

# 2010 Mathematics

## Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

### General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits **all the marks** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
  - a. The symbol  $(u-1)$  should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* in each of Section A(1) and Section A(2). Do not deduct any marks for *u* in Section B.
  - b. The symbol  $(pp-1)$  should be used to denote 1 mark deducted for *pp*. At most deduct **1 mark** for *pp* in each of Section A(1) and Section A(2). Do not deduct any marks for *pp* in Section B.
  - c. At most deduct 1 mark in each of Section A(1) and Section A(2).
  - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
<p>1. <math>a^{14} \left( \frac{b^3}{a^2} \right)^5</math></p> <p><math>= a^{14} \left( \frac{b^{15}}{a^{10}} \right)</math></p> <p><math>= a^{14-10} b^{15}</math></p> <p><math>= a^4 b^{15}</math></p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>----- (3)</p>	<p>for <math>\left( \frac{x}{y} \right)^m = \frac{x^m}{y^m}</math></p> <p>for <math>\frac{x^m}{x^n} = x^{m-n}</math></p>
<p>2. (a) <math>\frac{29x-22}{7} \leq 3x</math></p> <p><math>29x-22 \leq 21x</math></p> <p><math>29x-21x \leq 22</math></p> <p><math>8x \leq 22</math></p> <p><math>x \leq \frac{11}{4}</math></p>	<p>1M</p> <p>1A</p>	<p>for putting <math>x</math> on one side</p> <p><math>x \leq 2.75</math></p>
<div style="border: 1px solid black; padding: 5px;"> <p><math>\frac{29x-22}{7} \leq 3x</math></p> <p><math>\frac{29x}{7} - 3x \leq \frac{22}{7}</math></p> <p><math>\frac{8x}{7} \leq \frac{22}{7}</math></p> <p><math>x \leq \frac{11}{4}</math></p> </div>	<p>1M</p> <p>1A</p>	<p>for putting <math>x</math> on one side</p> <p><math>x \leq 2.75</math></p>
<p>(b) The required greatest integer is 2.</p>	<p>1A</p> <p>----- (3)</p>	
<p>3. (a) <math>m^2 + 12mn + 36n^2</math></p> <p><math>= (m+6n)^2</math></p> <p>(b) <math>m^2 + 12mn + 36n^2 - 25k^2</math></p> <p><math>= (m+6n)^2 - 25k^2</math></p> <p><math>= (m+6n+5k)(m+6n-5k)</math></p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>----- (3)</p>	<p>or equivalent</p> <p>for using the result of (a)</p> <p>or equivalent</p>

Solution	Marks	Remarks
<p>4. (a) The 2nd term  <math>= \tan \frac{180^\circ}{2+2}</math>  <math>= \tan 45^\circ</math>  <math>= 1</math></p> <p>(b) The two terms are <math>\sqrt{3}</math> and <math>\frac{1}{\sqrt{3}}</math>.</p>	<p>1A</p> <p>1A + 1A  ------(3)</p>	
<p>5. (a) <math>3(2c + 5d + 4) = 39d</math>  <math>2c + 5d + 4 = 13d</math>  <math>2c = 13d - 5d - 4</math>  <math>2c = 8d - 4</math>  <math>c = 4d - 2</math></p>	<p>1M for division</p> <p>1A or equivalent</p>	
<p><math>3(2c + 5d + 4) = 39d</math>  <math>6c + 15d + 12 = 39d</math>  <math>6c = 39d - 15d - 12</math>  <math>6c = 24d - 12</math>  <math>c = 4d - 2</math></p>	<p>1M for expanding</p> <p>1A or equivalent</p>	
<p>(b) If the value of <math>d</math> is decreased by 1, the value of <math>c</math> will be decreased by 4.</p>	<p>1M + 1M  ------(4)</p>	
<p>6. Let \$ <math>x</math> be the cost of a bottle of milk.</p> <p><math>3(2x) + 5x = 66</math></p> <p><math>11x = 66</math>  <math>x = 6</math></p> <p>Thus, the cost of a bottle of milk is \$ 6 .</p>	<p>1A + 1M + 1A</p> <p>1A</p>	<p>pp-1 for any undefined symbol</p> <p>{ 1A for <math>y = 2x</math>  + 1M for <math>3y + 5x</math></p> <p>u-1 for missing unit</p>
<p>Let \$ <math>x</math> be the cost of a bottle of milk  and \$ <math>y</math> be the cost of a bottle of orange juice. Then, we have</p> <p><math>\begin{cases} 3y + 5x = 66 \\ y = 2x \end{cases}</math></p> <p>So, we have <math>3(2x) + 5x = 66</math> .</p> <p>Solving, we have <math>x = 6</math> .</p> <p>Thus, the cost of a bottle of milk is \$ 6 .</p>	<p>} 1A + 1A</p> <p>1M for getting a linear equation in <math>x</math> or <math>y</math> only</p> <p>1A</p>	<p>pp-1 for any undefined symbol</p> <p>u-1 for missing unit</p>
<p>The cost of a bottle of milk</p> <p><math>\frac{66}{3(2) + 5}</math></p> <p><math>= \frac{66}{11}</math></p> <p><math>= \\$ 6</math></p>	<p>1M + 1A + 1A</p> <p>1A</p>	<p>{ 1M for fraction + 1A for numerator  + 1A for denominator</p> <p>u-1 for missing unit</p>
	<p>------(4)</p>	

