

只限教師參閱

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香港考試及評核局

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2005年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2005

數學 試卷一

MATHEMATICS PAPER 1

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2005-CE-MATH 1-1

只限教師參閱

FOR TEACHERS' USE ONLY

Hong Kong Certificate of Education Examination
Mathematics Paper 1

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
 - a. The symbol $(u-1)$ should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* in Section A. Do not deduct any marks for *u* in Section B.
 - b. The symbol $(pp-1)$ should be used to denote 1 mark deducted for *pp*. At most deduct **1 mark** for *pp* in each of Section A and Section B. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
 - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
 - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. Marks entered in the Page Total Box should be the NET total scored on that page.
8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’, ‘ft.’ stands for ‘follow through’ and ‘or equivalent’ means ‘accepting equivalent forms of the equation which has been simplified and without uncollected like terms’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $P = ab + 2bc + 3ac$ $ab + 3ac = P - 2bc$ $a(b + 3c) = P - 2bc$ $a = \frac{P - 2bc}{b + 3c}$	1M 1M 1A -----(3)	for putting a on one side for factorization or equivalent
2. $\frac{(x^3y)^2}{y^5}$ $= \frac{x^6y^2}{y^5}$ $= \frac{x^6}{y^{5-2}}$ $= \frac{x^6}{y^3}$	1M 1M 1A -----(3)	for $(ab)^n = a^n b^n$ or $(a^m)^n = a^{mn}$ for $\frac{b^m}{b^n} = b^{m-n}$ or $\frac{b^m}{b^n} = \frac{1}{b^{n-m}}$
3. (a) $4x^2 - 4xy + y^2$ $= (2x - y)^2$ (b) $4x^2 - 4xy + y^2 - 2x + y$ $= (2x - y)^2 - 2x + y$ (by (a)) $= (2x - y)^2 - (2x - y)$ $= (2x - y)(2x - y - 1)$	1A 1M 1A -----(3)	or equivalent for using the result of (a) or equivalent
4. $\frac{-3x+1}{4} > x-5$ $-3x+1 > 4x-20$ $-7x > -21$ $7x < 21$ $x < 3$ For $2x+1 \geq 0$, we have $x \geq \frac{-1}{2}$. Therefore, the solution of $\frac{-3x+1}{4} > x-5$ and $2x+1 \geq 0$ is $\frac{-1}{2} \leq x < 3$. Thus, all integers which satisfy both the inequalities $\frac{-3x+1}{4} > x-5$ and $2x+1 \geq 0$ are 0, 1 and 2.	1M 1A 1A -----(3)	for putting x on one side

Solution	Marks	Remarks
5. $n - 18 = \frac{2n}{5} + 18$ $\frac{3n}{5} = 36$ $n = 60$	1A + 1M 1A	1A for $\frac{2n}{5} + 1M$ for equating
Suppose that Susan and Teresa have $5k$ marbles and $2k$ marbles respectively. $5k - 18 = 2k + 18$ $3k = 36$ $k = 12$ $n = 5k$ $n = 60$	1A 1M 1A	for $5k$ and $2k$ pp-1 for any undefined symbol for equating
------(3)		
6. (a) Let $\$x$ be the marked price of the calculator. Then, we have $x = 160(1 + 25\%)$ $x = 200$ Thus, the marked price of the calculator is \$200 .	1A 1A	u-1 for missing unit
Let $\$x$ be the marked price of the calculator. Then, we have $\left(\frac{x - 160}{160}\right)(100\%) = 25\%$ $x - 160 = 40$ $x = 200$ Thus, the marked price of the calculator is \$200 .	1A 1A	accept without 100% u-1 for missing unit
(b) The selling price of the calculator $= 200(90\%)$ $= \$180$ The percentage profit $= \left(\frac{180 - 160}{160}\right)(100\%)$ $= 12.5\%$	1M 1A	for (a)(90%)
The selling price of the calculator $= 200(90\%)$ $= \$180$ The percentage profit $= \left(\frac{180 - 160}{200 - 160}\right)(25\%)$ $= 12.5\%$	1M 1A	for (a)(90%)
------(4)		
7. The common difference $= 8 - 5 = 3$ $\frac{n}{2}((2)(5) + (n-1)(3)) = 3925$ $3n^2 + 7n - 7850 = 0$ $n = 50$ or $n = \frac{-157}{3}$ (rejected since n is a positive integer) Thus, we have $n = 50$.	1A 1M 1M 1A -----(4)	can be absorbed for $\frac{n}{2}((2)(5) + (n-1)(d)) = 3925$ in the form $k_1n^2 + k_2n + k_3 = 0$ where $k_1 \neq 0$

