香港考試及評核局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2007年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2007

數學 試卷一
MATHEMATICS PAPER 1

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

只限教師參閱 FOR TEACHERS' USE ONLY

Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).

   a. The symbol *u-1* should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* in Section A. Do not deduct any marks for *u* in Section B.

   b. The symbol *pp-1* should be used to denote 1 mark deducted for *pp*. At most deduct ***1 mark*** for *pp* in each of Section A and Section B. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.

   c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.

   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
1. $5p-7=3(p+q)$		
$5p-7=3p+3q$	1M	for expanding
$5p-3p=3q+7$	1M	for putting p on one side
$2p=3q+7$		
$p=\dfrac{3q+7}{2}$	1A	or equivalent
$5p-7=3(p+q)$		
$\dfrac{5p-7}{3}=p+q$	1M	for division
$\dfrac{5p}{3}-p=q+\dfrac{7}{3}$	1M	for putting p on one side
$\dfrac{2p}{3}=q+\dfrac{7}{3}$		
$p=\dfrac{3q}{2}+\dfrac{7}{2}$	1A	or equivalent
	----------(3)	
2. $\dfrac{m^6}{m^9n^{-5}}$		
$=\dfrac{1}{m^3n^{-5}}$	1M	for $\dfrac{m^l}{m^k}=m^{l-k}$ or $\dfrac{m^l}{m^k}=\dfrac{1}{m^{k-l}}$
$=\dfrac{n^{-(-5)}}{m^3}$	1M	for $\dfrac{1}{x^{-k}}=x^{-(-k)}$ or $x^{-k}=\dfrac{1}{x^{-(-k)}}$
$=\dfrac{n^5}{m^3}$	1A	
	----------(3)	
3. (a) $r^2+10r+25$		
$=(r+5)^2$	1A	or equivalent
(b) $r^2+10r+25-s^2$		
$=(r+5)^2-s^2$	1M	for using the result of (a)
$=(r+5+s)(r+5-s)$	1A	or equivalent
$=(r+s+5)(r-s+5)$		
	----------(3)	

Solution	Marks	Remarks
4. The median		
$= 67$ kg	1A	u–1 for missing unit
The range		
$= 75 - 50$		
$= 25$ kg	1A	u–1 for missing unit
The standard deviation		
≈ 7.649546102		
≈ 7.65 kg	1A	u–1 for missing unit r.t. 7.65 kg
	----------(3)	
5. The discriminant		
$= 14^2 - 4(1)(k)$	1M	can be absorbed
$= 196 - 4k$	1A	can be absorbed
Since the equation has no real roots, we have		
$196 - 4k < 0$	1M	for discriminant < 0
$196 < 4k$		
$k > 49$	1A	
	----------(4)	
6. (a) The selling price of the vase		
$= 400(1 - 20\%)$	1M	can be absorbed
$= \$320$	1A	u–1 for missing unit
(b) The cost		
$= 320 - 70$		
$= \$250$		
The percentage profit		
$= \frac{70}{250}(100\%)$	1M	accept without 100 %
$= 28\%$	1A	
	----------(4)	

只限教師參閱　FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
7. Assume that x elderly patients consulted the doctor on that day.		pp−1 for any undefined symbol
$120x+160(67-x)=9000$	1A+1M+1A	$\begin{cases}\text{1A for } y=67-x \\ \text{+ 1M for } 120x+160y\end{cases}$
$10720-40x=9000$ $40x=1720$ $x=43$	1A	
Thus, 43 elderly patients consulted the doctor on that day.		
Assume that x elderly patients and y non-elderly patients consulted the doctor on that day. Then, we have		pp−1 for any undefined symbol
$\begin{cases} x+y=67 \\ 120x+160y=9000 \end{cases}$	1A + 1A	
So, we have $120x+160(67-x)=9000$.	1M	for getting a linear equation in x or y only
Solving, we have $x=43$.	1A	
Thus, 43 elderly patients consulted the doctor on that day.		
	----------(4)	
8. x $=180°-110°$ $=70°$	1A	u−1 for missing unit
y $=180°-90°-70°$ $=20°$	1A	u−1 for missing unit
z $=\angle CBF$ $=y$	1M	for finding $\angle CBF$
$=20°$	1A	u−1 for missing unit
	----------(4)	
9. (a) Let $\angle AOB=x°$.		pp−1 for any undefined symbol
$2\pi(40)\left(\frac{x}{360}\right)=16\pi$	1M + 1A	1M for $\frac{x}{360}$
$x=72$	1A	
Thus, we have $\angle AOB=72°$.		u−1 for missing unit
(b) The required area		
$=\pi(40)^2\left(\frac{72}{360}\right)$	1M	for $\pi(40)^2\frac{\text{(a)}}{360°}$
$=320\pi\text{ cm}^2$	1A	u−1 for missing unit
	----------(5)	

