HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2009

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)

This paper must be answered in English

INSTRUCTIONS

1. Write your Candidate Number in the space provided on Page 1.

2. Stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

3. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.

4. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

5. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

6. Write the question numbers of the questions you have attempted in Section B in the spaces provided on Page 1.

7. Unless otherwise specified, all working must be clearly shown.

8. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

9. The diagrams in this paper are not necessarily drawn to scale.

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Please stick the barcode label here.

Candidate Number						

	Marker's Use Only	Examiner's Use Only
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1–2		
3–4		
5–6		
7–8		
9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total	

Section B Question No.*	Marks	Marks
Section B Total		

To be filled in by the candidate

Checker's Use Only	Section B Total	

Checker No.	

1

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\dfrac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\dfrac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\dfrac{1}{3}$ × base area × height

Answers written on this page will not be marked.

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Make n the subject of the formula $\dfrac{3n-5m}{2}=4$.
(3 marks)

2. Simplify $\dfrac{x^2}{(x^{-7}y)^3}$ and express your answer with positive indices.
(3 marks)

Page total

3. Factorize

    (a) $a^2b + ab^2$,

    (b) $a^2b + ab^2 + 7a + 7b$.

    (3 marks)

4. Round off 405.504 to

    (a) the nearest integer,

    (b) 2 decimal places,

    (c) 2 significant figures.

    (3 marks)

Page total

5. The table below shows the distribution of the ages of all employees in a department of a company.

Age (x) Employee	$x < 30$	$30 \leq x < 40$	$x \geq 40$
Administrative officer	7	21	30
Clerk	53	57	32

If an employee is randomly selected from the department, find the probability that the selected employee is an administrative officer under the age of 40 .
(3 marks)

6. The total number of stamps owned by John and Mary is 300 . If Mary buys 20 stamps from a post office, the number of stamps owned by her will be 4 times that owned by John. Find the number of stamps owned by John.
(4 marks)

Page total

7. In a survey, there are 172 male interviewees. The number of female interviewees is 75% less than that of male interviewees. Find

    (a)   the number of female interviewees,

    (b)   the percentage of female interviewees in the survey.

    (4 marks)

8. In a polar coordinate system, O is the pole. The polar coordinates of the points P and Q are $(k, 123°)$ and $(24, 213°)$ respectively, where k is a positive constant. It is given that $PQ = 25$.

    (a)   Is $\triangle OPQ$ a right-angled triangle? Explain your answer.

    (b)   Find the perimeter of $\triangle OPQ$.

    (5 marks)

Page total

9. In Figure 1, the coordinates of the points A and B are $(-1, -2)$ and $(5, 2)$ respectively. A is translated vertically upward by 6 units to A'. B' is the reflection image of B with respect to the y-axis.
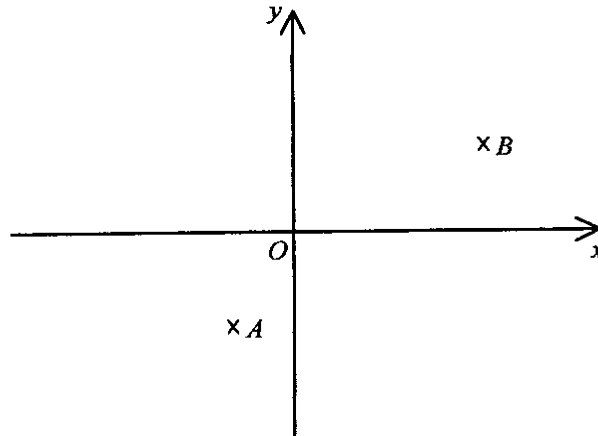

Figure 1

(a) Write down the coordinates of A' and B'.

(b) Is AB parallel to $A'B'$? Explain your answer.

(5 marks)

Page total

10.   The stem-and-leaf diagram below shows the distribution of the typing speed (in words per minute) of 20 students in a school before training.

<table>
<tr><td>Stem (tens)</td><td>Leaf (units)</td></tr>
<tr><td>1</td><td>2 2 6 7</td></tr>
<tr><td>2</td><td>1 1 3 3 3 5 7 8</td></tr>
<tr><td>3</td><td>2 4 5 5 8 9 9 9</td></tr>
</table>

(a)   Find the median, the range and the inter-quartile range of the above distribution.   (3 marks)

(b)   The box-and-whisker diagram below shows the distribution of the typing speed (in words per minute) of the 20 students after the training.

