評卷參考 *

Marking Scheme

香港考試及評核局
Hong Kong Examinations and Assessment Authority

2006年香港中學會考
Hong Kong Certificate of Education Examination 2006

數學　試卷一
Mathematics Paper 1

* 此部分只設英文版本

Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* in Section A. Do not deduct any marks for *u* in Section B.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***1 mark*** for *pp* in each of Section A and Section B. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
   c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\dfrac{(a^3)^5}{a^{-6}}$		
[redacted]	1M	for $(x^m)^n = x^{mn}$
[redacted]	1M	for $\dfrac{x^m}{x^n} = x^{m-n}$ or $x^{-n} = \dfrac{1}{x^n}$
$= a^{21}$	1A ——(3)	
2. (a) $x+1 < \dfrac{x+25}{6}$		
$6x+6 < x+25$		
$6x - x < 25 - 6$	1M	for putting x on one side
$5x < 19$		
$x < \dfrac{19}{5}$	1A	$x < 3.8$
(b) The required greatest integer is 3 .	1A ——(3)	
3. (a) $3b - ab$		
$= b(3-a)$	1A	
(b) $9 - a^2$		
$= (3+a)(3-a)$	1A	
(c) $9 - a^2 + 3b - ab$		
$= (3+a)(3-a) + b(3-a)$		
$= (3-a)(3+a+b)$	1A ——(3)	
4. The length of $\overset{\frown}{AB}$		
$= 2\pi(12)\left(\dfrac{150}{360}\right)$	1M + 1A	1M for $\dfrac{150}{360}$ + 1A for $2\pi(12)$
$= 10\pi$ cm	1A ——(3)	u−1 for missing unit

Solution	Marks	Remarks
5. $\angle AEB$		
$= \angle CBE$		
$= 70^\circ$		
$\angle ABE$		
$= \angle AEB$		
$= 70^\circ$	1A	u–1 for missing unit
$\angle BCD$		
$= 180^\circ - 70^\circ - 70^\circ$	1M	can be absorbed
$= 40^\circ$	1A	u–1 for missing unit
$\angle BAE$		
$= 180^\circ - 70^\circ - 70^\circ$	1M	can be absorbed
$= 40^\circ$		
$\angle BCD$		
$= \angle BAE$		
$= 40^\circ$	1A	u–1 for missing unit
	----------(3)	
6. (a) Let x kg be the weight of John. Then, we have		
$x(1+20\%) = 60$	1M	
$\frac{6x}{5} = 60$		
$x = 50$	1A	
Thus, the weight of John is 50 kg.		u–1 for missing unit
(b) The weight of Susan		
$= 60(1-20\%)$	1M	
$= 60(0.8)$		
$= 48$ kg		
$\neq 50$ kg		
Thus, Susan and John are not of the same weight.	1A	f.t.
	----------(4)	
7. (a) A'		
$= (7, 2)$	1A	pp–1 for missing '(' or ')'
B'		
$= (5, 5)$	1A	pp–1 for missing '(' or ')'
(b) $A'B'$		
$= \sqrt{(7-5)^2 + (2-5)^2}$	1M	for using distance formula (either one)
$= \sqrt{13}$		
AB		
$= \sqrt{(-5-(-2))^2 + (5-7)^2}$		
$= \sqrt{13}$		
Thus, the lengths of AB and $A'B'$ are equal.	1A	f.t.
To obtain B' is the same as rotating B clockwise through 90°.	1M	for reflection the same as rotation
So, $A'B'$ can be obtained by rotating AB clockwise through 90°.		
Thus, the lengths of AB and $A'B'$ are equal.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
8. (a) $\frac{2+3+5+8+11+11+12+15+19+k}{10}=11$	1M	
Thus, we have $k=24$.	1A	
(b) The required probability		
$=\frac{4}{10}$	1M + 1A	1M for numerator + 1A for denominator
$=\frac{2}{5}$	1A	0.4
	----------(5)	
9. (a) x		
$= 360° - 90° - 130° - 35° - 40° - 30°$	1M	for $360° - 90° - \theta$
$= 35°$	1A	u–1 for missing unit
(b) Her total expenditure		
$=1750\left(\frac{360}{35}\right)$	1M	
$=\$\,18\,000$	1A	u–1 for missing unit
(c) Her expenditure on travelling		
$=1750\left(\frac{35}{35}\right)$		
$=\$\,1\,750$	1A	u–1 for missing unit
Her expenditure on travelling		
$=18000\left(\frac{35}{360}\right)$		
$=\$\,1\,750$	1A	u–1 for missing unit
	----------(5)	

