Solution	Marks	Remarks
1. $mx = 2(m+c)$		
$mx = 2m + 2c$		
$mx - 2m = 2c$	1M	for putting m on one side
$m(x-2) = 2c$	1M	for factorization
$m = \dfrac{2c}{x-2}$	1A	
	---------(3)	
2. For $\dfrac{3-5x}{4} \ge 2-x$, we have		
$3-5x \ge 4(2-x)$		
$3-5x \ge 8-4x$		
$4x-5x \ge 8-3$	1M	for putting x on one side
$-x \ge 5$		
$x \le -5$	1A	
For $x+8>0$, we have		
$x > -8$		
So, the required solution is		
$x > -8$ and $x \le -5$.	1A	do not accept graphical solution
Thus, the required solution is		
$-8 < x \le -5$.		
	---------(3)	
3. (a) $x^2-(y-z)^2$		
$=(x+(y-z))(x-(y-z))$		
$=(x+y-z)(x-y+z)$	1A	
(b) $ab-ad-bc+cd$		
$=a(b-d)-c(b-d)$ [$b(a-c)-d(a-c)$]	1M	for taking out common factors
$=(a-c)(b-d)$	1A	
	---------(3)	
4. $4^{x+1}=8$		
$2^{2(x+1)}=2^3$	1M	for same base (2, 4 or 8 only)
$2^{2x+2}=2^3$		
$2x+2=3$	1M	for equating the powers
$2x=1$		
$x=\dfrac{1}{2}$	1A	
[Alternative] $4^{x+1}=8$		
$\log(4^{x+1})=\log 8$	1M	for taking log
$(x+1)\log 4=\log 8$		
$x+1=\dfrac{\log 8}{\log 4}$	1M	for putting log on one side
$x+1=\dfrac{3}{2}$		
$x=\dfrac{1}{2}$	1A	
	---------(3)	

Solution	Marks	Remarks
5. (a) The selling price of the handbag		
$= 400(1+20\%)(0.75)$ [$400(1+20\%)(1-25\%)$]	1A	
$= \$360$	1A	u–1 for missing unit
(b) Percentage loss		
$= \dfrac{400-360}{400} \times 100\%$	1M	accept without 100%
$= 10\%$	1A	
	(4)	
6. Let x be the number of first-class tickets sold. Then, the number of economy-class tickets sold is $3x$. Therefore, we have		
$x + 3x = 600$		
$4x = 600$	1A	
$x = 150$	1A	can be absorbed
The sum of money for the tickets sold		
$= (150)(850) + (3)(150)(500)$	1M	
$= \$352\,500$	1A	u–1 for missing unit r.t. \$353 000
[Alternative] The number of first-class tickets sold		
$= (600)\left(\dfrac{1}{1+3}\right)$	1A	
$= 150$		
The number of economy-class tickets sold		
$= 600 - 150$	1A	for either one and can be absorbed
$= 450$		
The sum of money for the tickets sold		
$= (150)(850) + (450)(500)$	1M	
$= \$352\,500$	1A	u–1 for missing unit r.t. \$353 000
	(4)	
7. (a) Common difference		
$= 5 - 2$		
$= 3$	1A	
The 10th term		
$= 2 + (10-1)(3)$		
$= 29$	1A	
(b) The sum of the first 10 terms		
$= \dfrac{10}{2}(2+29)$ [$\dfrac{10}{2}((2)(2)+(10-1)(3))$]	1M	for $\dfrac{10}{2}(2+(\text{a}))$
$= 155$	1A	
[Alternative] (a) Note that the arithmetic sequence is 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, … .	1A	
The 10th term $= 29$	1A	
(b) The sum of the first 10 terms		
$= 2+5+8+11+14+17+20+23+26+29$	1M	for $2+\cdots+(\text{a})$
$= 155$	1A	
	(4)	

