2009 Mathematics 數學

評卷參考 *
Marking Scheme

* 此部分只設英文版本

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* in each of Section A(1) and Section A(2). Do not deduct any marks for *u* in Section B.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***1 mark*** for *pp* in each of Section A(1) and Section A(2). Do not deduct any marks for *pp* in Section B.
   c. At most deduct 1 mark in each of Section A(1) and Section A(2).
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

* 此部分只設英文版本

Paper 1

Solution	Marks	Remarks
1. $\frac{3n-5m}{2}=4$		
$3n-5m=(2)(4)$	1M	
$3n=8+5m$	1M	for putting n on one side
$n=\frac{8+5m}{3}$	1A	or equivalent
$\frac{3n-5m}{2}=4$		
$\frac{3n}{2}-\frac{5m}{2}=4$	1M	
$\frac{3n}{2}=4+\frac{5m}{2}$	1M	for putting n on one side
$n=\frac{8}{3}+\frac{5m}{3}$	1A	or equivalent
	---------(3)	
2. $\frac{x^2}{(x^{-7}y)^3}$		
$=\frac{x^2}{x^{-21}y^3}$	1M	for $(ab)^k=a^kb^k$ or $(a^l)^k=a^{lk}$
$=\frac{x^{2+21}}{y^3}$	1M	for $\frac{1}{c^{-m}}=c^m$ or $\frac{c^n}{c^m}=c^{n-m}$
$=\frac{x^{23}}{y^3}$	1A	
	---------(3)	
3. (a) a^2b+ab^2		
$=ab(a+b)$	1A	or equivalent
(b) $a^2b+ab^2+7a+7b$		
$=ab(a+b)+7a+7b$	1M	for using the result of (a)
$=ab(a+b)+7(a+b)$		
$=(a+b)(ab+7)$	1A	or equivalent
	---------(3)	

Solution	Marks	Remarks
4. (a) 405.504 = 406 (correct to the nearest integer)	1A	
(b) 405.504 = 405.50 (correct to 2 decimal places)	1A	
(c) 405.504 = 410 (correct to 2 significant figures)	1A ---------(3)	
5. The required probability $= \frac{7+21}{7+21+30+53+57+32}$	1M + 1M	1M for numerator + 1M for denominator
$= \frac{28}{200}$		
$= \frac{7}{50}$	1A ---------(3)	0.14
6. Let x be the number of stamps owned by John. Then, the number of stamps owned by Mary is $(300-x)$.	1A	pp–1 for any undefined symbol can be absorbed
$4x = (300-x)+20$	1M + 1A	
$4x = 320-x$		
$5x = 320$		
$x = 64$ Thus, the number of stamps owned by John is 64 .	1A	
Let x and y be the numbers of stamps owned by John and Mary respectively.		pp–1 for any undefined symbol
$\begin{cases} x+y=300 \\ 4x=y+20 \end{cases}$	1A + 1A	
So, we have $4x = (300-x)+20$.	1M	for getting a linear equation in x or y only
Solving, we have $x = 64$.	1A	
Thus, the number of stamps owned by John is 64 .	---------(4)	

Solution	Marks	Remarks
7. (a) The required number $=172(1-75\%)$ $=43$	1M 1A	can be absorbed
(b) The required percentage $=\left(\frac{43}{43+172}\right)(100\%)$ $=20\%$	1M 1A	accept without 100 %
The required percentage $=\left(1-\frac{172}{43+172}\right)(100\%)$ $=20\%$	1M 1A	accept without 100 %
The required percentage $=\left(\frac{1-75\%}{1+(1-75\%)}\right)(100\%)$ $=20\%$	1M 1A	accept without 100 %
	---------(4)	
8. (a) $\angle POQ$ $=213°-123°$ $=90°$ Thus, ΔOPQ is a right-angled triangle.	1M 1A	for considering $\angle POQ$ f.t.
(b) $k^2+24^2=25^2$ $k=7$	1M 1A	for using Pythagoras' theorem
The perimeter of ΔOPQ $=7+24+25$ $=56$	1M	
	---------(5)	
9. (a) The coordinates of A' are $(-1, 4)$. The coordinates of B' are $(-5, 2)$.	1A 1A	pp–1 for missing '(' or ')' pp–1 for missing '(' or ')'
(b) The slope of AB $=\frac{2-(-2)}{5-(-1)}$ $=\frac{2}{3}$	1M 1A	either one
The slope of $A'B'$ $=\frac{4-2}{-1-(-5)}$ $=\frac{1}{2}$		either one
Since the slope of AB and the slope of $A'B'$ are different, AB is not parallel to $A'B'$.	1A	f.t.
	---------(5)	