Solution	Marks	Remarks
10(a) (i) Note that $f(1)=(1-a)(1-b)(1+1)-3$.		
$f(1)=1$		
$2(1-a)(1-b)-3=1$		
$(1-a)(1-b)=2$		
Thus, we have $(a-1)(b-1)=2$.	1	
(ii) Since a and b are positive integers and $a<b$, we have		
$a-1=1$ and $b-1=2$.		
Thus, we have $a=2$ and $b=3$.	1A + 1A	
	----------(3)	
(b) $f(x)-g(x)$		
$=(x-2)(x-3)(x+1)-3-(x^3-6x^2-2x+7)$		
$=x^3-4x^2+x+3-x^3+6x^2+2x-7$	1M	for expanding $f(x)$
$=2x^2+3x-4$	1M	
For $f(x)=g(x)$, we have $f(x)-g(x)=0$.		
So, we have $2x^2+3x-4=0$.		
Therefore, we have $x=\frac{-3\pm\sqrt{(3)^2-4(2)(-4)}}{2(2)}$.	1M	can be absorbed
Thus, the exact values of all the roots are $\frac{-3+\sqrt{41}}{4}$ and $\frac{-3-\sqrt{41}}{4}$.	1A	for both correct
	----------(4)	

Solution	Marks	Remarks
11. (a) The maximum absolute error is 0.5 cm .	1A	u–1 for missing unit
	----------(1)	
(b) The least possible area of the metal sheet		
$= (17.5)(11.5) + (14.5)(1.5)$	1M + 1A	1M for sum of several areas + 1A for one of the areas correct
$= 223 \text{ cm}^2$	1A	u–1 for missing unit
The least possible area of the metal sheet		
$= (14.5)(11.5 + 1.5) + (11.5)(17.5 - 14.5)$	1M + 1A	1M for sum of several areas + 1A for one of the areas correct
$= 223 \text{ cm}^2$	1A	u–1 for missing unit
The least possible area of the metal sheet		
$= (14.5)(11.5) + (14.5)(1.5) + (11.5)(17.5 - 14.5)$	1M + 1A	1M for sum of several areas + 1A for one of the areas correct
$= 223 \text{ cm}^2$	1A	u–1 for missing unit
The least possible area of the metal sheet		
$= (17.5)(11.5 + 1.5) - (1.5)(17.5 - 14.5)$	1M + 1A	1M for difference of areas + 1A for one of the areas correct
$= 223 \text{ cm}^2$	1A	u–1 for missing unit
	----------(3)	
(c) The actual area of the metal sheet		
$< (18.5)(12.5) + (15.5)(2.5)$	1A	for either area correct
$= 270 \text{ cm}^2$	1A	
The actual area of the metal sheet		
$< (15.5)(12.5 + 2.5) + (12.5)(18.5 - 15.5)$	1A	for either area correct
$= 270 \text{ cm}^2$	1A	
The actual area of the metal sheet		
$< (15.5)(12.5) + (15.5)(2.5) + (12.5)(18.5 - 15.5)$	1A	for either area correct
$= 270 \text{ cm}^2$	1A	
The actual area of the metal sheet		
$< (18.5)(12.5 + 2.5) - (2.5)(18.5 - 15.5)$	1A	for either area correct
$= 270 \text{ cm}^2$	1A	
Thus, by (b), we have $223 \le x < 270$.	1M + 1A	1M for end-points + 1A for inequality signs u–1 for having unit
	----------(4)	