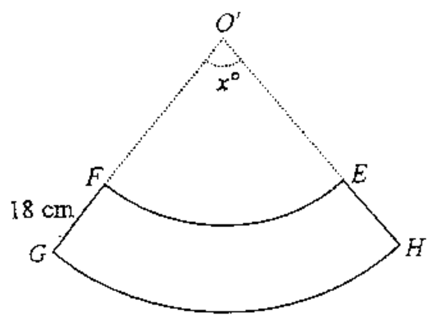
Solution	Marks	Remarks
8. (a) In ΔABC and ΔCDA,		
$\angle CAB = \angle ACD$ (alternate $\angle$s, $AB \parallel DC$)		[(內)錯角，$AB \parallel DC$]
$\angle ACB = \angle CAD$ (alternate $\angle$s, $BC \parallel AD$)		[(內)錯角，$BC \parallel AD$]
$AC = CA$ (common side)		[公共邊]
$\therefore$ $\Delta ABC \cong \Delta CDA$ (ASA)		
Marking Scheme :		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(b) $\Delta ABD \cong \Delta CDB$ $\Delta ABE \cong \Delta CDE$ $\Delta AED \cong \Delta CEB$		
Marking Scheme :		
Case 1 There are exactly three pairs of triangles and all of them are correct.	2	
Case 2 Any one pair is correct.	1	
	----------(4)	
9. (a) The shortest distance $= 100 \sin 60°$	1M	
$= 50\sqrt{3}$ ≈ 86.60254038 ≈ 87 km	1A	u–1 for missing unit
(b) The distance travelled by S between 1:00 a.m. and when it is nearest to L $= 100 \cos 60°$	1M	accept $\dfrac{87}{\tan 60°}$
$= 50$ km		
The distance travelled by S between 1:00 a.m. and when it is nearest to L $= \sqrt{100^2 - (50\sqrt{3})^2}$ $= \sqrt{2500}$ $= 50$ km	1M	for $\sqrt{100^2 - (a)^2}$
The time taken $= \dfrac{50}{20}$ $= 2.5$ hours	1M	
Therefore, S will be nearest to L at 3:30 a.m. .	1A	do not accept 2.5 hours after 1:00 a.m.
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $V = aL^2 + bL$, where a and b are constants.	1A	
When $L = 10$, $V = 30$, so we have $100a + 10b = 30$ $10a + b = 3$ .... (1) When $L = 15$, $V = 75$, so we have $225a + 15b = 75$ $15a + b = 5$ .... (2)	1M	for substitution (either one)
Solving (1) and (2), we have $\begin{cases} a = \dfrac{2}{5} \\ b = -1 \end{cases}$	1A	for both correct
$\therefore \quad V = \dfrac{2}{5}L^2 - L$	----------(3)	
(b) When $V \geq 30$, we have $\dfrac{2}{5}L^2 - L \geq 30$ (by(a)). Therefore, we have	1M	for putting the result of (a) into $V \geq 30$
$2L^2 - 5L - 150 \geq 0$	1M	in the form $k_1L^2 + k_2L + k_3 \geq 0$
$(2L + 15)(L - 10) \geq 0$ $L \geq 10$ or $L \leq -7.5$	1A	
Since $5 \leq L \leq 25$, we have $10 \leq L \leq 25$.	1A ----------(4)	accept '$L \geq 10$ and $L \leq 25$' but do not accept graphical solution

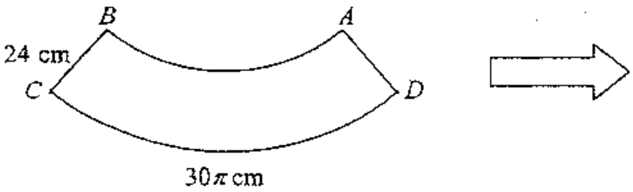
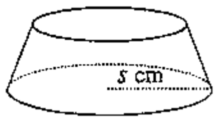
Solution	Marks	Remarks
11. (a) (i) The mode $= 10$	1A	
(ii) The median $= \frac{11+12}{2}$ $= 11.5$	1A	
(iii) The mean $= \frac{10+10+11+12+13+16}{6}$ $= 12$	1A	
(iv) The range $= 16-10$ $= 6$	1A	
	----------(4)	
(b) (i) The median will be the least when the four unknown data are at most equal to 10 . The least possible value of the median $= \frac{10+10}{2}$ $= 10$	1A	
The median will be the greatest when the four unknown data are at least equal to 16 . The greatest possible value of the median $= \frac{13+16}{2}$ $= 14.5$	1A	
(ii) The required mean $= \frac{(12)(6)+(11)(4)}{6+4}$	1M	for $(11)(4)$ = sum of the four unknown data
$= 11.6$	1A	
	----------(4)	