Solution	Marks	Remarks
10. (a) The median = 26 words per minute	1A	u–1 for missing unit
The range = 39 – 12 = 27 words per minute	1A	u–1 for missing unit
The inter-quartile range = 35 – 21 = 14 words per minute	1A ----------(3)	u–1 for missing unit
(b) (i) The inter-quartile range after the training = 47 – 35 = 12 words per minute	1M	for considering inter-quartile range
The inter-quartile range of the distribution of typing speed after the training is less than that before the training. Thus, the distribution after the training is not more dispersed than that before the training.	1A	f.t.
The range after the training = 52 – 27 = 25 words per minute	1M	for considering range
The range of the distribution of typing speed after the training is less than that before the training. Thus, the distribution after the training is not more dispersed than that before the training.	1A	f.t.
(ii) Note that the median of the distribution of the typing speed after the training is 40 words per minute which is greater than the maximum of the distribution of the typing speed before the training. Thus, the claim is agreed.	1M 1A	f.t.
Note that the minimum of the distribution of the typing speed after the training is 27 words per minute which is greater than the median of the distribution of the typing speed before the training. Thus, the claim is agreed.	1M 1A ----------(4)	f.t.

Solution	Marks	Remarks

11.

Marking Schemes for (a) and (b)(i):		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step and one correct reason.	1

(a) In ΔABC and ΔAED,

$AC = AD$	(given)		[已知]
$BC = ED$	(given)		[已知]
$\angle ACB + \angle BCE = \angle ADE + \angle CAD$	(ext. $\angle$ of Δ)		[Δ的外角]
$\angle BCE = \angle CAD$	(given)		[已知]
$\angle ACB = \angle ADE$			
$\Delta ABC \cong \Delta AED$	(SAS)		
		---------(3)	

(b) (i) In ΔABF and ΔDEA,

$\angle AFB = \angle DAE$	(alt. $\angle$s, $AD // BC$)		[(內)錯角，$AD // BC$]
$\angle FBA = \angle AED$	(by (a))		[由(a)]
$\angle FAB = \angle ADE$	($\angle$ sum of Δ)		[Δ內角和]
$\Delta ABF \sim \Delta DEA$	(AAA)		(AA) (equiangular) [等角]

(ii) ΔCBA and ΔCEF are similar to ΔABF. 1A + 1A

---------(5)

Solution	Marks	Remarks
12. (a) (i) The equation of the axis of symmetry is $x=11$.	1A	or equivalent
(ii) The coordinates of R are $(11, 23)$.	1A ----------(2)	pp–1 for missing '(' or ')'
(b) (i) $-2(x-11)^2+23=5$ $(x-11)^2=9$ $x-11=3$ or $x-11=-3$ $x=14$ or $x=8$	1M 1A	
$-2(x-11)^2+23=5$ $2x^2-44x+224=0$ $x^2-22x+112=0$ $(x-8)(x-14)=0$ $x=8$ or $x=14$	1M 1A	
The distance between P and Q $=14-8$ $=6$	 1M 1A	can be absorbed
(ii) The area of ΔPQR $=\frac{(6)(23-5)}{2}$ $=54$	 1M 1A	either one
The area of ΔPSQ $=\frac{(6)(5)}{2}$ $=15$		either one
The area of the quadrilateral $PRQS$ $=54+15$ $=69$	 1A	
The area of the quadrilateral $PRQS$ $=\frac{(6)(23)}{2}$ $=69$	 1M + 1A 1A	
	----------(7)	