Solution	Marks	Remarks
(a) The coordinates of M are $(4, 4)$.	1A	pp−1 for missing '(' or ')'
	----------(1)	
(b) The slope of $AB = \frac{8-0}{12-(-4)} = \frac{1}{2}$		
The slope of $CM = \frac{-1}{\frac{1}{2}} = -2$	1M	can be absorbed
The equation of CM is $y-4=-2(x-4)$		
$2x+y-12=0$	1A	or equivalent
Putting $y=0$ in $2x+y-12=0$, we have $x=6$.		
Thus, the coordinates of C are $(6, 0)$.	1A	pp−1 for missing '(' or ')'
	----------(3)	
(c) (i) The slope of $BD = \frac{8-0}{12-2} = \frac{4}{5}$		
The equation of BD is		
$y-0=\frac{4}{5}(x-2)$	1M	for point-slope form
$4x-5y-8=0$		
(ii) Solving $\begin{cases} 2x+y-12=0 \\ 4x-5y-8=0 \end{cases}$,	1M	
we have $x=\frac{34}{7}$ and $y=\frac{16}{7}$.	1A	
So, the coordinates of K are $\left(\frac{34}{7}, \frac{16}{7}\right)$.		pp−1 for missing '(' or ')'
The required ratio $=\frac{(AC)(4)}{2} : \frac{(AC)\left(\frac{16}{7}\right)}{2}$		
$=4:\frac{16}{7}$	1M	for y-coordinate of M : y-coordinate of K
$=7:4$	1A	accept $1:s$ and $t:1$ with s r.t. 0.571 and t r.t. 1.75
	----------(5)	

Solution	Marks	Remarks
13. (a) In Figure 7(a), let h cm be the height of the smaller cone. Then, we have		
$\frac{h}{h+8}=\frac{3}{6}$		
$h=8$	1A	can be absorbed
The volume of the frustum		
$=\frac{1}{3}\pi(6^2)(16)-\frac{1}{3}\pi(3^2)(8)$	1M	
$=168\pi$ cm^3	1A	can be absorbed
The volume of X		
$=\frac{2}{3}\pi(6^3)+168\pi$	1M	
$=312\pi$ cm^3	1A	u−1 for missing unit
The volume of Y		
$=\left(\sqrt{\frac{9}{4}}\right)^3(312\pi)$	1M	
$=\left(\frac{27}{8}\right)(312\pi)$		
$=1053\pi$ cm^3	1A	u−1 for missing unit
In Figure 7(a), let h cm be the height of the smaller cone. Then, we have		
$\frac{h}{h+8}=\frac{3}{6}$		
$h=8$	1A	can be absorbed
$\frac{\text{The volume of the frustum}}{\text{The volume of smaller cone}}=\frac{2^3-1^3}{1^3}=7$		
The volume of the frustum		
$=\frac{1}{3}\pi(3^2)(8)(7)$	1M	
$=168\pi$ cm^3	1A	can be absorbed
The volume of X		
$=\frac{2}{3}\pi(6^3)+168\pi$	1M	
$=312\pi$ cm^3	1A	u−1 for missing unit
The ratio of a linear measurement of X to the corresponding linear measurement of Y		
$=\sqrt{4}:\sqrt{9}$		
$=2:3$		
The radius of the top circular surface of $Y=3(\frac{3}{2})=4.5$ cm		
The radius of the hemisphere part of $Y=6(\frac{3}{2})=9$ cm		
The height of the frustum part of $Y=8(\frac{3}{2})=12$ cm		
The volume of Y		
$=\frac{1}{3}\pi(9^2)(24)-\frac{1}{3}\pi(4.5^2)(12)+\frac{2}{3}\pi(9^3)$	1M	
$=1053\pi$ cm^3	1A	u−1 for missing unit
	----------(7)	