Solution	Marks	Remarks
12. (a) The slope of BC		
$= \frac{3-0}{0-2}$		
$= \frac{-3}{2}$	1A	accept -1.5 or $-1\frac{1}{2}$
	----------(1)	
(b) The slope of AP		
$= \frac{-1}{-1.5}$	1M	can be absorbed
$= \frac{2}{3}$		
The equation of AP is:		
$\frac{y-0}{x-(-1)} = \frac{2}{3}$	1M	for point-slope form
$2x-3y+2=0$	1A	accept $y = \frac{2x}{3} + \frac{2}{3}$
	----------(3)	
(c) (i) Let the coordinates of H be $(0, h)$.		
Then, by (b), $2(0)-3h+2=0$	1M	for putting $x=0$ into (b)
$h = \frac{2}{3}$	1A	
Thus, the coordinates of H are $(0, \frac{2}{3})$.		
(ii) The slope of $AC = \frac{3-0}{0-(-1)} = 3$	1A	
Suppose the altitude from B to AC cuts AC at Q.		
The slope of $BQ = \frac{-1}{3}$	1M	or, the slope of $BH = \frac{0-\frac{2}{3}}{2-0} = \frac{-1}{3}$
The equation of BQ is: $\frac{y-0}{x-2} = \frac{-1}{3}$		
$x+3y-2=0$		
Note that $0+(3)(\frac{2}{3})-2 = 2-2 = 0$		
Hence, the three altitudes pass through the same point H.	1	
Note that the slope of $BH = \frac{0-\frac{2}{3}}{2-0} = \frac{-1}{3}$	1M	
and the slope of $AC = \frac{3-0}{0-(-1)} = 3$	1A	
$\because$ (the slope of BH)(the slope of AC) $= (\frac{-1}{3})(3) = -1$		
$\therefore$ $BH \perp AC$		
Hence, the three altitudes pass through the same point H.	1	
	----------(5)	

Solution	Marks	Remarks
13. (a) (i) $\frac{x}{360}=\frac{30\pi}{(2\pi)(56+24)}$	1M	for $\frac{x}{360}=\frac{30\pi}{2\pi r}$
$x=67.5$	1A	u−1 for having unit
$30\pi=(56+24)(\frac{x\pi}{180})$	1M	for $30\pi=r\left(\frac{x\pi}{180}\right)$
$x=67.5$	1A	u−1 for having unit
(ii) The required area = area of sector ODC − area of sector OAB $=\left(\frac{67.5}{360}\right)\left((56+24)^2\pi\right)-\left(\frac{67.5}{360}\right)\left(56^2\pi\right)$	1M	for either one
$=1200\pi-588\pi$ $=612\pi\text{ cm}^2$	1A	u−1 for missing unit
The required area = area of sector ODC − area of sector OAB $=\frac{1}{2}(56+24)^2\left(\frac{67.5\pi}{180}\right)-\frac{1}{2}(56^2)\left(\frac{67.5\pi}{180}\right)$	1M	for either one
$=1200\pi-588\pi$ $=612\pi\text{ cm}^2$	1A	u−1 for missing unit
	----------(4)	
(b) (i) The required area $=(612\pi)(\frac{18}{24})^2$	1M	for $((a)(ii))(\frac{18}{24})^2$
$=\frac{1377}{4}\pi\text{ cm}^2$	1A	accept $344.25\pi\text{ cm}^2$ or $344\frac{1}{4}\pi\text{ cm}^2$ u−1 for missing unit