Solution	Marks	Remarks
7. (a) The number of badges owned by Tom $= 50(1 - 30\%)$ $= 35$ Thus, Tom has 35 badges.	1M 1A	
(b) Note that $50 + 35 = 85$ which is an odd number. Thus, they will not have the same number of badges.	1M 1A	f.t.
Note that $\frac{50 + 35}{2} = 42.5$ which is not an integer. Thus, they will not have the same number of badges.	1M 1A	f.t.
Assume Mary gives $x$ badges to Tom so that they will have the same number of badges. Then, we have $50 - x = 35 + x$ . Solving, we have $x = 7.5$ . Since 7.5 is not an integer, they will not have the same number of badges.	1M 1A	f.t.
If Mary gives 7 of her badges to Tom, then the number of badges owned by Mary is greater than that owned by Tom. If Mary gives 8 of her badges to Tom, then the number of badges owned by Mary is less than that owned by Tom. Thus, they will not have the same number of badges.	1M 1A	----- } either one ----- f.t.
-----(4)		
8. (a) The estimated total amount $= 16 + 24 + 32$ $= \$72$	1M + 1A 1A	1M for either correct + 1A for all u-1 for missing unit
(b) By (a), the actual total amount they have is greater than \$72 . Thus, they have enough money to buy the football.	1A	f.t.
-----(4)		
9. (a) Note that $\angle ABC + \angle BAC + \angle ACB = 180^\circ$ and $\angle CAE + \angle ACD = 180^\circ$ . Then, we have $\angle ABC + \angle BAC + \angle ACB + \angle CAE + \angle ACD = 360^\circ$ . Hence, we have $\angle ABC + (\angle BAC + \angle CAE) + (\angle ACB + \angle ACD) = 360^\circ$ . Therefore, we have $\angle ABC + \angle BAE + \angle BCD = 360^\circ$ . $\begin{aligned} \angle ABC &= 360^\circ - \angle BAE - \angle BCD \\ &= 360^\circ - 108^\circ - 126^\circ \\ &= 126^\circ \end{aligned}$	1M	for either one
(b) In $\triangle ABC$ and $\triangle DCB$ , $AB = DC$ ( given ) $\angle ABC = 126^\circ$ ( by (a) ) $\angle DCB = 126^\circ$ ( given ) $\angle ABC = \angle DCB$ $BC = CB$ ( common side ) $\triangle ABC \cong \triangle DCB$ ( SAS )	1A	u-1 for missing unit
<b>Marking Scheme:</b>		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
-----(5)		

Solution	Marks	Remarks
<p>10. (a) Let <math>C = ax + bx^2</math>, where <math>a</math> and <math>b</math> are non-zero constants.  When <math>x = 4</math>, <math>C = 96</math>, we have  <math>4a + 16b = 96</math>  <math>a + 4b = 24</math>  When <math>x = 5</math>, <math>C = 145</math>, we have  <math>5a + 25b = 145</math>  <math>a + 5b = 29</math>  Solving, we have <math>b = 5</math>.  Hence, we have <math>a = 4</math> and <math>b = 5</math>.  Thus, we have <math>C = 4x + 5x^2</math>.</p>	<p>1A  1M   1M 1A</p>	<p>     for either substitution   for eliminating one variable for both correct</p>
-----(4)		
<p>(b) <math>4x + 5x^2 = 288</math>  <math>5x^2 + 4x - 288 = 0</math>  <math>(5x - 36)(x + 8) = 0</math>  <math>x = \frac{36}{5}</math> or <math>x = -8</math> (rejected)  Thus, the perimeter of the tablecloth is <math>\frac{36}{5}</math> metres.</p>	<p>1M  1M 1A</p>	<p>for using (a)   7.2 u-1 for missing unit</p>
-----(3)		

Solution	Marks	Remarks	
<p>11. (a) The mean  <math display="block">= \frac{550}{22}</math> <math display="block">= 25</math></p> <p>The median  <math display="block">= 26</math></p> <p>The range  <math display="block">= 31 - 18</math> <math display="block">= 13</math></p> <p>(b) (i) Let <math>x</math> be the mean age of the three new players.  <math display="block">\frac{550 - 2(31) + 3x}{23} = 25</math> <math display="block">x = 29</math> Thus, the mean age of the three new players is 29 .</p>	<p>1A</p> <p>1A</p> <p>1A</p> <p>----- (3)</p> <p>1M + 1A</p> <p>1A</p>	<p>pp-1 for any undefined symbol</p>	
<table border="1" style="width: 100%;"> <tr> <td style="padding: 5px;"> The mean age of the three new players  <math display="block">= \frac{2(31) + 25}{3}</math> <math display="block">= 29</math> </td> </tr> </table>	The mean age of the three new players $= \frac{2(31) + 25}{3}$ $= 29$	<p>1M + 1A</p> <p>1A</p>	
The mean age of the three new players $= \frac{2(31) + 25}{3}$ $= 29$			
<p>(ii) Two sets of possible ages of the three new players are <math>\{25, 31, 31\}</math>  and <math>\{26, 30, 31\}</math> .</p>	<p>1A + 1A</p> <p>----- (5)</p>	<p>accept 27, 29, 31 and 28, 28, 31</p>	