Solution	Marks	Remarks
10. (a) The maximum absolute error		
$= 0.5$ cm		
The least possible length of the metal wire		
$= 5 - 0.5$	1M	
$= 4.5$ cm	1A	u−1 for missing unit
	----------(2)	
(b) (i) The maximum absolute error		
$= 0.05$ m		
The actual length of this metal wire		
$< 2.0 + 0.05$	1M	accept $\leq 2.0 + 0.05$
$= 2.05$ m		
$= 205$ cm		
< 206 cm		
Thus, it is not possible that the actual length of this metal wire exceeds 206 cm .	1A	f.t.
If the actual length of this metal wire exceeds 206 cm , then the measured length of this metal wire correct to the nearest 0.1 m will be at least 2.1 m .	1M	
Thus, it is not possible that the actual length of this metal wire exceeds 206 cm .	1A	f.t.
(ii) Let n be the number of pieces of shorter metal wires.		
$n < \frac{205}{4.5}$	1M	accept $n \leq \frac{206}{(a)}$
$n < 45.55555556$		
$n < 45.6$	1A	accept $n < 46$ and $n \leq 45.8$
Therefore, the greatest possible value of n is 45 .		
Thus, it is not possible to cut this metal wire in that way.	1A	f.t.
Note that		
$(46)(4.5)$	1M	for $46(a)$
$= 207$	1A	
> 205		accept $207 > 206$
Thus, it is not possible to cut this metal wire in that way.	1A	f.t.
	----------(5)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
11. (a) Let r cm be the radius of the water surface. Then, we have		
$\frac{8}{24}=\frac{r}{18}$	1M	for considering the ratio
$r=6$	1A	
The volume of water		
$=\frac{1}{3}\pi(6)^2(8)$	1M	
$=96\pi\text{ cm}^3$	1A	u−1 for missing unit
The volume of the vessel		
$=\frac{1}{3}\pi(18)^2(24)$	1M	
$=2592\pi\text{ cm}^3$	1A	
Let $V\text{ cm}^3$ be the volume of water. Then, we have		pp−1 for any undefined symbol
$\frac{V}{2592\pi}=\left(\frac{8}{24}\right)^3$	1M	
$V=96\pi$	1A	
Thus, the volume of water is $96\pi\text{ cm}^3$.		u−1 for missing unit
	----------(4)	
(b) (i) The slant height of the wet curved surface		
$=\sqrt{6^2+8^2}$	1M	for using Pythagoras' theorem
$=10\text{ cm}$		
The required area		
$=\pi(6)(10)$	1M	
$=60\pi\text{ cm}^2$	1A	u−1 for missing unit
The slant height of the vessel		
$=\sqrt{18^2+24^2}$	1M	for using Pythagoras' theorem
$=30\text{ cm}$		
The curved surface area of the vessel		
$=\pi(18)(30)$	1M	
$=540\pi\text{ cm}^2$		
Let $S\text{ cm}^2$ be the wet curved surface area. Then, we have		pp−1 for any undefined symbol
$\frac{S}{540\pi}=\left(\frac{8}{24}\right)^2$		
$S=60\pi$	1A	
Thus, the required area is $60\pi\text{ cm}^2$.		u−1 for missing unit
(ii) Since the two vessels have the same ratio of height to base radius, they are similar.		
Thus, the required area is $60\pi\text{ cm}^2$.	1M	for (b)(ii) = (b)(i) u−1 for missing unit
	----------(4)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
12. (a) $\frac{k}{17}=\frac{63}{153}$	1M	
$k=7$	1A	
	----------(2)	
(b) The number of students in class A		
$=17\left(\frac{360}{153}\right)$	1M	
$=\frac{360}{9}$		
$=40$	1A	
The number of students in class A		
$=7\left(\frac{360}{63}\right)$	1M	for (a)$\left(\frac{360}{63}\right)$
$=\frac{360}{9}$		
$=40$	1A	
	----------(2)	
(c) The number of students having 1 key		
$=40-12-17-7$	1M	can be absorbed
$=4$		
The required probability		
$=\frac{4}{40}$	1M	for denominator using (b)
$=\frac{1}{10}$	1A	0.1
	----------(3)	
(d) There is a modification needed on the bar chart and the modification is reducing the scale of the vertical axis of the bar chart by half.	1A	
There is a modification needed on the bar chart and the modification is doubling the height of each bar.	1A	
However, there is no modification needed on the pie chart.	1A	
	----------(2)	