Solution	Marks	Remarks
$\because\ \frac{FO'}{BO}=\frac{FG}{BC}$ $\therefore\ \frac{FO'}{56}=\frac{18}{24}$ Hence, $FO'=42$ cm		
The required area $=\frac{1}{2}(42+18)^2\left(\frac{67.5\pi}{180}\right)-\frac{1}{2}(42^2)\left(\frac{67.5\pi}{180}\right)$	1M	
$=\frac{1377}{4}\pi\text{ cm}^2$	1A	accept $344.25\pi\text{ cm}^2$ or $344\frac{1}{4}\pi\text{ cm}^2$ u−1 for missing unit

Solution	Marks	Remarks
(ii) $2\pi r = \left(\frac{18}{24}\right)(30\pi)$	1M+1M	1M for $\left(\frac{18}{24}\right)(30\pi)$ + 1M for equating $2\pi r$
$2\pi r = 22.5\pi$		
$r = \frac{45}{4}$	1A	accept 11.25 r.t. 11.3
		
$\because\ 2\pi s = 30\pi$	1M	for equating $2\pi s$
$\therefore\ s = 15$		
$\because\ r = \left(\frac{FG}{BC}\right)s$		
$\therefore\ r = \left(\frac{18}{24}\right)(15)$	1M	
Thus, $r = \frac{45}{4}$	1A	accept 11.25 r.t. 11.3
	---------(5)	

Solution	Marks	Remarks
14. (a) By cosine formula, we have		
$\cos\angle OAC = \dfrac{3^2+6^2-4^2}{(2)(3)(6)}$	1A	
$\angle OAC \approx 36.33605751°$		
$\angle OAC \approx 36.3°$	1A	u−1 for missing unit
	-----(2)	
(b) (i) $\tan 40° = \dfrac{BC}{4}$; $BC = 4\tan 40°$	1A	for either one correct
$BC \approx 3.356398525$		
$BC \approx 3.36$ m		
$\tan 30° = \dfrac{4\tan 40°}{CD}$	1M	accept $\tan 30° \approx \dfrac{3.36}{CD}$
$CD \approx 5.813452775$		
$CD \approx 5.81$ m	1A	u−1 for missing unit r.t. 5.81
(ii) By cosine formula, we have		
$\cos\angle CAD = \dfrac{6^2+8^2-CD^2}{(2)(8)(6)}$	1M	with CD substituted
$\cos\angle CAD \approx 0.689622571$		
$\angle CAD \approx 46.39976047°$		
$\angle CAD \approx 46.4°$	1A	u−1 for missing unit r.t. 46.4°
(iii) By sine formula, we have		
$\dfrac{CE}{\sin\angle EAC} = \dfrac{6}{\sin\theta}$ and $\dfrac{ED}{\sin\angle EAD} = \dfrac{8}{\sin(180°-\theta)}$	1M	for either one with angle substituted
So, $\dfrac{6\sin 36.33605751°}{\sin\theta} + \dfrac{8\sin 10.06370296°}{\sin(180°-\theta)} \approx CD$	1M	with CD substituted
$\dfrac{3.5551215}{\sin\theta} + \dfrac{1.397944043}{\sin\theta} \approx 5.813452775$		
$\sin\theta \approx 0.85200065$	1M	for making $\sin\theta$ the subject
$\theta \approx 58.42994248°$		
$\theta \approx 58.4°$ ($\because \theta$ is acute)	1A	u−1 for missing unit r.t. 58.4°
By cosine formula, we have		
$\cos\angle ACD = \dfrac{6^2+CD^2-8^2}{(2)(6)(CD)}$	2M	with CD substituted
$\cos\angle ACD \approx 0.083086497$		
$\angle ACD \approx 85.23400001°$		
$\because$ $\angle EAC + \angle ACD + \theta = 180°$ ($\angle$ sum of Δ)		
$36.33605751° + 85.23400001° + \theta = 180°$	1M	
$\therefore$ $\theta \approx 58.42994248°$		
Thus, $\theta \approx 58.4°$	1A	u−1 for missing unit r.t. 58.4°
	-----(9)	