Typing speed (words per minute)

27   35   40   47   52
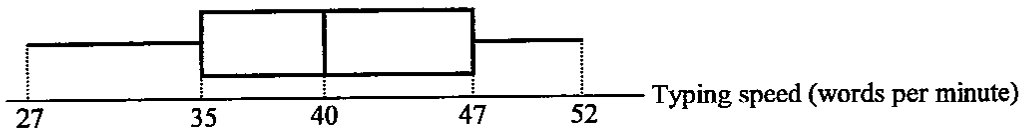
(i)   Is the distribution of the typing speed after the training more dispersed than that before the training? Explain your answer.

(ii)   The trainer claims that not less than half of these students show improvement in their typing speed after the training. Do you agree? Explain your answer.

(4 marks)

Page total

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11. In Figure 2, C is a point lying on DE. AE and BC intersect at F. It is given that $AC = AD$, $BC = DE$ and $\angle BCE = \angle CAD$.
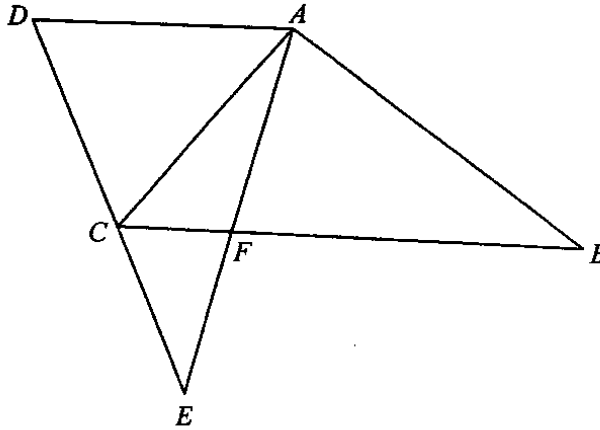

Figure 2

(a) Prove that $\triangle ABC \cong \triangle AED$. (3 marks)

(b) If $AD /\!/ BC$,

(i) prove that $\triangle ABF \sim \triangle DEA$;

(ii) write down two other triangles which are similar to $\triangle ABF$.

(5 marks)

Page total

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12. In Figure 3, R is the vertex of the graph of $y = -2(x-11)^2 + 23$.

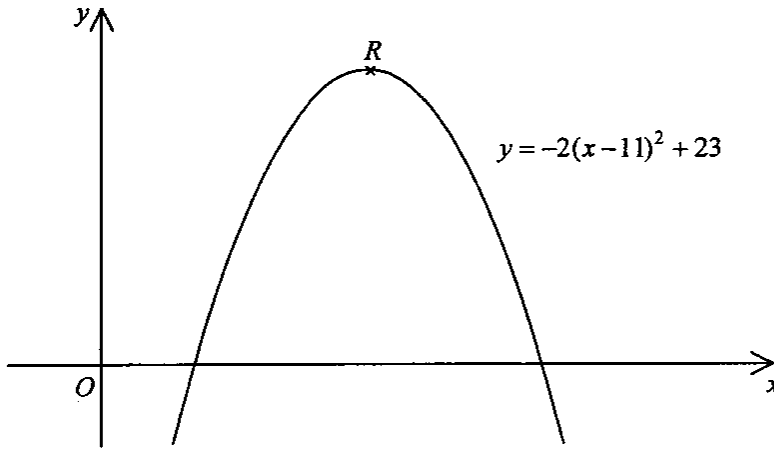

$y = -2(x-11)^2 + 23$

Figure 3

(a) Write down

    (i)   the equation of the axis of symmetry of the graph,

    (ii)   the coordinates of R .

<div align="right">(2 marks)</div>

(b) It is given that $P(p, 5)$ and $Q(q, 5)$ are two distinct points lying on the graph. Find

    (i)   the distance between P and Q ;

    (ii)   the area of the quadrilateral $PRQS$, where S is a point lying on the x-axis.

<div align="right">(7 marks)</div>

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13. (a) The height and the base radius of an inverted right circular conical container are 18 cm and 12 cm respectively.

   (i) Find the capacity of the circular conical container in terms of π.

   (ii) Figure 4(a) shows a frustum which is made by cutting off the lower part of the container. The height of the frustum is 6 cm . Find the volume of the frustum in terms of π.
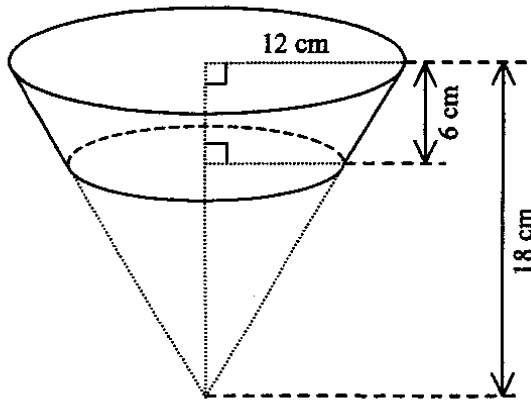
   (4 marks)