Solution	Marks	Remarks
8. $x = \frac{180(6-2)}{6}$ $= 120$ $y = \frac{180-x}{2}$ $= \frac{180-120}{2}$ $= 30$ $z = 180 - 2y$ $= 180 - (2)(30)$ $= 120$	1A 1M 1A 1M 1A	u-1 for having unit u-1 for having unit u-1 for having unit u-1 for having unit
$z = (180)(5-2) - 2x - 2(x-y)$ $= 540 - 4x + 2y$ $= 540 - (4)(120) + (2)(30)$ $= 120$	1M 1A	u-1 for having unit
------(5)		
9. (a) $2\pi(OA)\left(\frac{100}{360}\right) = 10\pi$ $OA = 18 \text{ cm}$	1M 1A	for $\frac{100}{360}$ u-1 for missing unit
$(OA)\left(\frac{100\pi}{180}\right) = 10\pi$ $OA = 18 \text{ cm}$	1M 1A	for $\frac{100\pi}{180}$ u-1 for missing unit
(b) The area of sector $OABC$ $= \frac{100}{360}\pi(18)^2$ $\approx 282.7433388 \text{ cm}^2$ The area of $\triangle OAC$ $= \frac{1}{2}(18)^2 \sin 100^\circ$ $\approx 159.538856 \text{ cm}^2$ The required area $\approx 282.7433388 - 159.538856$ ≈ 123.2044828 $\approx 123 \text{ cm}^2$	1M 1M 1A	for $\frac{100}{360}\pi(a)^2$ for $\frac{1}{2}(a)^2 \sin 100^\circ$ u-1 for missing unit r.t. 123 cm^2
The area of sector $OABC$ $= \frac{1}{2}(18)^2\left(\frac{100}{180}\pi\right)$ $\approx 282.7433388 \text{ cm}^2$ The area of $\triangle OAC$ $= \frac{1}{2}(2(18 \sin 50^\circ))(18 \cos 50^\circ)$ $\approx 159.538856 \text{ cm}^2$ The required area $\approx 282.7433388 - 159.538856$ ≈ 123.2044828 $\approx 123 \text{ cm}^2$	1M 1M 1A	for $\frac{1}{2}(a)^2\left(\frac{100}{180}\pi\right)$ for $\frac{1}{2}(2((a) \sin 50^\circ))((a) \cos 50^\circ)$ u-1 for missing unit r.t. 123 cm^2
------(5)		

Solution	Marks	Remarks
<p>10. (a) Let $f(x) = ax^3 + bx$, where a and b are non-zero constants.</p> <p>Since $f(2) = -6$, we have $8a + 2b = -6$ $4a + b = -3$ (1)</p> <p>Since $f(3) = 6$, we have $27a + 3b = 6$ $9a + b = 2$ (2)</p> <p>Solving (1) and (2), we have</p> $\begin{cases} a = 1 \\ b = -7 \end{cases}$ <p>Thus, we have $f(x) = x^3 - 7x$.</p> <p>(b) (i) $g(x) = f(x) - 6$ $g(3) = f(3) - 6 = 6 - 6 = 0$ Thus, by Factor Theorem, $x - 3$ is a factor of $g(x)$.</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>----- (4)</p> <p>1A</p>	<p>for substitution (either one)</p> <p>for solving simultaneous equations and can be absorbed</p> <p>for both correct</p>
<div style="border: 1px solid black; padding: 5px;"> $g(x) = x^3 - 7x - 6$ $g(3) = 3^3 - 7(3) - 6 = 0$ Thus, by Factor Theorem, $x - 3$ is a factor of $g(x)$. </div>	<p>1A</p>	<p>accept using division correctly</p>
<p>(ii) $g(x) = (x-3)(x^2 + 3x + 2)$ $= (x-3)(x+1)(x+2)$</p>	<p>1M + 1A</p> <p>1A</p> <p>----- (4)</p>	<p>1M for $(x-3)(ax^2 + bx + c)$</p>