Solution	Marks	Remarks
<p>12. (a) The slope of <math>AB</math></p> $= \frac{24-18}{6-(-2)}$ $= \frac{3}{4}$ <p>The equation of the straight line passing through <math>A</math> and <math>B</math> is</p> $y-24 = \frac{3}{4}(x-6)$ $3x-4y+78=0$	<p>1M 1A ----- (2)</p>	<p>or equivalent</p>
<p>(b) Let <math>(c, 0)</math> be the coordinates of <math>C</math>.</p> <p>Note that the slope of <math>AC</math> is <math>\frac{24-0}{6-c}</math>.</p> <p>Then, we have <math>\left(\frac{24}{6-c}\right)\left(\frac{3}{4}\right) = -1</math>.</p> <p>Solving, we have <math>c = 24</math>.</p> <p>Thus, the coordinates of <math>C</math> are <math>(24, 0)</math>.</p>	<p>1M 1M 1A</p>	<p>pp-1 for missing '(' or ')'</p>
<p>The slope of <math>AC</math> is <math>\frac{-4}{3}</math>.</p> <p>The equation of <math>AC</math> is <math>4x+3y-96=0</math>.</p> <p>Putting <math>y=0</math> in <math>4x+3y-96=0</math>, we have <math>x=24</math>.</p> <p>Thus, the coordinates of <math>C</math> are <math>(24, 0)</math>.</p>	<p>1M 1M 1A</p>	<p>for putting <math>y=0</math> pp-1 for missing '(' or ')'</p>
<p>(c)</p> $AB = \sqrt{(6-(-2))^2 + (24-18)^2}$ $= 10 \text{ units}$ $AC = \sqrt{(6-24)^2 + (24-0)^2}$ $= 30 \text{ units}$ <p>The area of <math>\triangle ABC</math></p> $= \frac{(10)(30)}{2}$ $= 150 \text{ square units}$ <p>(d) The ratio of the area of <math>\triangle ABD</math> to the area of <math>\triangle ACD</math> is <math>r:1</math>.</p> $\frac{90}{150-90} = \frac{r}{1}$ $r = \frac{3}{2}$	<p>1M 1A ----- (2)</p>	<p>either one</p> <p>1.5</p>
<p>The ratio of the area of <math>\triangle ABC</math> to the area of <math>\triangle ABD</math> is <math>(r+1):r</math>.</p> $\frac{150}{90} = \frac{r+1}{r}$ $r = \frac{3}{2}$	<p>1M 1A</p>	<p>1.5</p>
	<p>----- (2)</p>	

Solution	Marks	Remarks
<p>13. (a) Let <math>M</math> be the mid-point of <math>BC</math>. Then, we have <math>BM = 8</math> cm .</p> $AM = \sqrt{17^2 - 8^2} = 15 \text{ cm}$ <p>The area of <math>\triangle ABC</math></p> $= \frac{(16)(15)}{2} = 120 \text{ cm}^2$ <p>(b) The volume of the wooden block <math>ABCDEF</math></p> $= (120)(20) = 2400 \text{ cm}^3$ <p>(c) (i) The volume of the wooden block <math>APQRES</math></p> $= 2400 \left(\frac{4}{16}\right)^2 = 150 \text{ cm}^3$	<p>1M</p> <p>1A</p> <p>-----(2)</p> <p>1M</p> <p>1A</p> <p>-----(2)</p> <p>1M</p> <p>1A</p>	<p>for Pythagoras' theorem</p> <p>u-1 for missing unit</p> <p>u-1 for missing unit</p> <p>u-1 for missing unit</p>
<p>The height of the triangular base <math>APQ</math></p> $= 15 \left(\frac{4}{16}\right) = \frac{15}{4} \text{ cm}$ <p>The volume of the wooden block <math>APQRES</math></p> $= \frac{1}{2}(4)\left(\frac{15}{4}\right)(20) = 150 \text{ cm}^3$	<p>1M</p> <p>1A</p>	<p>for using ratio</p> <p>u-1 for missing unit</p>
<p>(ii) Note that <math>\frac{\text{the volume of } APQRES}{\text{the volume of } ABCDEF} = \frac{1}{16}</math> and <math>\left(\frac{PQ}{BC}\right)^3 = \frac{1}{64}</math>.</p> <p>Also note that the two ratios are not equal. Thus, the two blocks are not similar.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for finding either ratio</p> <p>for comparing two ratios f.t.</p>
<p>Note that <math>\frac{PQ}{BC} = \frac{1}{4}</math> and <math>\frac{QR}{CD} = 1</math>.</p> <p>Also note that the two ratios are not equal. Thus, the two blocks are not similar.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for finding either ratio</p> <p>for comparing two ratios f.t.</p>
<p>Note that <math>\frac{\text{the area of parallelogram } AQRE}{\text{the area of parallelogram } ACDE} = \frac{1}{4}</math> and <math>\left(\frac{PQ}{BC}\right)^2 = \frac{1}{16}</math>.</p> <p>Also note that the two ratios are not equal. Thus, the two blocks are not similar.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for finding either ratio</p> <p>for comparing two ratios f.t.</p>
<p>-----(5)</p>		