Solution	Marks	Remarks
(b) The volume of X'		
$= 312\pi + \frac{4}{3}\pi(1^3)$		
$= \frac{940\pi}{3}$ cm^3		
The volume of Y'		
$= 1053\pi + \frac{4}{3}\pi(2^3)$		
$= \frac{3191\pi}{3}$ cm^3		
$\frac{\text{The volume of } X'}{\text{The volume of } Y'}$		
$= \frac{940}{3191}$		
$\neq \frac{8}{27}$	1M	
Thus, X' and Y' are not similar.	1A	f.t.
The ratio of a linear measurement of X to the corresponding linear measurement of Y		
$= \sqrt{4} : \sqrt{9}$		
$= 2:3$		
But the ratio of the radius of the sphere fixed onto X to the radius of the sphere fixed onto Y is $1:2$, which is not equal to $2:3$.	1M	
Thus, X' and Y' are not similar.	1A	f.t.
$\frac{\text{The volume of the sphere fixed on } X}{\text{The volume of the sphere fixed on } Y}$		
$= \left(\frac{1}{2}\right)^3$		
$= \frac{1}{8}$		
$\neq \frac{8}{27}$	1M	
Thus, X' and Y' are not similar.	1A	f.t.
$\frac{\text{The surface area of the sphere fixed on } X}{\text{The surface area of the sphere fixed on } Y}$		
$= \left(\frac{1}{2}\right)^2$		
$= \frac{1}{4}$		
$\neq \frac{4}{9}$	1M	
Thus, X' and Y' are not similar.	1A	f.t.
	----------(2)	

Solution	Marks	Remarks
14. (a) (i) Some statistics are tabulated as follows:		

	Class A	Class B
The lower quartile	18 marks	11 marks
The upper quartile	39 marks	25 marks

Solution	Marks	Remarks
(table above)	1A	for either row or either column
The inter-quartile range of the score distribution of the students of class A		
$= 39 - 18$	1M	for either one
$= 21$ marks	1A	for both correct
The inter-quartile range of the score distribution of the students of class B		
$= 25 - 11$		
$= 14$ marks		
(ii) By (a)(i), the inter-quartile range of the score distribution of the students of class B is less than that of class A. Thus, the score distribution of the students of class B is less dispersed than class A.	1M	
	----------(4)	
(b) (i) The required probability		
$= (\frac{28}{50})(\frac{27}{49})(\frac{22}{48})(3)$	1M + 1M	1M for $(\frac{p}{q})(\frac{p-1}{q-1})(\frac{50-p}{q-2})$, $p < q$ + 1M for the 3 cases
$= \frac{297}{700}$	1A	r.t. 0.424
(ii) The required probability		
$= \left((\frac{18}{50})(\frac{17}{49})(\frac{22}{48}) + (\frac{10}{50})(\frac{9}{49})(\frac{22}{48})\right)(3)$	1M	for $(\frac{p}{q})(\frac{p-1}{q-1})(\frac{r}{q-2})$, $p < q$ and $r < q-2$
$= \frac{1089}{4900}$	1A	r.t. 0.222
(iii) The required probability		
$= \dfrac{\frac{1089}{4900}}{\frac{297}{700}}$	1M	for $\frac{\text{(b)(ii)}}{\text{(b)(i)}}$
$= \frac{11}{21}$	1A	r.t. 0.524
The required probability		
$= \frac{(18)(17)(22) + (10)(9)(22)}{(28)(27)(22)}$	1M	for denominator $= (28)(27)(22)$ or denominator $= (28)(27)$
$= \frac{11}{21}$	1A	r.t. 0.524
	----------(7)	