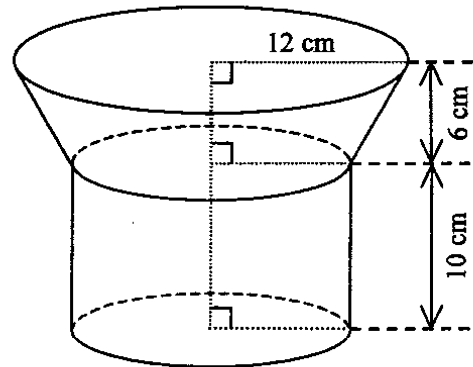Figure 4(a)   Figure 4(b)

   (b) Figure 4(b) shows a vessel which is held vertically. The vessel consists of two parts with a common base: the upper part is the frustum shown in Figure 4(a) and the lower part is a right circular cylinder of height 10 cm . Some water is poured into the vessel. The vessel now contains 884π cm^3 of water.

   (i) Find the depth of water in the vessel.

   (ii) If a piece of metal of volume 1000 cm^3 is then put into the vessel and the metal is totally immersed in the water, will the water overflow? Explain your answer.

   (5 marks)

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SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14. The frequency distribution table below shows the lifetime (in hours) of a batch of randomly chosen light bulbs of brand A and a batch of randomly chosen light bulbs of brand B.

Lifetime (x hours)	Frequency	
	Brand A	Brand B
$1000 \leq x < 1100$	8	4
$1100 \leq x < 1200$	50	12
$1200 \leq x < 1300$	42	40
$1300 \leq x < 1400$	10	36
$1400 \leq x < 1500$	10	28

(a) According to the above frequency distribution, which brand of light bulbs is likely to have a longer lifetime? Explain your answer.　(2 marks)

(b) If the lifetime of a light bulb is not less than 1300 hours, then the light bulb is classified as *good*. Otherwise, it is classified as *acceptable*.

　　(i)　If a light bulb is randomly chosen from the batch of light bulbs of brand A, find the probability that the chosen light bulb is *acceptable*.

　　(ii)　If two light bulbs are randomly chosen from the batch of light bulbs of brand A, find the probability that at least one of the two chosen light bulbs is *good*.

　　(iii)　The following two methods describe how two light bulbs are chosen from the two batches of light bulbs.

　　　Method 1:　One batch is randomly selected from the two batches of light bulbs and two light bulbs are then randomly chosen from the selected batch.

　　　Method 2:　One light bulb is randomly chosen from each of the two batches of light bulbs.

　　　Which one of the above two methods should be adopted in order to have a greater chance of choosing at least one *good* light bulb? Explain your answer.
　　　　　　　　(9 marks)

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15. In a city, the taxi fare is charged according to the following table:

Distance travelled	Taxi fare
The first 2 km (under 2 km will be counted as 2 km)	$ 30
Every 0.2 km thereafter (under 0.2 km will be counted as 0.2 km)	$ 2.4

Assume that there are no other extra fares.

(a) A hired taxi in the city travels a distance of x km , where $x \geq 2$.

　　(i) Suppose that x is a multiple of 0.2 . Prove that the taxi fare is $ $(6+12x)$.

　　(ii) Suppose that x is not a multiple of 0.2 . Is the taxi fare $ $(6+12x)$? Explain your answer.

(2 marks)

(b) If a hired taxi in the city travels a distance of 3.1 km , find the taxi fare. (3 marks)

(c) In the city, a taxi is hired for 99 journeys. The 1st journey covers a distance of 3.1 km . Starting from the 2nd journey, the distance covered by each journey is 0.5 km longer than that covered by the previous journey. The taxi driver claims that the total taxi fare will not exceed $ 33000 . Is the claim correct? Explain your answer. (6 marks)