Solution	Marks	Remarks
15. (a) (i) The required area $= \frac{1}{2}k(1-k)\sin 60^\circ$	1M	for $\frac{1}{2}ab\sin 60^\circ$
$= \frac{\sqrt{3}}{4}k(1-k)$ m^2	1A	u−1 for missing unit
(ii) By cosine formula, we have $x^2 = k^2 + (1-k)^2 - 2k(1-k)\cos 60^\circ$	1M	for $x^2 = a^2 + b^2 - 2ab\cos 60^\circ$
$x^2 = 3k^2 - 3k + 1$ $x = \sqrt{3k^2 - 3k + 1}$	1A	
(iii) By symmetry, $A_1B_1 = B_1C_1 = C_1A_1 = x$ m . Thus, $A_1B_1C_1$ is an equilateral triangle.	1	
	----------(5)	
(b) (i) $\because \frac{A_2B_1}{A_1B_0} = \frac{x(1-k)}{1-k} = x = \frac{xk}{k} = \frac{B_2B_1}{B_1B_0}$ $\angle A_2B_1B_2 = 60^\circ = \angle A_1B_0B_1$ $\therefore \Delta A_1B_0B_1 \sim \Delta A_2B_1B_2$ (ratio of 2 sides, inc.∠)		[兩邊成比例且夾角相等]
Marking Scheme :		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(ii) $\because \Delta A_1B_0B_1 \sim \Delta A_2B_1B_2 \sim \Delta A_3B_2B_3 \sim \ldots$ $\therefore$ their areas form a geometric sequence with a common ratio x^2.	1A	can be absorbed
So, the total area $= \frac{\sqrt{3}}{4}k(1-k) + \frac{\sqrt{3}}{4}k(1-k)x^2 + \frac{\sqrt{3}}{4}k(1-k)x^4 + \cdots$		
$= \dfrac{\frac{\sqrt{3}}{4}k(1-k)}{1-x^2}$	1M	for $\frac{\text{(a)(i)}}{1-r}$
$= \dfrac{\frac{\sqrt{3}}{4}k(1-k)}{3k-3k^2}$ (by (a)(ii))	1M	
$= \frac{\sqrt{3}}{12}$ m^2	1A	u−1 for missing unit
	----------(6)	

Solution	Marks	Remarks
16. (a) The required probability $= \left(1-\frac{1}{10}\right)\left(\frac{1}{2}\right)$	1M	for $\left(1-\frac{1}{10}\right)P_1$, where $0<P_1<1$
$= \frac{9}{20}$	1A	0.45
	----------(2)	
(b) (i) The required probability $= \left(1-\frac{2}{25}\right)\left(\frac{1}{2}\right)$	1M	for $\left(1-\frac{2}{25}\right)P_2$, where $0<P_2<1$
$= \frac{23}{50}$	1A	0.46
(ii) (1) The required probability $= \left(\frac{2}{3}\right)\left(\frac{9}{20}\right)+\left(\frac{1}{3}\right)\left(\frac{23}{50}\right)$	1M+1M+1A	1M for $\left(\frac{2}{3}\right)$(a) + 1M for $\left(\frac{1}{3}\right)$((b)(i))
$= \frac{34}{75}$	1A	r.t. 0.453

(2)

Transportation	Transportation Cost	Transportation Cost + \$15 Lunch
Bus and Train	\$12	\$27
Train and Train	\$15	\$30