Solution	Marks	Remarks
13. (a) (i) The capacity of the container		
$=\frac{1}{3}\pi(12)^2(18)$	1M	
$=864\pi$ cm^3	1A	u–1 for missing unit
(ii) Let r cm be the base radius of the lower part of the container.		
Then, we have $\frac{18-6}{18}=\frac{r}{12}$.	1M	
Solving, we have $r=8$.		
The volume of frustum		
$=864\pi-\frac{1}{3}\pi(8)^2(18-6)$		
$=864\pi-256\pi$		
$=608\pi$ cm^3	1A	u–1 for missing unit
Let V cm^3 be the volume of the frustum.		
Then, we have $\frac{864\pi-V}{864\pi}=\left(\frac{18-6}{18}\right)^3$.	1M	
So, we have $\frac{864\pi-V}{864\pi}=\frac{8}{27}$.		
Solving, we have $V=608\pi$.	1A	
Thus, the volume of the frustum is 608π cm^3.		u–1 for missing unit
	----------(4)	
(b) (i) The capacity of the lower part of the vessel		
$=\pi(8)^2(10)$	1M	
$=640\pi$ cm^3		
Let h cm be the depth of water in the vessel.		
$\left(\frac{h-10+(18-6)}{18}\right)^3=\frac{(884\pi-640\pi)+(864\pi-608\pi)}{864\pi}$	1M	
$\left(\frac{h+2}{18}\right)^3=\frac{125}{216}$		
$\frac{h+2}{18}=\frac{5}{6}$		
$h=13$	1A	
Thus, the depth of water in the vessel is 13 cm.		u–1 for missing unit
(ii) The volume not occupied by water in the vessel		
$=608\pi+640\pi-884\pi$		
$=364\pi$ cm^3		
Note that the volume of metal is 1000 cm^3.		
So, the volume of metal is less than 364π cm^3.	1M	for comparison
Thus, the water will not overflow.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
14. (a) The mean lifetime of the batch of light bulbs of brand A $= \frac{(1050)(8)+(1150)(50)+(1250)(42)+(1350)(10)+(1450)(10)}{120}$ $= 1220$ h	1A	either one
The mean lifetime of the batch of light bulbs of brand B $= \frac{(1050)(4)+(1150)(12)+(1250)(40)+(1350)(36)+(1450)(28)}{120}$ $= 1310$ h		
Since the mean lifetime of the batch of light bulbs of brand A is less than that of brand B, light bulbs of brand B are likely to have a longer lifetime.	1A	f.t.
Note that the median of the lifetime of the batch of light bulbs of brand A lies in the class $1200 \le x < 1300$.	1A	either one
Also note that the median of the lifetime of the batch of light bulbs of brand B lies in the class $1300 \le x < 1400$.		
Since the median of the lifetime of the batch of light bulbs of brand A is less than that of brand B, light bulbs of brand B are likely to have a longer lifetime.	1A	f.t.
Note that the modal class of the lifetime of the batch of light bulbs of brand A is the class $1100 \le x < 1200$.	1A	either one
Also note that the modal class of the lifetime of the batch of light bulbs of brand B is the class $1200 \le x < 1300$.		
Therefore, light bulbs of brand B are likely to have a longer lifetime.	1A	f.t.
	----------(2)	
(b) (i) The required probability $= \frac{8+50+42}{120}$	1M	for numerator
$= \frac{5}{6}$	1A	r.t. 0.833
(ii) The required probability $= 1-\left(\frac{5}{6}\right)\left(\frac{99}{119}\right)$	1M + 1A	for $1-((b)(i))p$ where $p<1$
$= \frac{73}{238}$	1A	r.t. 0.307
(iii) The required probability by adopting Method 1 $= \frac{1}{2}\left(\frac{73}{238}\right)+\frac{1}{2}\left(1-\left(\frac{56}{120}\right)\left(\frac{55}{119}\right)\right)$	1M + 1A	for $\frac{1}{2}((b)(ii))+\frac{1}{2}\left(1-\left(\frac{r}{120}\right)\left(\frac{r-1}{119}\right)\right)$ where $0<r<120$
$= \frac{779}{1428}$	1A	r.t. 0.546
The required probability by adopting Method 2 $= 1-\left(\frac{5}{6}\right)\left(\frac{56}{120}\right)$		either one
$= \frac{11}{18}$		r.t. 0.611
Since $\frac{779}{1428} < \frac{11}{18}$, Method 2 will be adopted.	1A	f.t.
	----------(9)	