Solution	Marks	Remarks
13. (a) The equation of AB is		
$y-3=\frac{-4}{3}(x-10)$	1M	for point-slope form
$4x+3y-49=0$	1A	or equivalent
	----------(2)	
(b) Since $A(4,h)$ lies on $4x+3y-49=0$, we have $4(4)+3h-49=0$.	1M	for substitution
Thus, we have $h=11$.	1A	
Since $\frac{h-3}{4-10}=\frac{-4}{3}$, we have $h-3=8$.	1M	for equating slopes
Thus, we have $h=11$.	1A	
	----------(2)	
(c) (i) The value of k		
$=-2$	1A	
(ii) The area of ΔABC		
$=\frac{1}{2}(10-(-2))(11-3)$		
$=48$ square units	1A	
AC		
$=\sqrt{(4-(-2))^2+(11-3)^2}$	1M	for $AC=\sqrt{(4-(c)(i))^2+((b)-3)^2}$
$=10$		
Since the area of ΔABC is $\frac{1}{2}(BD)(AC)$, we have		
$\frac{1}{2}(BD)(10)=48$	1M	for equating areas
$BD=\frac{48}{5}$ units	1A	9.6
By the results of (b) and (c)(i), the slope of AC is $\frac{4}{3}$.		
The equation of AC is		
$y-3=\frac{4}{3}(x-(-2))$		
$4x-3y+17=0$		
The slope of BD is $\frac{-3}{4}$.	1M	for finding the slope of BD
The equation of BD is		
$y-3=\frac{-3}{4}(x-10)$		
$3x+4y-42=0$		
Therefore, the coordinates of D are $\left(\frac{58}{25},\frac{219}{25}\right)$.		
$BD=\sqrt{\left(10-\frac{58}{25}\right)^2+\left(3-\frac{219}{25}\right)^2}$	1M	for using distance formula
$=\frac{48}{5}$ units	1A	9.6
	----------(5)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
14. (a) (i) $f(-3)=0$	1M	
$4(-3)^3+k(-3)^2-243=0$		
$k=39$	1A	
(ii) $f(x)=(x+3)(4x^2+27x-81)$	1M+1A	1M for $(x+3)(lx^2+mx+n)$
$=(x+3)(x+9)(4x-9)$	1A	
	----------(5)	
(b) (i) Let $C=ax^3+bx^2$, where a and b are non-zero constants.	1A	
Since $x=5.5$, $C=7\,381$, we have		
$a(5.5)^3+b(5.5)^2=7\,381$		
$11a+2b=488$ .... (1)	1M	for substitution (either one)
Since $x=6$, $C=9\,072$, we have		
$a(6)^3+b(6)^2=9\,072$		
$6a+b=252$ .... (2)		
Solving (1) and (2), we have $a=16$ and $b=156$.	1A	for both correct
Thus, we have $C=16x^3+156x^2$.		
(ii) $16x^3+156x^2=972$	1M	for (b)(i) $=972$
$4x^3+39x^2-243=0$		
$f(x)=0$		
$(x+3)(x+9)(4x-9)=0$	1M	for using the result of (a)(ii)
$x=\frac{9}{4}$ $x=-3$ (rejected) or $x=-9$ (rejected)	1A	
Thus, the required length is $\frac{9}{4}$ cm .		2.25 cm
	----------(6)	