Solution	Marks	Remarks
11. (a) The required probability $= \frac{1}{2}$	1A -----(1)	0.5
(b) The required probability $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{1}{8}$	1M 1A -----(2)	for (a) $p_1 p_2$, $0 < p_1, p_2 < 1$ 0.125
(c) The required probability $= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right)$ $= \frac{3}{4}$	1M+1M 1A	1M for $(1 - (a))$ + 1M for $(a)p_3$, $0 < p_3 < 1$ 0.75
The required probability $= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{3}{4}$	1M+1M 1A	1M for $(1 - p_3)$ + 1M for $p_3 = (a)p_4$, $0 < p_3, p_4 < 1$ 0.75
(d) The required probability $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{1}{8}$	1M 1A -----(2)	for (a) $p_4 \left(\frac{1}{2}\right)$, $0 < p_4 < 1$ 0.125

Solution	Marks	Remarks
<p>12. (a) Since the volume of the right circular cone is equal to the volume of the hemisphere, we have</p> $\frac{1}{3}\pi(h-4)^2h = \frac{2}{3}\pi(h-4)^3$ $h = 2(h-4) \quad (\because h \neq 4)$ $h = 8$	<p>1M+1M</p> <p>1A</p> <p>------(3)</p>	<p>1M for $\frac{\pi}{3}r^2h$ + 1M for $\frac{2}{3}\pi r^3$</p> <p>u-1 for having unit</p>
<p>(b) The length of the slant edge of the right circular cone</p> $= \sqrt{8^2 + 4^2}$ $= \sqrt{80}$ $= 4\sqrt{5} \text{ cm}$	<p>1M</p>	<p>for $\sqrt{(a)^2 + ((a)-4)^2}$</p>
<p>The total surface area of the solid</p> <p>= the curved surface area of the cone + the surface area of the hemisphere</p> $= \pi(4)(4\sqrt{5}) + 2\pi(4^2)$ $= 16(\sqrt{5} + 2)\pi$ ≈ 212.9280006 $\approx 213 \text{ cm}^2$	<p>1M</p> <p>1A</p> <p>------(3)</p>	<p>for $\pi((a)-4)l + 2\pi((a)-4)^2$</p> <p>u-1 for missing unit</p>
<p>(c) The increase in the total surface area</p> $= 2\left(\frac{(8)(8)}{2} + \frac{\pi(4^2)}{2}\right)$ ≈ 114.2654825 $\approx 114 \text{ cm}^2$	<p>1M</p> <p>1A</p> <p>------(2)</p>	<p>for $\left(\frac{(a)(2(a)-8)}{2} + \frac{\pi((a)-4)^2}{2}\right)$</p> <p>u-1 for missing unit</p>