Solution	Marks	Remarks
15. (a) (i) The taxi fare $= 30 + \frac{x-2}{0.2}(2.4)$ $= 30 + 12(x-2)$ $= \$(6+12x)$	1	
(ii) If x is not a multiple of 0.2, then the distance used for calculating the taxi fare is the estimated distance after rounding up to the nearest 0.2 km instead of the actual distance travelled. Thus, the taxi fare is not $\$(6+12x)$.	1A ---------(2)	f.t.
(b) Note that the distance used for calculating the taxi fare is 3.2 km. The taxi fare $= 6 + (12)(3.2)$ $= \$44.4$	1A 1M 1A ---------(3)	can be absorbed
(c) The taxi fare for the 2nd journey $= 6 + (12)(3.6)$ $= \$49.2$	1A	can be absorbed
The distance covered by the 99th journey $= 3.1 + (99-1)(0.5)$ $= 52.1$ km		
The taxi fare for the 99th journey $= 6 + (12)(52.2)$ $= \$632.4$		
The total taxi fare $= 44.4 + 49.2 + 56.4 + 61.2 + \cdots + 625.2 + 632.4$ $= (44.4 + 56.4 + \cdots + 632.4) + (49.2 + 61.2 + \cdots + 625.2)$	1M	for splitting into two arithmetic sequences
$= \frac{(44.4+632.4)(50)}{2} + \frac{(49.2+625.2)(49)}{2}$ $= 16\,920 + 16\,522.8$ $= \$33\,442.8$ $> \$33\,000$	1M+1A+1A	1M for summing either arithmetic sequence + 1A for correct number of terms in each series
Thus, the claim is incorrect.	1A	f.t
The taxi fare for the 2nd journey $= 6 + (12)(3.6)$ $= \$49.2$	1A	can be absorbed
The total taxi fare $= 44.4 + 49.2 + 56.4 + 61.2 + \cdots + 625.2 + 632.4$ $= (44.4 + 56.4 + \cdots + 632.4) + (49.2 + 61.2 + \cdots + 625.2)$	1M	for splitting into two arithmetic sequences
$= \frac{\big((2)(44.4)+(50-1)(12)\big)(50)}{2} + \frac{\big((2)(49.2)+(49-1)(12)\big)(49)}{2}$ $= 16\,920 + 16\,522.8$ $= \$33\,442.8$ $> \$33\,000$	1M+1A+1A	1M for summing either arithmetic sequence + 1A for correct number of terms in each series
Thus, the claim is incorrect.	1A ---------(6)	f.t.

Solution	Marks	Remarks
16 (a) (i) The slope of L_1		
$=\dfrac{24-16}{12-8}$		
$=2$		
The equation of L_1 is		
$y-16=2(x-8)$	1M	
$2x-y=0$	1A	or equivalent
The slope of L_2		either one
$=\dfrac{-1}{2}$		
The equation of L_2 is		
$y-24=\dfrac{-1}{2}(x-12)$		
$x+2y-60=0$	1A	or equivalent
(ii) The system of inequalities is $\begin{cases} x \geq 8 \\ y \geq 10 \\ 2x \geq y \\ x+2y \leq 60 \end{cases}$	1A+1A+1A	or equivalent
	---------(6)	
(b) Let x and y be the numbers of square tables and round tables placed respectively.		
Now, the constraints are $x \geq 8$, $y \geq 10$, $2x \geq y$ and $x+2y \leq 60$.		
Denote the total profit on the dining tables by $\$P$.		
Then, we have $P=4\,000x+6\,000y$.	1A	
Note that the vertices of the shaded region are $(12,24)$, $(40,10)$, $(8,10)$ and $(8,16)$.		
At $(12,24)$, we have $P=(4\,000)(12)+(6\,000)(24)=192\,000$.	1M + 1M	1M for testing a point + 1M for testing four points
At $(40,10)$, we have $P=(4\,000)(40)+(6\,000)(10)=220\,000$.		
At $(8,10)$, we have $P=(4\,000)(8)+(6\,000)(10)=92\,000$.		
At $(8,16)$, we have $P=(4\,000)(8)+(6\,000)(16)=128\,000$.		
So, the greatest value of P is $220\,000$.	1A	
Thus, the claim is disagreed.	1A	f.t.
Let x and y be the numbers of square tables and round tables placed respectively.		
Now, the constraints are $x \geq 8$, $y \geq 10$, $2x \geq y$ and $x+2y \leq 60$.		
Denote the total profit on the dining tables by $\$P$.		
Then, we have $P=4\,000x+6\,000y$.	1A	
Draw the straight line $2x+3y=k$ on Figure 5, where k is a constant.	1M + 1M	1M for sliding straight line + 1M for straight line with negative slope
It is found that P attains its greatest value at $(40,10)$.		
So, the greatest value of P is $220\,000$.	1A	
Thus, the claim is disagreed.	1A	f.t.
	---------(5)	