Solution	Marks	Remarks
<p>14. (a) (i) The required probability</p> $= \binom{8}{10} \binom{7}{9}$ $= \frac{28}{45}$ <p>(ii) The required probability</p> $= 2 \binom{2}{10} \binom{8}{9}$ $= \frac{16}{45}$ <p>(iii) The required probability</p> $= \frac{16}{45} + \frac{28}{45}$ $= \frac{44}{45}$	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p><math>\left\{ \begin{array}{l} \text{for } \binom{p}{m} \binom{q}{m-1}, \\ p &lt; m \text{ and } q &lt; m-1 \end{array} \right.</math></p> <p>r.t. 0.622</p> <p><math>\left\{ \begin{array}{l} \text{for } 2 \binom{s}{n} \binom{t}{n-1}, \\ s &lt; n \text{ and } t &lt; n-1 \end{array} \right.</math></p> <p>r.t. 0.356</p> <p>for (a)(i) + (a)(ii)</p> <p>r.t. 0.978</p>
<p>The required probability</p> $= 1 - \binom{2}{10} \binom{1}{9}$ $= \frac{44}{45}$	<p>1M</p> <p>1A</p>	<p>r.t. 0.978</p>
----- (6)		
<p>(b) (i) Note that the mean results of Alice and Betty are 275 seconds and 272 seconds respectively So, the mean result of Betty is better than that of Alice. Thus, Betty is likely to get a better result.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>f.t.</p>
<p>Note that the median results of Alice and Betty are 279.5 seconds and 272.5 seconds respectively. So, the median result of Betty is better than that of Alice. Thus, Betty is likely to get a better result.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>f.t.</p>
<p>By comparing each result of Alice with each result of Betty, there are altogether 100 outcomes. Alice gets a better result in 38 outcomes out of the 100 outcomes. Betty gets a better result in 61 outcomes out of the 100 outcomes. Thus, Betty is likely to get a better result.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>f.t.</p>
<p>(ii) Alice gets three results which are better than 267 seconds but Betty gets only one result which is better than 267 seconds. Thus, Alice has a greater chance of breaking the record.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
----- (5)		

Solution	Marks	Remarks
<p>15. (a) By sine formula</p> $\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \angle BAC}$ $\frac{AB}{\sin(180^\circ - 73^\circ - 59^\circ)} = \frac{24}{\sin 73^\circ}$ $AB = \frac{24 \sin 48^\circ}{\sin 73^\circ}$ $AB \approx 18.65041003 \text{ cm}$ $AB \approx 18.7 \text{ cm}$	<p>1M</p> <p>1A</p> <p>(2)</p>	<p>r.t. 18.7 cm</p>
<p>(b) (i) By cosine formula</p> $BD^2 = AB^2 + AD^2 - 2(AB)(AD) \cos \angle BAD$ $BD^2 \approx (18.65041003)^2 + (18.65041003)^2 - 2(18.65041003)^2 \cos 92^\circ$ $BD \approx 26.83196445 \text{ cm}$ $BD \approx 26.8 \text{ cm}$	<p>1M</p> <p>1A</p>	<p>accept <math>BD = 2AB \sin \frac{\angle BAD}{2}</math></p> <p>r.t. 26.8 cm</p>
<p>(ii) Let <math>Q</math> be the foot of perpendicular from <math>B</math> to <math>AC</math>.</p> $\sin \angle BAC = \frac{BQ}{AB}$ $BQ$ $\approx 18.65041003 \sin 73^\circ$ $\approx 17.83547581 \text{ cm}$	<p>1M</p>	
<p>The angle between the plane <math>ABC</math> and the plane <math>ACD</math> is <math>\angle BQD</math>.</p> $\sin \frac{\angle BQD}{2} = \frac{BD}{BQ}$ $\sin \frac{\angle BQD}{2} \approx \frac{13.41598223}{17.83547581}$ $\angle BQD \approx 97.56395848^\circ$ $\angle BQD \approx 97.6^\circ$ <p>Thus, the required angle is <math>97.6^\circ</math>.</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>for identifying the required angle</p> <p>r.t. <math>97.6^\circ</math></p>
$\cos \angle BQD = \frac{BQ^2 + DQ^2 - BD^2}{2(BQ)(DQ)}$ $\cos \angle BQD \approx \frac{(17.83547581)^2 + (17.83547581)^2 - (26.83196445)^2}{2(17.83547581)(17.83547581)}$ $\angle BQD \approx 97.56395848^\circ$ $\angle BQD \approx 97.6^\circ$ <p>Thus, the required angle is <math>97.6^\circ</math>.</p>	<p>1M</p> <p>1A</p>	<p>for using cosine formula</p> <p>r.t. <math>97.6^\circ</math></p>
<p>(iii) Note that <math>\sin \frac{\angle BPD}{2} = \frac{BD}{2BP}</math> and <math>BD</math> is a constant.</p> <p>The length of <math>BP</math> is the shortest when <math>P</math> is at <math>Q</math>.</p> <p>Note also that the length of <math>BP</math> varies inversely as <math>\sin \frac{\angle BPD}{2}</math>.</p> <p>Thus, <math>\angle BPD</math> increases from <math>\angle BAD</math> (<math>92^\circ</math>) to <math>\angle BQD</math> (<math>97.6^\circ</math>) and then decreases to <math>\angle BCD</math> (<math>68.0^\circ</math>).</p>	<p>1M</p> <p>1M</p> <p>1A</p> <p>(9)</p>	