Solution	Marks	Remarks
<p>13. (a) Putting $y=0$ in $2x-y+4=0$, we have $x=-2$. Thus, the coordinates of A are $(-2, 0)$. Putting $x=0$ in $2x-y+4=0$, we have $y=4$. Thus, the coordinates of B are $(0, 4)$.</p> <p>(b) \therefore the slope of L_1 is 2. \therefore the slope of L_2 is $\frac{-1}{2}$. Thus, the equation of L_2 is $y = \frac{-x}{2} + 4$$x + 2y - 8 = 0$</p> <p>(c) Putting $y=0$ in $x+2y-8=0$, we have $x=8$. So, the coordinates of C are $(8, 0)$. Therefore, we have $OC:AC=4:5$. The required ratio $= 4^2:(5^2-4^2)$ $= 16:9$</p>	<p>1A</p> <p>1A</p> <p>------(2)</p> <p>1M</p> <p>1M+1A</p> <p>------(3)</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>pp-1 for missing '(' or ')'</p> <p>pp-1 for missing '(' or ')'</p> <p>can be absorbed</p> <p>1M for slope-intercept form or point-slope form + 1A or equivalent</p> <p>for finding the coordinates of C</p> <p>can be absorbed</p> <p>for $OC^2:(AC^2-OC^2)$ accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
<p>Putting $y=0$ in $x+2y-8=0$, we have $x=8$. So, the coordinates of C are $(8, 0)$. Let the coordinates of D be (a, b). Then, we have $b=2a$ and $a+2b-8=0$. Solving, the coordinates of D are $(1.6, 3.2)$. The area of $\triangle ODC$ $= \frac{(8)(3.2)}{2}$$= 12.8$ The area of $\triangle ABC$ $= \frac{(10)(4)}{2}$$= 20$ The required ratio $= 12.8:(20-12.8)$ $= 12.8:7.2$ $= 16:9$</p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>for finding the coordinates of C</p> <p>can be absorbed</p> <p>accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
<p>Let the coordinates of D be (a, b). Then, we have $b=2a$ and $a+2b-8=0$. Solving, the coordinates of D are $(1.6, 3.2)$. $OD^2 = (1.6)^2 + (3.2)^2 = 12.8$ and $AB^2 = 2^2 + 4^2 = 20$ The required ratio $= 12.8:(20-12.8)$ $= 16:9$</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>can be absorbed for either one</p> <p>accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
	<p>------(4)</p>	

Solution	Marks	Remarks
14. (a) $\sin 30^\circ = \frac{BE}{120}$ $BE = 60 \text{ cm}$ $\cos 30^\circ = \frac{CE}{120}$ $CE = 60\sqrt{3} \text{ cm}$	1A 1A -----(2)	r.t. 104 cm $CE \approx 103.9230485 \text{ cm}$
(b) By sine formula, we have $\frac{AB}{\sin 40^\circ} = \frac{120}{\sin 60^\circ}$ $AB \approx 89.06726388$ $AB \approx 89.1 \text{ cm}$ and $\frac{AC}{\sin 80^\circ} = \frac{120}{\sin 60^\circ}$ $AC \approx 136.4589651$ $AC \approx 136 \text{ cm}$	1M 1A+1A -----(3)	for either one AB r.t. 89.1 cm AC r.t. 136 cm
(c) $CD = \sqrt{AC^2 - AD^2}$ $CD \approx \sqrt{136.4589651^2 - 100^2}$ $CD \approx 92.84960504 \text{ cm}$ $DE = \sqrt{AB^2 - (AD - BE)^2}$ $DE \approx \sqrt{89.06726388^2 - (100 - 60)^2}$ (by (a)) $DE \approx 79.58000688 \text{ cm}$	1M 1M	for $\sqrt{AB^2 - (100 - BE)^2}$
By cosine formula, we have $\cos \angle CDE = \frac{DE^2 + CD^2 - CE^2}{2DE \cdot CD}$ $\cos \angle CDE \approx \frac{79.58000688^2 + 92.84960504^2 - (60\sqrt{3})^2}{2(79.58000688)(92.84960504)}$ (by (a)) $\angle CDE \approx 73.67434913^\circ$ $\angle CDE \approx 73.7^\circ$	1M 1A	r.t. 73.7°
The required distance $= CD \sin \angle CDE$ $\approx 92.84960504 \sin 73.67434913^\circ$ ≈ 89.10586658 $\approx 89.1 \text{ cm}$	1M 1A	r.t. 89.1 cm
Let $x \text{ cm}$ be the shortest distance from C to DE . Then, we have $\frac{1}{2}(x)(DE) = \frac{1}{2}(CD)(DE) \sin \angle CDE$ $x = CD \sin \angle CDE$ $x \approx 92.84960504 \sin 73.67434913^\circ$ $x \approx 89.10586658$ $x \approx 89.1$ Thus, the required distance is 89.1 cm.	1M 1A	r.t. 89.1
	-----(6)	

