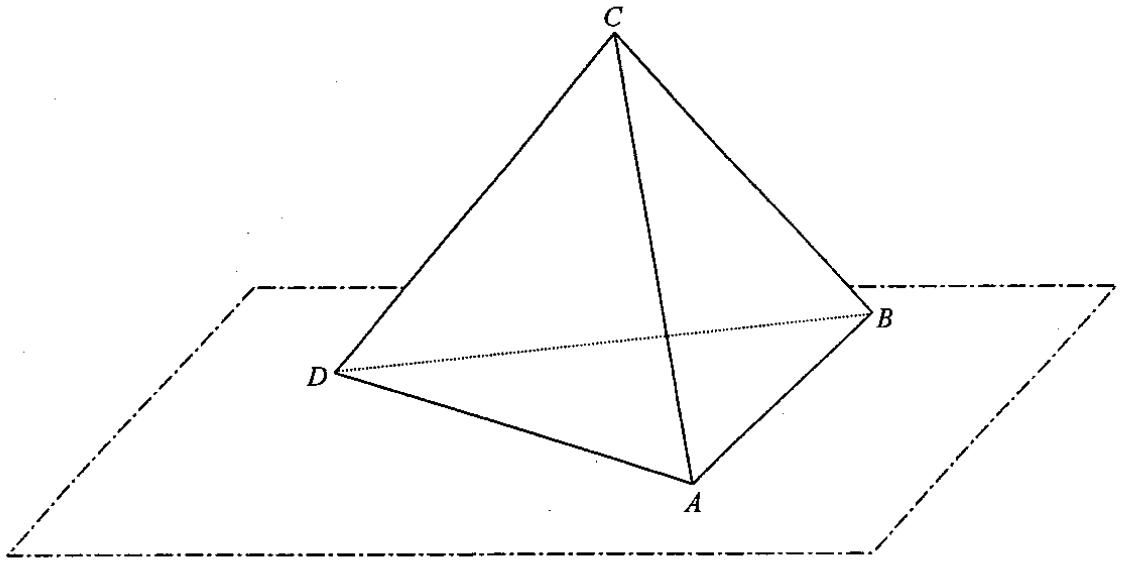

Figure 4(b)

(i) Find the distance between B and D on the horizontal ground.

(ii) Find the angle between the plane ABC and the plane ACD.

(iii) Let P be a movable point on the slant edge AC. Describe how $\angle BPD$ varies as P moves from A to C. Explain your answer.

(9 marks)

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16. Let $f(x)=\frac{1}{2}x-\frac{1}{144}x^2-6$.

(a) (i) Using the method of completing the square, find the coordinates of the vertex of the graph of $y=f(x)$.

(ii) If the graph of $y=g(x)$ is obtained by translating the graph of $y=f(x)$ leftwards by 4 units and upwards by 5 units, find $g(x)$.

(iii) If the graph of $y=h(x)$ is obtained by translating the graph of $y=2^{f(x)}$ leftwards by 4 units and upwards by 5 units, find $h(x)$.

(7 marks)

(b) A researcher performs an experiment to study the relationship between the number of bacteria A (u hundred million) and the temperature (s °C) under some controlled conditions. From the data of u and s recorded in Table 1(a), the researcher suggests using the formula $u=2^{f(s)}$ to describe the relationship.

s	a_1	a_2	a_3	a_4	a_5	a_6	a_7
u	b_1	b_2	b_3	b_4	b_5	b_6	b_7

Table 1(a)

(i) According to the formula suggested by the researcher, find the temperature at which the number of the bacteria is 8 hundred million.

(ii) The researcher then performs another experiment to study the relationship between the number of bacteria B (v hundred million) and the temperature (t °C) under the same controlled conditions and the data of v and t are recorded in Table 1(b).

t	a_1-4	a_2-4	a_3-4	a_4-4	a_5-4	a_6-4	a_7-4
v	b_1+5	b_2+5	b_3+5	b_4+5	b_5+5	b_6+5	b_7+5

Table 1(b)

Using the formula suggested by the researcher, propose a formula to express v in terms of t.

(4 marks)

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17. Figure 5(a) shows the circle passing through the four vertices of the square $ABCD$. A rectangular coordinate system is introduced in Figure 5(a) so that the coordinates of A and B are $(0, 0)$ and $(8, 6)$ respectively.

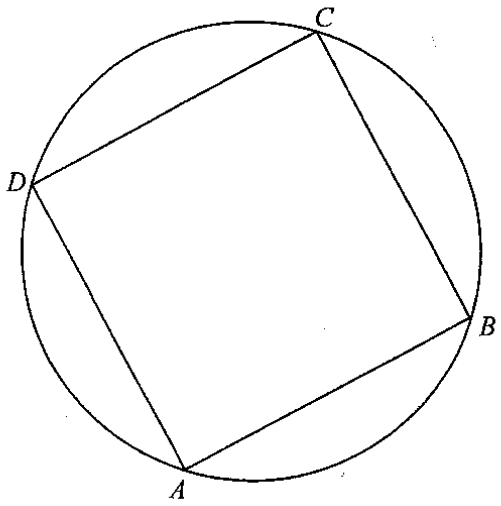

Figure 5(a)

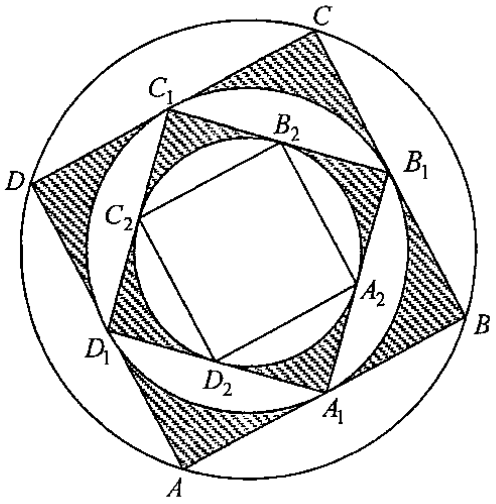

Figure 5(b)

(a) (i) Using a suitable transformation, or otherwise, write down the coordinates of D. Hence, or otherwise, find the coordinates of the centre of the circle $ABCD$.

(ii) Find the radius of the circle $ABCD$.

(5 marks)

(b) A student uses the circle $ABCD$ of Figure 5(a) to design a logo for the class association. The process of designing the logo starts by constructing the inscribed circle of the square $ABCD$ such that the inscribed circle touches AB, BC, CD and DA at A_1, B_1, C_1 and D_1 respectively. The region between the square $ABCD$ and its inscribed circle is shaded as shown in Figure 5(b). The inscribed circle of the square $A_1B_1C_1D_1$ is then constructed such that this inscribed circle touches A_1B_1, B_1C_1, C_1D_1 and D_1A_1 at A_2, B_2, C_2 and D_2 respectively. The region between the square $A_1B_1C_1D_1$ and its inscribed circle is also shaded. The process is carried on until the region between the square $A_9B_9C_9D_9$ and its inscribed circle is shaded.

(i) Find the ratio of the area of the circle $A_1B_1C_1D_1$ to the area of the circle $ABCD$.

(ii) Suppose that the ratio of the total area of all the shaded regions to the area of the circle $ABCD$ is $p:1$. The student thinks that the design of the logo is good when p lies between 0.2 and 0.3. According to the student, is the design of the logo good? Explain your answer.

(6 marks)

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