Solution	Marks	Remarks
(a) Let $C = aA + \frac{bA^2}{n}$ where a and b are non-zero constants.	1A	pp−1 for writing $C \propto aA + \frac{bA^2}{n}$
When $A = 50$ and $n = 500$, $C = 350$, we have		
$50a + \frac{(50^2)b}{500} = 350$	1M	for substitutions (either one)
$10a + b = 70$ .... (1)		
When $A = 20$ and $n = 400$, $C = 100$, we have		
$20a + \frac{(20^2)b}{400} = 100$		
$20a + b = 100$ .... (2)		
Solving (1) and (2), we have $a = 3$ and $b = 40$.	1A	for both correct
Thus, we have $C = 3A + \frac{40A^2}{n}$.		
	---------(3)	
(b) (i) Note that $P = 8A - \left(3A + \frac{40A^2}{n}\right)$.		
Thus, we have $P = 5A - \frac{40A^2}{n}$.	1M	for $P = 8A - $(a) and simplified
(ii) When $P : n = 5 : 32$,		
$\frac{5n}{32} = 5A - \frac{40A^2}{n}$		
$256A^2 - 32An + n^2 = 0$	1M	for $\alpha A^2 + \beta An + \gamma n^2 = 0$ or equivalent
$(16A - n)^2 = 0$		
Thus, we have $A : n = 1 : 16$.	1A	accept 0.0625
(iii) When $n = 500$ and $P = 100$, we have $5A - \frac{40A^2}{500} = 100$.		
So, we have $2A^2 - 125A + 2500 = 0$.		
$\Delta = (-125)^2 - 4(2)(2500)$ $= -4375$ < 0	1M	accept attempting to solve the quadratic equation or find the greatest value of P
Thus, it is impossible to make a profit of \$ 100.	1A	f.t.
(iv) When $n = 400$,		
$P = 5A - \frac{40A^2}{400}$		
$= 5A - \frac{A^2}{10}$		
$= \frac{-1}{10}(A^2 - 50A)$		
$= \frac{-1}{10}(A^2 - 50A + 25^2 - 25^2)$	1M	for completing the square
$= \frac{-1}{10}(A - 25)^2 + \frac{125}{2}$	1A	
Thus, the greatest profit is \$ 62.5.	1A	
	---------(8)	

Solution	Marks	Remarks
16. (a) (i) $\angle BAS = \angle BCS = 90^\circ$ ($\angle$ in semi-circle)		[半圓上的圓周角]
Produce CH to meet AB at K .		
$\angle BKC = \angle BOA = 90^\circ$ (orthocentre of Δ)		[Δ垂心]
Hence, $\angle BAS = 90^\circ = \angle BKC$		
and $\angle BCS = 90^\circ = \angle BOA$.		
So, $AS // HC$ and $SC // AH$. (corr. $\angle$s equal)		[同位角等] [同側(旁)內角互補] (int. $\angle$s supp.) [(內)錯角等] (alt. $\angle$s equal)
Thus, $AHCS$ is a parallelogram.		
$\angle BAS = \angle BCS = 90^\circ$ ($\angle$ in semi-circle)		[半圓上的圓周角]
So, $SA \perp AB$ and $SC \perp BC$.		
$CH \perp AB$ and $AH \perp BC$ (orthocentre of Δ)		[Δ垂心]
So, $AS // HC$ and $SC // AH$. (corr. $\angle$s equal)		[同位角等] [同側(旁)內角互補] (int. $\angle$s supp.) [(內)錯角等] (alt. $\angle$s equal)
Thus, $AHCS$ is a parallelogram.		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
(ii) $\because$ $AHCS$ is a parallelogram. (by(a)(i))		
$\therefore$ $AH = SC$ (opp. sides, // gram)		[// 四邊形對邊]
$\because$ $GR \perp BC$		
$\therefore$ $BR = RC$ (line from centre $\perp$ chord bisects chord)		[圓心至弦的垂線平分弦] [通過圓心垂直於弦的線平分弦]
Note that $BG = GS$.		
Hence, we have $SC = 2GR$. (mid-point thm.)		[中點定理]
Thus, we have $AH = 2GR$.		
$\because$ $AHCS$ is a parallelogram. (by(a)(i))		
$\therefore$ $AH = SC$ (opp. sides, // gram)		[// 四邊形對邊]
$\angle BRG = 90^\circ = \angle BCS$		
$\angle GBR = \angle SBC$ (common $\angle$)		[公共角]
$\angle BGR = 180^\circ - \angle BRG - \angle GBR$ ($\angle$ sum of Δ)		[Δ內角和]
$= 180^\circ - \angle BCS - \angle SBC$		
$= \angle BSC$ ($\angle$ sum of Δ)		[Δ內角和]
Hence, $\Delta BGR \sim \Delta BSC$. (AAA)		[等角] (AA) (equiangular)
So, we have $\frac{BG}{GR} = \frac{BS}{SC}$.		
Note that $BG = GS$.		
Therefore, we have $BS = 2BG$.		
Hence, we have $SC = 2GR$.		
Thus, we have $AH = 2GR$.		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
	----------(5)	