Solution	Marks	Remarks
The required probability $= 1-\frac{34}{75}$	2M	for $1-$(b)(ii)(1)
$= \frac{41}{75}$	1A	r.t. 0.547
The required probability = P(John will spend more than a total of \$30) = P(John will spend more than a total of \$22.5 for the morning trip and lunch) $= \left(\frac{2}{3}\right)\left(\frac{1}{10}+\left(1-\frac{1}{10}\right)\left(\frac{1}{2}\right)\right)+\left(\frac{1}{3}\right)\left(\frac{2}{25}+\left(1-\frac{2}{25}\right)\left(\frac{1}{2}\right)\right)$		
$= \frac{1}{15}+\frac{3}{10}+\frac{2}{75}+\frac{23}{150}$	1A+1A	1A for either one correct + 1A for all correct
$= \frac{41}{75}$	1A	r.t. 0.547
The required probability = P(John will spend more than a total of \$30) = P(John will spend more than a total of \$22.5 for the morning trip and lunch) $= \left(\frac{2}{3}\right)\left(\frac{1}{10}\right)\left(\frac{1}{2}\right)+\left(\frac{2}{3}\right)\left(\frac{1}{10}\right)\left(\frac{1}{2}\right)+\left(\frac{2}{3}\right)\left(\frac{9}{10}\right)\left(\frac{1}{2}\right)+$ $\left(\frac{1}{3}\right)\left(\frac{2}{25}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{25}\right)\left(\frac{1}{2}\right)+\left(\frac{1}{3}\right)\left(\frac{23}{25}\right)\left(\frac{1}{2}\right)$		
$= \frac{1}{30}+\frac{1}{30}+\frac{3}{10}+\frac{1}{75}+\frac{1}{75}+\frac{23}{150}$	1A+1A	1A for either one correct + 1A for all correct
$= \frac{41}{75}$	1A	r.t. 0.547
	----------(9)	

Solution	Marks	Remarks
17. (a) (i) In ΔNPM and ΔNKP,		
$\angle NPM = \angle NKP$ ($\angle$ in alt. segment)		[交錯弓形的圓周角]、[弦切角定理]
$\angle MNP = \angle PNK$ (common angle)		[公共角]
$\angle NMP = \angle NPK$ ($\angle$ sum of Δ)		[△內角和]
$\therefore \Delta NPM \sim \Delta NKP$ (AAA)		[等角] (AA) (equiangular)
So, $\frac{NP}{NK} = \frac{NM}{NP}$		
Thus, we can conclude that $NP^2 = NK \cdot NM$.		

Marking Scheme :	
Case 1 Any correct proof with correct reasons.	3
Case 2 Any correct proof without reasons.	2
Case 3 Any one line except the first line and the conclusion.	1

Solution	Marks	Remarks
(ii) $\because NP^2 = NK \cdot NM$ and $ON^2 = NK \cdot NM$ (by (a)(i))		
$\therefore NP^2 = ON^2$		
$\therefore NP = ON$	1	
$\because RS \parallel OP$		
$\therefore \Delta KRM \sim \Delta KON$ (AAA) and $\Delta KMS \sim \Delta KNP$ (AAA)		
$\therefore \frac{RM}{ON} = \frac{KM}{KN}$ and $\frac{MS}{NP} = \frac{KM}{KN}$		
$\therefore \frac{RM}{ON} = \frac{MS}{NP}$	1	
$\therefore RM = MS$		
	----------(5)	
(b) (i) $\because FM = 2a$, $MG = 2(p-a)$	1A	for either one correct
$\therefore FG = 2a + 2(p-a)$ $= 2p$	1A	
$\because$ x-coordinate of F $= -a$; x-coordinate of G $= a + 2(p-a)$ $= 2p - a$	1A	for either one correct
$\therefore FG = (2p-a) - (-a)$ $= 2p$	1A	
(ii) $F = (-a, b)$	1A	
$\because FG = 2OP$ (by (b)(i)) and $FG \parallel OP$ (given)		
$\therefore O$ is the mid-point of F and Q.		
Thus, $Q = (a, -b)$	1A	
(iii) $\because$ x-coordinate of Q $= a =$ x-coordinate of M		
$\therefore MQ \perp RS$	1	
$\therefore RM = MS$ (by (a)(ii))		
$\therefore \Delta QMR \cong \Delta QMS$ (SAS)		
Thus, $QR = QS$		
Hence, ΔQRS is an isosceles triangle.	1	
	----------(6)	