Solution	Marks	Remarks
16. (a) (i) $f(x)$ $= \frac{1}{2}x - \frac{1}{144}x^2 - 6$ $= \frac{-1}{144}(x^2 - 72x) - 6$ $= \frac{-1}{144}(x^2 - 72x + 36^2 - 36^2) - 6$ $= \frac{-1}{144}(x - 36)^2 + 3$ Thus, the coordinates of the vertex are $(36, 3)$ .	1M 1M 1A	
(ii) $g(x)$ $= f(x+4) + 5$ $= \frac{-1}{144}((x+4) - 36)^2 + 3 + 5$ $= \frac{-1}{144}(x - 32)^2 + 8$	1A 1A	accept $\frac{-1}{144}x^2 + \frac{4}{9}x + \frac{8}{9}$
(iii) $h(x)$ $= 2^{f(x+4)} + 5$ $= 2^{\frac{-1}{144}(x-32)^2 + 3} + 5$	1A 1A -----(7)	accept $2^{\frac{-1}{144}x^2 + \frac{4}{9}x - \frac{37}{9}} + 5$
(b) (i) $2^{f(s)} = 8$ $2^{f(s)} = 2^3$ $f(s) = 3$ $\frac{-1}{144}(s - 36)^2 + 3 = 3$ (by (a)(i)) $s = 36$ Thus, the required temperature is $36^\circ\text{C}$ .	1M 1A	
$2^{f(s)} = 8$ $2^{f(s)} = 2^3$ $f(s) = 3$ $\frac{1}{2}s - \frac{1}{144}s^2 - 6 = 3$ $s^2 - 72s + 1296 = 0$ $(s - 36)^2 = 0$ $s = 36$ Thus, the required temperature is $36^\circ\text{C}$ .	1M 1A	
(ii) $v$ $= h(t)$ $= 2^{\frac{-1}{144}(t-32)^2 + 3} + 5$	1A 1M -----(4)	accept $v = 2^{f(t+4)} + 5$ for using (a)(iii)

Solution	Marks	Remarks
<p>17. (a) (i) By rotating <math>B</math> anticlockwise through <math>90^\circ</math> with respect to <math>A</math>, the coordinates of <math>D</math> are <math>(-6, 8)</math>.  The coordinates of the centre of the circle <math>ABCD</math>  = the coordinates of the mid-point of <math>BD</math>  <math>= \left( \frac{8 + (-6)}{2}, \frac{6 + 8}{2} \right)</math>  <math>= (1, 7)</math></p> <p>(ii) The radius of the circle <math>ABCD</math>  <math>= \sqrt{(1-0)^2 + (7-0)^2}</math>  <math>= 5\sqrt{2}</math> units</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>r.t. 7.07 units</p>
$AB = \sqrt{6^2 + 8^2} = 10 \text{ units}$ $BD = \sqrt{AB^2 + AD^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2} \text{ units}$ <p>The radius of the circle <math>ABCD = \frac{BD}{2} = 5\sqrt{2}</math> units</p>	<p>1M</p> <p>1A</p>	<p>-----</p> <p>either one</p> <p>r.t. 7.07 units</p>
<p>(b) (i) The radius of the circle <math>A_1B_1C_1D_1</math>  <math>= \frac{2(5\sqrt{2})\sin 45^\circ}{2}</math>  <math>= 5</math> units</p> <p>The required ratio  <math>= 5^2 : (5\sqrt{2})^2</math>  <math>= 1 : 2</math></p> <p>(ii) The areas of the shaded regions form a geometric sequence.</p> <p>The total area of all the shaded regions</p> $= \left( 10^2 - \pi \left( \frac{10}{2} \right)^2 \right) + \frac{1}{2} \left( 10^2 - \pi \left( \frac{10}{2} \right)^2 \right) + \dots + \left( \frac{1}{2} \right)^9 \left( 10^2 - \pi \left( \frac{10}{2} \right)^2 \right)$ $= \left( 10^2 - \pi \left( \frac{10}{2} \right)^2 \right) \left( 1 + \left( \frac{1}{2} \right) + \left( \frac{1}{2} \right)^2 + \dots + \left( \frac{1}{2} \right)^9 \right)$ $= \left( 10^2 - \pi \left( \frac{10}{2} \right)^2 \right) \left( \frac{1 - \left( \frac{1}{2} \right)^{10}}{1 - \frac{1}{2}} \right)$ <p><math>\approx 42.8784529</math> square units</p> $\approx \frac{42.8784529}{\pi(5\sqrt{2})^2}$ <p><math>\approx 0.272972709</math></p> <p>Thus, the design of the logo is good.</p>	<p>----- (5)</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>----- (6)</p>	<p>accept 0.5 : 1</p> <p>for sum of geometric sequence</p> <p>f.t.</p>