Solution	Marks	Remarks
(b) (i) Let the equation of the required circle be $x^2+y^2+Dx+Ey+F=0$.		
$\because$ the coordinates of B and C are $(-6, 0)$ and $(4, 0)$ respectively.		
$\therefore \begin{cases}(-6)^2+(0)^2+D(-6)+E(0)+F=0 \\ (4)^2+(0)^2+D(4)+E(0)+F=0\end{cases}$	1M	for either one
So, we have $D=2$ and $F=-24$.	1A	for both correct
$\because$ the coordinates of A are $(0,12)$.		
$\therefore$ $(0)^2+(12)^2+D(0)+E(12)+F=0$		
So, we have $E=-10$.	1A	
Thus, the equation of the circle is $x^2+y^2+2x-10y-24=0$.		$(x+1)^2+(y-5)^2=50$
(ii) Note that the coordinates of G are $(-1,5)$.		
So, the coordinates of R are $(-1,0)$.		
Then, we have $GR=5$.		
By (a)(ii), we have $AH=10$.		
Thus, the coordinates of the H are $(0,2)$.	1A	
(iii) The slope of $GH=\dfrac{5-2}{-1-0}=-3$		
The slope of $BG=\dfrac{5-0}{-1-(-6)}=1$		
$\because$ (the slope of GH) (the slope of BG) $=-3\neq-1$	1M	for testing whether $GH\perp BG$
$\therefore$ $\angle BGH\neq 90^\circ$		
Note that $\angle BOH=90^\circ$.		
So, we have $\angle BGH+\angle BOH\neq 180^\circ$.		
Thus, B, O, H and G are not concyclic.	1A	f.t.
For the equation of the circle which passes through B, H and O,		
$\because$ $\angle BOH=90^\circ$		
$\therefore$ the centre of the circle is the mid-point of BH.		
So, the coordinates of the centre are $(-3,1)$.		
The radius $=\sqrt{(-3-0)^2+(1-2)^2}=\sqrt{10}$		
So, the equation of the circle which passes through B, H and O is $(x+3)^2+(y-1)^2=10$.		
Note that $(-1+3)^2+(5-1)^2=20\neq10$.	1M	for testing whether the fourth point lies on the circle
G does not lie on the circle which passes through B, H and O.		
Thus, B, O, H and G are not concyclic.	1A	f.t.
Let the equation of the circle which passes through B, H and O be $x^2+y^2+Dx+Ey+F=0$.		
Since the coordinates of O are $(0,0)$, we have $F=0$.		
$\because$ the coordinates of B and H are $(-6, 0)$ and $(0, 2)$ respectively.		
$\therefore$ $(-6)^2+(0)^2+D(-6)+E(0)=0$, $(0)^2+(2)^2+D(0)+E(2)=0$.		
Hence, we have $D=6$ and $E=-2$.		
So, the equation of the circle which passes through B, H and O is $x^2+y^2+6x-2y=0$.		
Note that $(-1)^2+5^2+6(-1)-2(5)=10\neq0$.	1M	for testing whether the fourth point lies on the circle
G does not lie on the circle which passes through B, H and O.		
Thus, B, O, H and G are not concyclic.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
17. (a) By cosine formula, we have		
$\cos\angle BAD = \dfrac{AC^2 + AB^2 - BC^2}{2(AC)(AB)}$	1M	accept using Pythagoras' theorem twice
$\cos\angle BAD = \dfrac{90^2 + 40^2 - 60^2}{2(90)(40)}$		
$\cos\angle BAD = \dfrac{61}{72}$		
$\angle BAD \approx 32.08918386°$		
AD $= 40\cos\angle BAD$ $= 40(\dfrac{61}{72})$ $= \dfrac{305}{9}$ cm	1A	r.t. 33.9 cm, $AD \approx 33.88888889$ cm
	---------(2)	
(b) (i) (1) CD $= CA - AD$ $= 90 - \dfrac{305}{9}$ $= \dfrac{505}{9}$ cm ≈ 56.11111111 cm		
By sine formula, we have $\dfrac{CD}{\sin\angle DAC} = \dfrac{AD}{\sin\angle DCA}$ $\dfrac{505}{9\sin 62°} = \dfrac{305}{9\sin\angle DCA}$ $\angle DCA \approx 32.22634992°$ $\angle ADC$ $\approx 180° - 32.22634992° - 62°$ $\approx 85.77365008°$ By sine formula, we have $\dfrac{AC}{\sin\angle ADC} = \dfrac{CD}{\sin\angle DAC}$	1M	for either one
$\dfrac{AC}{\sin 85.77365008°} \approx \dfrac{56.11111111}{\sin 62°}$ $AC \approx 63.37695244$ cm		
Thus, the required distance is 63.4 cm .	1A	r.t. 63.4 cm
By cosine formula, we have $CD^2 = AD^2 + AC^2 - 2(AD)(AC)\cos\angle DAC$	1M	
$(\dfrac{505}{9})^2 = (\dfrac{305}{9})^2 + AC^2 - 2(\dfrac{305}{9})(AC)\cos 62°$ $AC^2 - 2(15.90986963)(AC) - 2000 \approx 0$ $AC \approx 63.37695244$ cm		
Thus, the required distance is 63.4 cm .	1A	r.t. 63.4 cm