Solution	Marks	Remarks
<p>15. (a) The mean = 122 marks</p> <p>The mean deviation $\frac{38+36+32+29+22+19+(3)(2)+1+(2)(12)+14+15+22+(3)(24)+36}{20}$ = 18.3 marks</p> <p>The standard deviation = 22 marks</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>------(4)</p>	
<p>(b) The total number of the top 20% students in the music test = (20)(20%) = 4</p> <p>The least score for the top 20% students in the music test = 146 marks</p> <p>The score obtained by Mary = 122 + (22)(1) = 144 < 146</p> <p>Thus, Mary is not one of the top 20% students in the music test.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>can be absorbed</p> <p>must show reasons</p>
<p>The total number of the top 20% students in the music test = (20)(20%) = 4</p> <p>The least score for the top 20% students in the music test = 146 marks</p> <p>The standard score of the least score for the top 20% students in the music test $= \frac{146 - 122}{22}$ $= \frac{12}{11}$ > 1</p> <p>Thus, Mary is not one of the top 20% students in the music test.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>can be absorbed</p> <p>must show reasons</p>
<p>(c) (i) The required probability = $\frac{1}{20}$</p> <p>(ii) The required probability $= 2 \left(\left(\frac{1}{20}\right)\left(\frac{1}{19}\right) + \left(\frac{1}{20}\right)\left(\frac{1}{19}\right) \right)$ $= \frac{1}{95}$ </p>	<p>1A</p> <p>1M+1M</p> <p>1A</p> <p>------(4)</p>	<p>0.05</p> <p>1M for $\left(\frac{1}{n}\right)\left(\frac{1}{n-1}\right)$ where $n \geq 10$ + 1M for the four cases r.t. 0.0105</p>

Solution	Marks	Remarks
<p>16. (a) (i) The required interest $= (200\ 000)\left(\frac{6\%}{12}\right)$ $= \\$1\ 000$</p> <p>(ii) The required amount $= 200\ 000 + 1\ 000 - x$ $= \\$ (201\ 000 - x)$</p> <p>(iii) The required amount $= 200\ 000\left(1 + \frac{6\%}{12}\right)^n - x\left(1 + \frac{6\%}{12}\right)^{n-1} - x\left(1 + \frac{6\%}{12}\right)^{n-2} - \dots - x$ $= 200\ 000(1.005)^n - x[(1.005)^{n-1} + (1.005)^{n-2} + \dots + 1]$ $= 200\ 000(1.005)^n - x\left[\frac{(1.005)^n - 1}{1.005 - 1}\right]$ $= \\$ \{200\ 000(1.005)^n - 200x[(1.005)^n - 1]\}$</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1</p> <p>----- (6)</p>	<p></p> <p>pp-1 for missing '(' or ')'</p> <p>for sum of GP</p> <p>pp-1 for missing '(', ')', '[',]', '{' or '}'</p>
<p>(b) (i) Assume that Peter has not yet fully repaid the loan after paying the nth instalment but the loan is fully repaid after Peter has paid the $(n+1)$th instalment. Then, by (a)(iii),</p> $0 < 200\ 000(1.005)^n - (200)(1\ 800)[(1.005)^n - 1] \leq \frac{1\ 800}{1 + \frac{6\%}{12}}$ $360\ 000 \leq 160\ 000(1.005)^{n+1} < 360\ 000(1.005)$ $2.25 \leq (1.005)^{n+1} < 2.26125$ $\frac{\log(2.25)}{\log(1.005)} \leq n+1 < \frac{\log(2.26125)}{\log(1.005)}$ $162.5911713 \leq n+1 < 163.5911713$ <p>Thus, the required number of the months is 163 .</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>accept either inequality</p> <p>for taking log</p>
<p>Let the required time be n months. By (a)(iii), we have</p> $200\ 000(1.005)^n - (200)(1\ 800)[(1.005)^n - 1] \leq 0$ $360\ 000 \leq 160\ 000(1.005)^n$ $(1.005)^n \geq \frac{9}{4}$ $n \geq \frac{\log(2.25)}{\log(1.005)}$ $n \geq 162.5911713$ <p>Thus, the required number of the months is 163 .</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>accept using (a)(iii) = 0</p> <p>for taking log</p>
<p>(ii) By(a)(i), the monthly instalment of \$ 900 is less than the loan interest of \$1 000 for the 1st month. Therefore, the loan can never be fully repaid. Thus, the bank refuses his request.</p>	<p>1M</p> <p>1A</p>	<p>for comparing the result of (a)(i)</p>
<p>If the loan can be fully repaid in m months, then by (a)(iii),</p> $200\ 000(1.005)^m - (200)(900)[(1.005)^m - 1] \leq 0$ $180\ 000 \leq -20\ 000(1.005)^m$ <p>which has no solution. Therefore, the loan can never be fully repaid. Thus, the bank refuses his request.</p>	<p>1M</p> <p>1A</p>	<p>for mentioning no solution</p>
	<p>----- (5)</p>	