## 2010 Mathematics

**試卷二**  
**Paper 2**

<b>題號</b>	<b>答案</b>	<b>題號</b>	<b>答案</b>
<b>Question No.</b>	<b>Key</b>	<b>Question No.</b>	<b>Key</b>
1.	C (75)	31.	D (59)
2.	B (59)	32.	B (36)
3.	D (80)	33.	C (79)
4.	B (76)	34.	A (60)
5.	D (72)	35.	A (67)
6.	C (85)	36.	A (84)
7.	D (56)	37.	C (44)
8.	A (75)	38.	A (41)
9.	C (60)	39.	C (51)
10.	D (53)	40.	D (57)
11.	B (49)	41.	A (73)
12.	D (52)	42.	C (56)
13.	A (38)	43.	B (61)
14.	D (59)	44.	C (41)
15.	B (85)	45.	B (51)
16.	A (55)	46.	B (37)
17.	B (73)	47.	D (30)
18.	C (47)	48.	D (34)
19.	C (61)	49.	B (41)
20.	A (49)	50.	C (63)
21.	B (66)	51.	A (38)
22.	D (62)	52.	D (44)
23.	A (24)	53.	A (62)
24.	B (65)	54.	B (77)
25.	A (81)		
26.	C (55)		
27.	C (80)		
28.	D (80)		
29.	B (48)		
30.	C (46)		

註：括號內數字為答對百分率。

Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.

## 2010 Mathematics

### Candidates' Performance

#### Paper 1

Candidates generally performed better in Section A than in Section B. More candidates were willing to attempt those parts of the questions in Section B that fall within the Foundation Part of the Whole Syllabus.

#### Section A(1) (Compulsory)

Question Number	Performance in General
1	Good. Some candidates wrongly thought that $a^{14}\left(\frac{b^3}{a^2}\right)^5 = \frac{a^{14}b^{15}}{a^2}$ and hence gave $a^{12}b^{15}$ as the answer. Quite a number of candidates wrote an '=' sign at the beginning of their working.
2 (a)	Very good. A few candidates wrongly thought that $\frac{29x-22}{7} = \frac{7x}{7} = x$ .
(b)	Good. Some candidates mistakenly gave 3 as the answer.
3 (a)	Very good. A few candidates mistakenly gave $(m+6)^2$ or $(m+12n)^2$ as the answer.
(b)	Good. Some candidates mistakenly gave $(m+6n+25k)(m+6n-25k)$ as the answer.
4 (a)	Very good. Most candidates could obtain the correct answer.
(b)	Good. Some candidates mistakenly gave 1.73 and 0.577 as the answer.
5 (a)	Very good. A few candidates wrongly made $d$ the subject of the formula and hence gave $d = \frac{c+2}{4}$ as the answer.
(b)	Good. Some candidates tried to substitute numeric values for $d$ to find the answer.
6	Very good. A few candidates wrongly thought that the cost of a bottle of milk is the same as the cost of 2 bottles of orange juice. Some candidates did not write down the unit.
7 (a)	Good. Some candidates wrongly thought that the number of badges owned by Tom is 30% more than that owned by Mary.
(b)	Good. Some candidates could not explain the answer clearly.
8 (a)	Good. Some candidates did not know the meaning of 'rounding down'.
(b)	Poor. Most candidates could not give an acceptable explanation and they could not clearly point out that the actual total amount owned by the students is not less than \$72.
9 (a)	Good. Some candidates wrongly thought that $\angle ACD = \angle BAE = 108^\circ$ .
(b)	Good. Some candidates could not complete the proof concisely.

**Section A(2) (Compulsory)**

Question Number	Performance in General
10 (a)	Very good. Most candidates could obtain the correct answers but some of them did not show the steps of solving equations.
(b)	Good. Many candidates could obtain the correct answer but many of them did not write down the unit.
11 (a)	Very good. Most candidates could obtain the correct answer.
(b) (i)	Good. Many candidates could give the correct answer but some candidates did not show the method clearly.
(ii)	Fair. Many candidates only gave one set of possible ages of the three new players.
12 (a)	Good. Some candidates mistakenly thought that the slope of the straight line passing through $A$ and $B$ was $\frac{6 - (-2)}{24 - 18}$ .
(b)	Good. Some candidates mistakenly thought that the $x$ -coordinate of $C$ was zero.
(c)	Fair. Many candidates tried to find the length of $BC$ and the corresponding height of $\triangle ABC$ but many of them could not correctly find the height.
(d)	Poor. Many candidates wrongly thought that $\frac{90}{150} = \frac{r}{1}$ while many other candidates wrongly thought that $\frac{90}{150 - 90} = \left(\frac{r}{1}\right)^2$ .
13 (a)	Very good. Most candidates could obtain the correct answer but some candidates could not correctly use Pythagoras' theorem.
(b)	Good. Some candidates wrongly thought that the height of the wooden block $ABCDEF$ was 16 cm instead of 20 cm.
(c) (i)	Poor. Most candidates mistakenly thought that the ratio of the volume of the wooden block $APQRES$ to the volume of the wooden block $ABCDEF$ was equal to $4^3 : 16^3$ instead of $4^2 : 16^2$ .
(ii)	Poor. Most candidates could not correctly apply the relations between lengths, areas and volumes of similar solids to explain the answer.