Solution	Marks	Remarks
15. (a) (i) The required probability		
$=\dfrac{48}{80}$		
$=\dfrac{3}{5}$	1A	0.6
(ii) The required probability		
$=\dfrac{12}{80}$		
$=\dfrac{3}{20}$	1A	0.15
(iii) The required probability		
$=\dfrac{48+4}{80}$	1M	for $\dfrac{52}{l}$, where $l \geq 53$
$=\dfrac{13}{20}$	1A	0.65
(iv) The required probability		
$=\dfrac{12}{48}$	1M	accept $\dfrac{\text{(a)(ii)}}{\text{(a)(i)}}$
$=\dfrac{1}{4}$	1A	0.25
	----------(6)	
(b) (i) The required probability		
$=\left(\dfrac{16}{80}\right)\left(\dfrac{15}{79}\right)$	1M	for $\left(\dfrac{k}{80}\right)\left(\dfrac{k-1}{79}\right)$, where $k \geq 2$
$=\dfrac{3}{79}$	1A	r.t. 0.0380
(ii) The probability of dressing shirts of the same size		
$=\left(\dfrac{28}{80}\right)\left(\dfrac{27}{79}\right)+\left(\dfrac{36}{80}\right)\left(\dfrac{35}{79}\right)+\dfrac{3}{79}$	1M	for $\left(\dfrac{m}{80}\right)\left(\dfrac{m-1}{79}\right)+\left(\dfrac{n}{80}\right)\left(\dfrac{n-1}{79}\right)+$(b)(i) where $m, n \geq 2$
$=\dfrac{141}{395}$	1A	f.t. (r.t. 0.4)
$<\dfrac{1}{2}$		
Thus, the probability of dressing shirts of the same size is not greater than that of dressing shirts of different sizes.	1A	f.t.
The probability of dressing shirts of different sizes		
$=2\left[\left(\dfrac{28}{80}\right)\left(\dfrac{36}{79}\right)+\left(\dfrac{28}{80}\right)\left(\dfrac{16}{79}\right)+\left(\dfrac{36}{80}\right)\left(\dfrac{16}{79}\right)\right]$	1M	for $\left(\dfrac{m}{80}\right)\left(\dfrac{n}{79}\right)+\left(\dfrac{m}{80}\right)\left(\dfrac{k}{79}\right)+\left(\dfrac{n}{80}\right)\left(\dfrac{k}{79}\right)$ where m, n, k are all distinct
$=\dfrac{254}{395}$	1A	f.t. (r.t. 0.6)
$>\dfrac{1}{2}$		
Thus, the probability of dressing shirts of the same size is not greater than that of dressing shirts of different sizes.	1A	f.t.
	----------(5)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
16 (a) Note that $\dfrac{AB+BC+AC}{2}=\dfrac{9+5+6}{2}=10$ cm .		
The required area		
$=\sqrt{10(10-9)(10-5)(10-6)}$	1M	accept using $\dfrac{1}{2}ab\sin C$
$=10\sqrt{2}\ \text{cm}^2$	1A	
$\approx 14.14213562\ \text{cm}^2$		
$\approx 14.1\ \text{cm}^2$		r.t. $14.1\ \text{cm}^2$
The required volume		
$=(10\sqrt{2})(20)+\dfrac{1}{3}(10\sqrt{2})(23-20)$	1M	
$=210\sqrt{2}\ \text{cm}^3$	1A	
$\approx 296.9848481\ \text{cm}^3$		
$\approx 297\ \text{cm}^3$		r.t. $297\ \text{cm}^3$
	----------(4)	
(b) DE		
$=\sqrt{(23-20)^2+6^2}$	1M	for using Pythagoras' theorem — either one
$=\sqrt{45}$ cm		
DF		
$=\sqrt{(23-20)^2+5^2}$		
$=\sqrt{34}$ cm		
By cosine formula, we have		
$\cos\angle DFE=\dfrac{DF^2+EF^2-DE^2}{2(DF)(EF)}$	1M	
$\cos\angle DFE=\dfrac{34+9^2-45}{2(\sqrt{34})(9)}$		
$\cos\angle DFE=\dfrac{35\sqrt{34}}{306}$		
$\angle DFE$		
$\approx 48.16875177°$		
$\approx 48.2°$	1A	r.t. $48.2°$
The shortest distance from D to EF		
$=DF\sin\angle DFE$	1M	
$\approx\sqrt{34}\sin 48.16875177°$		
≈ 4.344714399		
≈ 4.34 cm	1A	r.t. 4.34 cm
	----------(5)	
(c) The area of $\Delta DEF=\dfrac{1}{2}(EF)(DF\sin\angle DFE)$	1M	for finding the area of ΔDEF
$\approx\dfrac{1}{2}(9)(4.344714399)$		
$\approx 19.5512148\ \text{cm}^2$		
$<20\ \text{cm}^2$		
So, the area of the triangle DEF is less than the area of the metal plate. Thus, the given metal plate cannot be fixed in that way.	1A	f.t.
	----------(2)	