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16. (a) In Figure 5, the straight lines L_1 and L_2 are perpendicular to each other. The equations of the straight lines L_3 and L_4 are $x = 8$ and $y = 10$ respectively. It is given that L_1 and L_2 intersect at the point $(12, 24)$ while L_1 and L_3 intersect at the point $(8, 16)$.
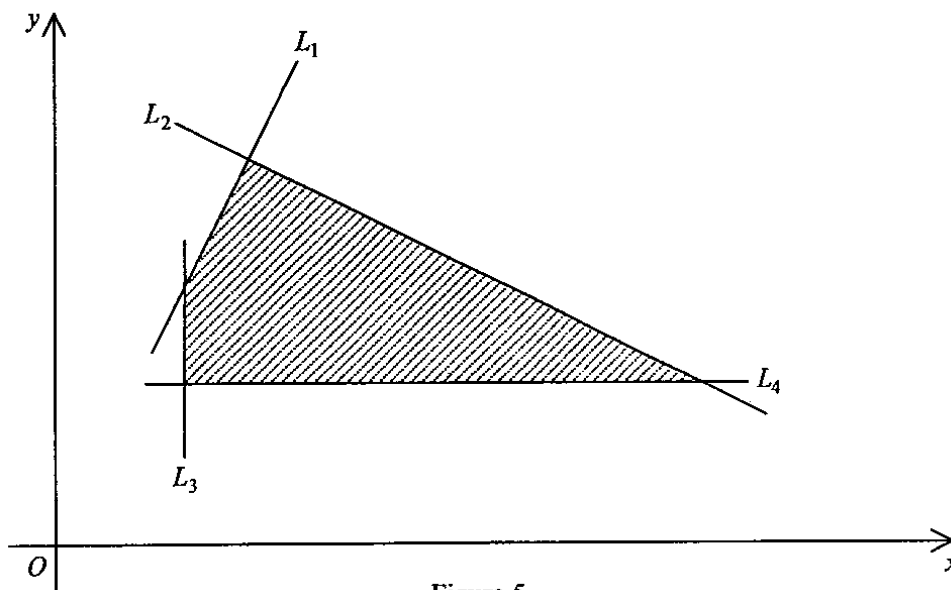

Figure 5

(i) Find the equations of L_1 and L_2 .

(ii) In Figure 5, the shaded region (including the boundary) represents the solution of a system of inequalities. Write down the system of inequalities.

(6 marks)

(b) There are two kinds of dining tables placed in a restaurant: square tables and round tables. The manager of the restaurant wants to place at least 8 square tables and 10 round tables. Moreover, the number of round tables placed is not more than 2 times that of the square tables placed. Each square table occupies a floor area of 4 m^2 and each round table occupies a floor area of 8 m^2 . The floor area occupied by the dining tables in the restaurant is at most 240 m^2 . On a certain day, the profits on a square table and a round table are $4000 and $6000 respectively. The manager claims that the total profit on the dining tables can exceed $230000 that day. Do you agree? Explain your answer.

(5 marks)

Page total

17. Figure 6 shows a geometric model fixed on the horizontal ground. The model consists of two thin triangular metal plates ABE and CDE, where D lies on AB and CE is perpendicular to the thin metal plate ABE. It is given that A, B, C and D lie on the horizontal ground. It is found that $AC = 28$ cm , $BC = 25$ cm , $BD = 6$ cm , $BE = 24$ cm and $\angle ABC = 57°$.
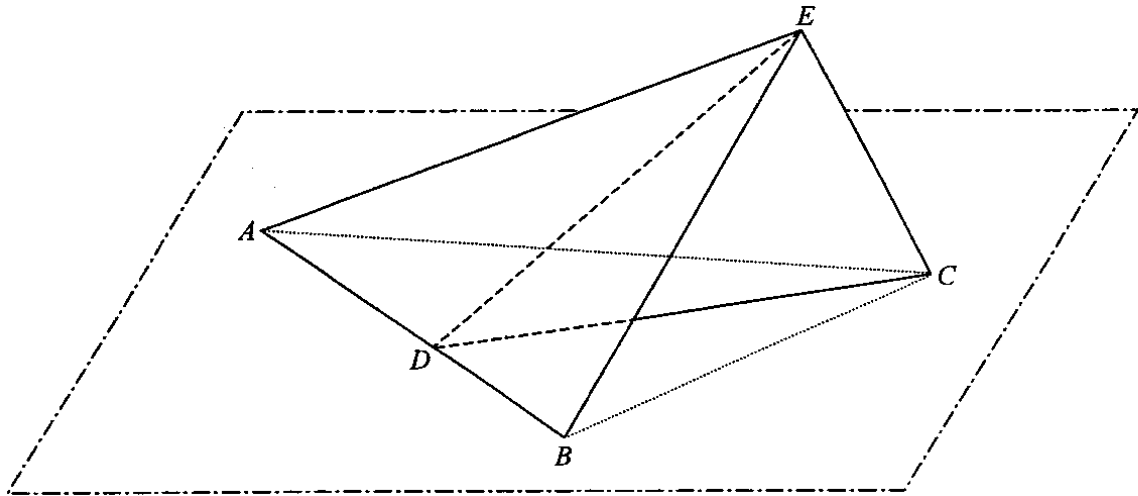

Figure 6

(a) Find

   (i)   the length of CD,

   (ii) $\angle BAC$,

   (iii) the area of $\triangle ABC$,

   (iv)   the shortest distance from E to the horizontal ground.

(9 marks)

(b) A student claims that the angle between DE and the horizontal ground is $\angle CDE$. Do you agree? Explain your answer.

(2 marks)

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