**Section B (A choice of 3 out of 4 questions)**

Question Number	Popularity %	Performance in General
14 (a)(i)	95	Good. Some candidates wrongly treated the problem as independent events instead of dependent events.
(ii)		Good. Some candidates wrongly gave $\frac{8}{45}$ or $\frac{32}{45}$ as the answer.
(iii)		Fair. Many candidates wrongly calculated the probability that 'only one' of the best two results is selected instead of the probability that 'at most one' of the best two results is selected.
(b) (i)		Good. Some candidates wrongly compared the ranges and standard deviations of the results of Alice and Betty while some candidates mistakenly thought that a longer time was better than a shorter time and hence wrongly concluded that Alice was likely to get a better result.
(ii)		Very good. A few candidates wrongly compared the means of the results of Alice and Betty.
15 (a)	92	Very good. A few candidates mistakenly thought that the quadrilateral $ABCD$ was a rhombus and hence wrongly thought that $\angle ABD = \angle CBD = 29.5^\circ$ .
(b) (i)		Very good. A few candidates mistakenly thought that $\angle BAD$ was $146^\circ$ instead of $92^\circ$ .
(ii)		Good. Some candidates mistakenly thought that the required angle was $\angle BED$ where $E$ was the mid-point of $AC$ instead of the foot of the perpendicular from $B$ to $AC$ .
(iii)		Poor. Most candidates could only describe the change but could not give an acceptable explanation.
16 (a) (i)	68	Fair. Many candidates could not find the coordinates of the vertex of the graph of $y = f(x)$ by the method of completing the square while some other candidates used the formula instead of the method of completing the square to find the coordinates of the vertex of the graph of $y = f(x)$ .
(ii)		Good. Many candidates knew that $g(x) = f(x+4)+5$ but gave wrong answers from careless calculation.
(iii)		Poor. Most candidates wrongly thought that $h(x) = 2^{f(x+4)+5}$ .
(b) (i)		Good. Some candidates wrongly thought that $s = 3$ followed from $2^{f(s)} = 8$ .
(ii)		Poor. Most candidates could not give the correct answer.



Question Number	Popularity %	Performance in General
17 (a) (i)	45	Fair. Many candidates could not obtain the correct coordinates of $D$ by using a suitable transformation while many other candidates could not find the equation of the straight line $CD$ .
(ii)		Good. Many candidates could find the radius of the circle $ABCD$ correctly.
(b) (i)		Fair. Many candidates could obtain the correct areas of the circle $A_1B_1C_1D_1$ and the circle $ABCD$ but some of them could not correctly give the required ratio.
(ii)		Poor. Most candidates could not correctly use the formula for the summation of a geometric sequence to find the total area of all the shaded regions.

#### General recommendations

Candidates are advised to:

1. revise fundamental mathematics topics like change of subject, percentages, factorization, estimation, indices, congruency and similarity;
2. show all working and explain clearly how to get the conclusion from the premise;
3. define any symbols used;
4. write down the unit of the answer if necessary;
5. identify different situations in probability problems;
6. have a better understanding of statistical terms and their applications;
7. practise more on problems involving geometric proofs;
8. develop a better spatial sense, such as distinguishing right-angled triangles from non right-angled triangles in 3D diagrams;
9. make use of the memory space in calculators for carrying more significant figures throughout the working in solving trigonometric problems; and
10. trace the co-relation between different parts of a question, particularly in the long questions.

**Paper 2**

The paper consisted of 54 multiple-choice items. Section A comprised 36 questions on the Foundation Part and Section B 18 questions on the Whole Syllabus. Post-examination analysis revealed the following:

1. Candidates' performance on Items 1, 3, 4, 5, 6, 8, 15, 17, 25, 27, 28, 33, 36, 41 and 54 was good. Over 70% of the candidates answered them correctly.
2. Candidates' performance on Item 23 was unsatisfactory. Less than 30% of the candidates gave the correct answer.
3. In Item 13, many candidates were not familiar with the applications of percentage to real-life problems and hence gave wrong answers.

Q.13 If the price of a magazine is 60% higher than the price of a newspaper, then the price of the newspaper is

- \* A. 37.5% lower than the price of the magazine. (38%)
- B. 40% lower than the price of the magazine. (19%)
- C. 60% lower than the price of the magazine. (25%)
- D. 62.5% lower than the price of the magazine. (18%)

4. In Item 23, many candidates mistakenly thought that the number of planes of reflection of a cube was not 9, and hence wrongly gave Option C as the answer.

Q.23 Which of the following statements about a cube must be true?

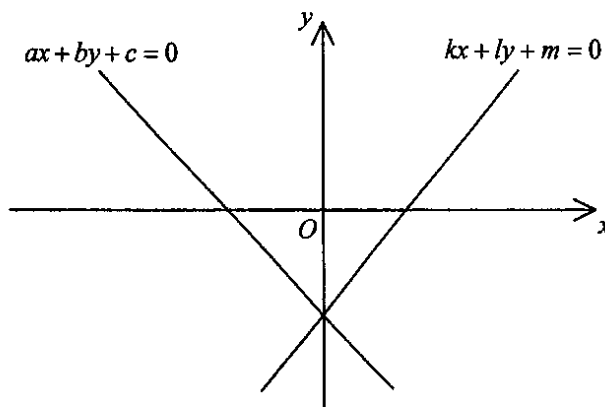
- I. The number of planes of reflection is 9.
- II. All the axes of rotational symmetry intersect at the same point.
- III. The angle between any two intersecting axes of rotational symmetry is  $90^\circ$ .