Solution	Marks	Remarks
17. (a) (i) By cosine formula, we have		
$CD^2 = BC^2 + BD^2 - 2(BC)(BD)\cos\angle ABC$	1M	
$CD^2 = 25^2 + 6^2 - 2(25)(6)\cos 57°$		
$CD \approx 22.30713539$		
Thus, the length of CD is 22.3 cm .	1A	r.t. 22.3 cm
(ii) By sine formula, we have		
$\frac{\sin\angle BAC}{BC} = \frac{\sin\angle ABC}{AC}$	1M	
$\frac{\sin\angle BAC}{25} = \frac{\sin 57°}{28}$		
$\angle BAC \approx 48.48766126°$		
$\angle BAC \approx 48.5°$	1A	r.t. 48.5°
(iii) The area of ΔABC		
$= \frac{1}{2}(AC)(BC)\sin\angle ACB$	1M	accept Heron's formula
$\approx \frac{1}{2}(28)(25)\sin(180° - 57° - 48.48766126°)$		
≈ 337.2907934		
$\approx 337 \text{ cm}^2$	1A	r.t. 337 cm^2
(iv) CE		
$= \sqrt{BC^2 - BE^2}$	1M	either
$= \sqrt{25^2 - 24^2}$		
$= 7$ cm		
AE		
$= \sqrt{AC^2 - CE^2}$		
$= \sqrt{28^2 - 7^2}$		
$= \sqrt{735}$ cm		
By cosine formula, we have		
$AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos\angle ACB$		
$AB^2 \approx 28^2 + 25^2 - 2(28)(25)\cos(180° - 57° - 48.48766126°)$		
$AB \approx 32.17385288$ cm		
Let $s = \frac{AB + AE + BE}{2}$.		
The area of ΔABE		
$= \sqrt{s(s - AB)(s - AE)(s - BE)}$		
$\approx 317.9377429 \text{ cm}^2$		
Let h cm be the shortest distance from E to the horizontal ground.		
$\frac{h}{3}(\text{The area of } \Delta ABC) = \frac{1}{3}(\text{The area of } \Delta ABE)(CE)$	1M	
$\frac{h}{3}(337.2907934) \approx \frac{1}{3}(317.9377429)(7)$		
$h \approx 6.598354428$		
$h \approx 6.60$	1A	
Thus, the required distance is 6.60 cm .		r.t. 6.60 cm
	---------(9)	

Solution	Marks	Remarks
(b) DE $=\sqrt{CD^2-CE^2}$ $\approx\sqrt{22.30713539^2-7^2}$ ≈ 21.1803751 cm Let d be the perpendicular distance from E to CD. Then, we have $\frac{d(CD)}{2}=\frac{(CE)(DE)}{2}$. Therefore, we have $d=\frac{(CE)(DE)}{CD}$. The perpendicular distance from E to CD $=\frac{(CE)(DE)}{CD}$ $\approx\frac{(7)(21.1803751)}{22.30713539}$ ≈ 6.646421565 cm So, the perpendicular distance from E to CD and the shortest distance from E to the horizontal ground are different. Thus, the claim is disagreed.	1M 1A	accept equating sine ratios f.t.
DE $=\sqrt{CD^2-CE^2}$ $\approx\sqrt{22.30713539^2-7^2}$ ≈ 21.1803751 cm Let θ be the angle between DE and the horizontal ground. $\sin\theta=\frac{\text{The shortest distance from } E \text{ to the hortizontal ground}}{DE}$ $\sin\theta\approx\frac{6.598354429}{21.1803751}$ $\theta\approx 18.1515512°$ $\sin\angle CDE=\frac{CE}{CD}$ $\sin\angle CDE\approx\frac{7}{22.30713539}$ $\angle CDE\approx 18.28844266°$ So, $\angle CDE$ and the angle between DE and the horizontal ground are different. Thus, the claim is disagreed.	1M 1A ---------(2)	for identifying the required angle f.t.

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卷二
Paper 2

題號 Question No.	答案 Key	題號 Question No.	答案 Key
1.	B (95)	31.	B (43)
2.	D (81)	32.	A (60)
3.	D (71)	33.	D (38)
4.	A (89)	34.	C (74)
5.	A (52)	35.	C (81)
6.	A (82)	36.	C (86)
7.	B (92)	37.	D (29)
8.	D (61)	38.	C (48)
9.	B (33)	39.	D (45)
10.	C (72)	40.	D (48)
11.	B (84)	41.	A (45)
12.	C (52)	42.	B (44)
13.	C (51)	43.	A (40)
14.	D (62)	44.	A (47)
15.	D (82)	45.	B (39)
16.	A (45)	46.	C (58)
17.	B (60)	47.	B (30)
18.	B (74)	48.	B (33)
19.	A (53)	49.	C (63)
20.	C (84)	50.	C (32)
21.	D (49)	51.	B (57)
22.	B (28)	52.	D (35)
23.	B (49)	53.	C (38)
24.	C (44)	54.	A (62)
25.	D (48)		
26.	A (83)		
27.	A (75)		
28.	C (79)		
29.	A (65)		
30.	D (53)		

註： 括號內數字爲答對百分率。
Note: Figures in brackets indicate the percentages of candidates choosing the correct answers.