Solution	Marks	Remarks
(2) s $=\frac{1}{2}(AB+BC+AC)$ $\approx\frac{1}{2}(40+60+63.37695244)$ ≈ 81.6884762 cm		
The area of ΔABC $=\sqrt{s(s-AB)(s-BC)(s-AC)}$	1M	with s defined
$\approx\sqrt{s(s-40)(s-60)(s-63.376952)}$ $\approx 1162.961055\text{ cm}^2$ $\approx 1160\text{ cm}^2$	1A	r.t. 1160 cm^2
(3) The area of ΔADC $=\frac{1}{2}(AD)(AC)\sin\angle DAC$	1M	
$\approx\frac{1}{2}(33.888888889)(63.37695244)\sin 62°$ $\approx 948.1861616\text{ cm}^2$		
BD $=AB\sin\angle BAD$ $\approx 40\sin 32.08918386°$ ≈ 21.24954611 cm		
Let H be the projection of D on the horizontal plane. Then, the height of the tetrahedron $ABCD$ is DH. So, we have $\frac{DH}{3}$(area of ΔABC) $=\frac{BD}{3}$(area of ΔADC)	1M	
$\frac{1}{3}DH(1162.961055)\approx\frac{1}{3}(21.24954611)(948.1861616)$ $DH\approx 17.32519373$ cm Thus, the required height is 17.3 cm .	1A	r.t. 17.3 cm
(ii) The volume of the tetrahedron $ABCD$ $=\frac{(AD)(CD)(BD)\sin\angle ADC}{6}$ So, the volume of the tetrahedron varies directly as $\sin\angle ADC$.	1M	for either one
When $\angle ADC$ increases from 30° to 90°, the volume of the tetrahedron $ABCD$ increases. When $\angle ADC$ increases from 90° to 150°, the volume of the tetrahedron $ABCD$ decreases.	1A	
	----------(9)	