- \* A. I and II only (24%)
- B. I and III only (11%)
- C. II and III only (53%)
- D. I, II and III (12%)

5. In Item 32, many candidates overlooked that the  $x$ -intercept of  $kx + ly + m = 0$  is positive, and hence gave wrong answers.

Q.32 In the figure, the two straight lines intersect at a point on the negative  $y$ -axis. Which of the following must be true?

- I.  $ac > 0$
- II.  $km > 0$
- III.  $am = ck$
- IV.  $bm = cl$



- A. I and III only (17%)
- \* B. I and IV only (36%)
- C. II and III only (25%)
- D. II and IV only (22%)

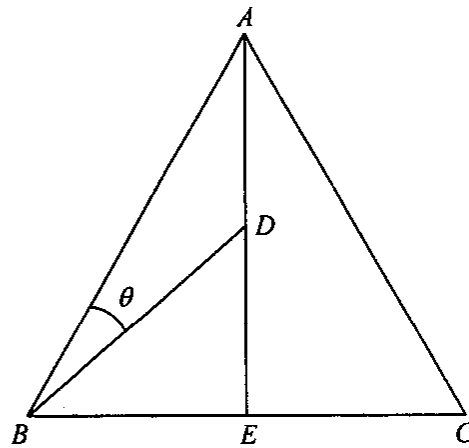
6. In Item 46, many candidates overlooked that  $\cos^2 45^\circ = 0.5$  and  $\cos^2 90^\circ = 0$ , therefore they mistakenly thought that there were 45 pairs of  $\cos^2 \theta$  and  $\sin^2 \theta$ , and hence wrongly gave Option C as the answer.

Q.46  $\cos^2 1^\circ + \cos^2 2^\circ + \cos^2 3^\circ + \dots + \cos^2 89^\circ + \cos^2 90^\circ =$

- |      |        |       |
|------|--------|-------|
| A.   | 44 .   | (9%)  |
| * B. | 44.5 . | (37%) |
| C.   | 45 .   | (36%) |
| D.   | 45.5 . | (18%) |

7. In Item 47, many candidates were unable to apply cosine formula in  $\triangle ABD$ , and hence gave wrong answers.

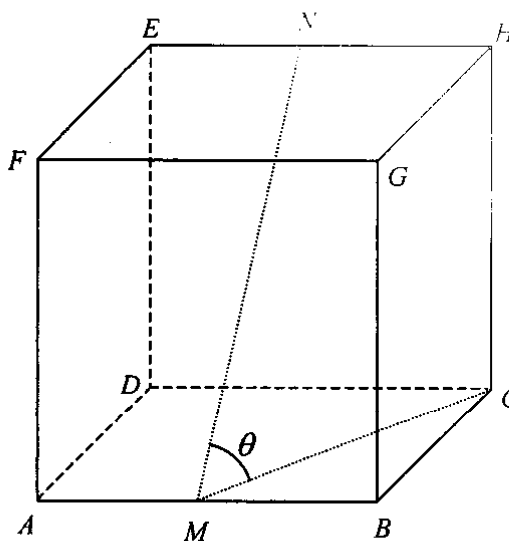
Q.47 In the figure,  $AD$  is produced to meet  $BC$  at  $E$ . If  $AB = BC = AC$ ,  $BE = CE$  and  $AD = DE$ , find  $\sin \theta$ .



- |      |                        |       |
|------|------------------------|-------|
| A.   | $\frac{\sqrt{3}}{5}$   | (22%) |
| B.   | $\frac{\sqrt{3}}{10}$  | (23%) |
| C.   | $\frac{\sqrt{21}}{7}$  | (25%) |
| * D. | $\frac{\sqrt{21}}{14}$ | (30%) |

8. In Item 48, many candidates were unable to apply Pythagoras' Theorem, and hence gave wrong answers

Q.48 In the figure,  $ABCDEFGH$  is a cube. If  $M$  and  $N$  are the mid-points of  $AB$  and  $EH$  respectively, then  $\cos \theta =$



- A.  $\frac{\sqrt{6}}{4}$  . (16%)
- B.  $\frac{\sqrt{6}}{5}$  . (23%)
- C.  $\frac{\sqrt{10}}{4}$  . (27%)
- \* D.  $\frac{\sqrt{10}}{5}$  . (34%)

9. In Item 51, many candidates overlooked that  $AB$  is a diameter of the circle, and hence gave wrong answers.

Q.51 Let  $O$  be the origin. If  $A$  and  $B$  are points lying on the  $x$ -axis and the  $y$ -axis respectively such that the equation of the circumcircle of  $\Delta OAB$  is  $x^2 + y^2 - 16x - 12y = 0$ , then the equation of the straight line passing through  $A$  and  $B$  is

- \* A.  $3x + 4y - 48 = 0$  . (38%)
- B.  $3x + 4y + 48 = 0$  . (20%)
- C.  $4x + 3y - 48 = 0$  . (27%)
- D.  $4x + 3y + 48 = 0$  . (15%)