Solution	Marks	Remarks						
<p>17. (a) (i) Note that $\angle QRP = 90^\circ$ (\angle in semi-circle)</p> <p>In $\triangle OQR$ and $\triangle ORP$,</p> <p>$\therefore \angle QRO = 90^\circ - \angle PRO$</p> <p>$\angle RPO = 90^\circ - \angle PRO$ (\angle sum of Δ)</p> <p>$\therefore \angle QRO = \angle RPO$</p> <p>$\angle QOR = 90^\circ = \angle ROP$ (given)</p> <p>$\angle OQR = \angle ORP$ (\angle sum of Δ)</p> <p>Therefore, $\triangle OQR \sim \triangle ORP$ (AAA)</p> <p>So, we have $\frac{OR}{OQ} = \frac{OP}{OR}$.</p> <p>Thus, we can conclude that $OR^2 = OP \cdot OQ$.</p>		<p>[半圓上的圓周角]</p> <p>[Δ內角和]</p> <p>[已知]</p> <p>[Δ內角和]</p> <p>[等角] (AA) (equiangular)</p>						
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Case 1 Any correct proof with correct reasons.	3							
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Case 3 Incomplete proof with any one correct step and one correct reason.	1							
<p>(ii) In $\triangle MON$ and $\triangle POR$,</p> <p>$\angle MNO = \angle PRO$ (\angles in the same segment)</p> <p>$\therefore \angle MON = 90^\circ$ (\angle in semi-circle)</p> <p>$\angle POR = 90^\circ$ (given)</p> <p>$\therefore \angle MON = \angle POR$</p> <p>$\angle OMN = \angle OPR$ (\angle sum of Δ)</p> <p>Therefore, $\triangle MON \sim \triangle POR$ (AAA)</p>		<p>[同弓形內的圓周角]</p> <p>[對同弧的圓周角]</p> <p>[半圓上的圓周角]</p> <p>[已知]</p> <p>[Δ內角和]</p> <p>[等角] (AA) (equiangular)</p>						
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Case 1 Any correct proof with correct reasons.	2							
Case 2 Any correct proof without reasons.	1							
<p>(b) (i) By (a)(i), we have $OR^2 = OP \cdot OQ$</p> <p>$OR^2 = (4)(9)$</p> <p>$OR = 6$</p> <p>Thus, the coordinates of R are $(0, 6)$.</p>	1A	pp-1 for missing '(' or ')'						
<p>(ii) Note that $PR = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}$.</p> <p>By (a)(ii), we have $\frac{MN}{2\sqrt{13}} = \frac{3\sqrt{13}}{6}$.</p> <p>Therefore, we have $MN = \frac{13}{2}$.</p> <p>Hence, the radius of the circle $MONR$ is $\frac{13}{4}$.</p> <p>Let the coordinates of the centre of the circle $MONR$ be (a, b).</p> <p>Then, we have</p> <p>$b = \frac{OR}{2} = \frac{6}{2} = 3$ and</p> <p>$a = -\sqrt{\left(\frac{13}{4}\right)^2 - 3^2} = \frac{-5}{4}$.</p> <p>So, the coordinates of the centre of the circle $MONR$ are $\left(\frac{-5}{4}, 3\right)$.</p>	1M 1M 1A	3.25 -1.25 pp-1 for missing '(' or ')'						
<p>------(5)</p> <p>------(6)</p>								