只限教師參閱 FOR TEACHERS' USE ONLY

Solution	Marks	Remarks
17. **Marking Scheme for (a)(i) and (a)(ii):**		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
(a) (i) In ΔABG and ΔDBG ,		
$\angle ABG = \angle DBG$ (in-centre of Δ)		[Δ內心]
$BG = BG$ (common side)		[公共邊]
$AB = BD$ (given)		[已知]
$\Delta ABG \cong \Delta DBG$ (SAS)		
(ii) In ΔAGI and ΔABE ,		
$\angle IAG = \angle EAB$ (in-centre of Δ)		[Δ內心]
$\angle ABE = 90°$ ($\angle$ in semi-circle)		[半圓上的圓周角]
$\angle AGI = \angle DGI$ (by (a)(i))		
$\angle AGI + \angle DGI = 180°$ (adj. $\angle$s on st. line)		[直線上的鄰角]
$\angle AGI = \angle DGI = 90°$ (by (a)(i))		
$\angle AGI = \angle ABE$		
$\angle AIG = \angle AEB$ ($\angle$ sum of Δ)		[Δ內角和]
$\Delta AGI \sim \Delta ABE$ (AAA)		[等角] (AA) (equiangular)
Thus, we have $\dfrac{GI}{AG} = \dfrac{BE}{AB}$.		
	----------(6)	
(b) (i) Let the coordinates of G be $(a, 0)$.		
$a = \dfrac{-25+11}{2} = -7$	1A	
Thus, the coordinates of G are $(-7, 0)$.		
(ii) Note that $AG = 11-(-7) = 18$.	1M	can be absorbed
By (a)(ii), we have $\dfrac{GI}{AG} = \dfrac{BE}{AB} = \dfrac{1}{2}$.		
So, we have $GI = \left(\dfrac{BE}{AB}\right) AG$		
$= \dfrac{1}{2}(18)$	1M	for using (a)(ii)
$= 9$	1A	
So, the coordinates of I are $(-7, 9)$.		
The equation of the inscribed circle is		
$(x+7)^2 + (y-9)^2 = 9^2$	1A	
$x^2 + y^2 + 14x - 18y + 49 = 0$		
	----------(5)	