2001-CE
MATH

PAPER 1

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2001

MATHEMATICS PAPER 1

Question-Answer Book

8.30 am – 10.30 am (2 hours)

This paper must be answered in English

1. Write your Candidate Number, Centre Number and Seat Number in the spaces provided on this cover.

2. This paper consists of THREE sections, A(1), A(2) and B. Each section carries 33 marks.

3. Attempt ALL questions in Sections A(1) and A(2), and any THREE questions in Section B. Write your answers in the spaces provided in this Question-Answer Book. Supplementary answer sheets will be supplied on request. Write your Candidate Number on each sheet and fasten them with string inside this book.

4. Write the question numbers of the questions you have attempted in Section B in the spaces provided on this cover.

5. Unless otherwise specified, all working must be clearly shown.

6. Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.

7. The diagrams in this paper are not necessarily drawn to scale.

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Candidate Number							
Centre Number							
Seat Number							

	Marker's Use Only	**Examiner's Use Only**
	Marker No.	Examiner No.
Section A Question No.	Marks	Marks
1–2		
3–4		
5–6		
7		
8–9		
10		
11		
12		
13		
Section A Total		

Checker's Use Only	Section A Total		

Section B Question No.*	Marks	Marks
Section B Total		

To be filled in by the candidate.

Checker's Use Only	Section B Total		

Checker No.	

Page total

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3}\times$ base area $\times$ height

SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Simplify $\dfrac{m^3}{(mn)^2}$ and express your answer with positive indices. (3 marks)

2. Let $f(x) = x^3 - x^2 + x - 1$. Find the remainder when $f(x)$ is divided by $x - 2$. (3 marks)

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3. Find the perimeter of the sector in Figure 1. (3 marks)

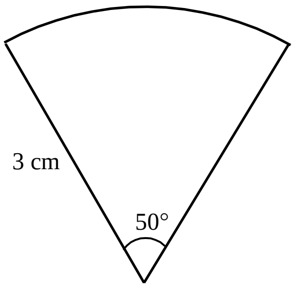

Figure 1

4. Solve $x^2 + x - 6 > 0$ and represent the solution in Figure 2. (3 marks)

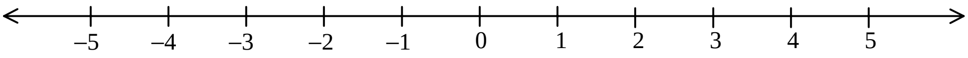

Figure 2

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5. In Figure 3, AC is a diameter of the circle. Find $\angle DAC$. (4 marks)

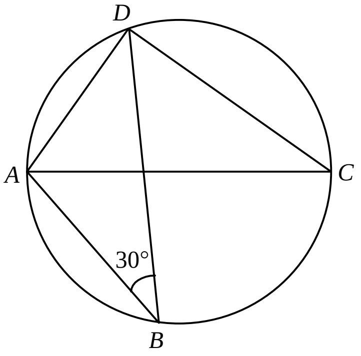

Figure 3

6. Make x the subject of the formula $y = \frac{1}{2}(x+3)$. (4 marks)

If the value of y is increased by 1 , find the corresponding increase in the value of x .

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7. Two points A and B are marked in Figure 4. (4 marks)

(a) Write down the coordinates of A and B.

(b) Find the equation of the straight line joining A and B.

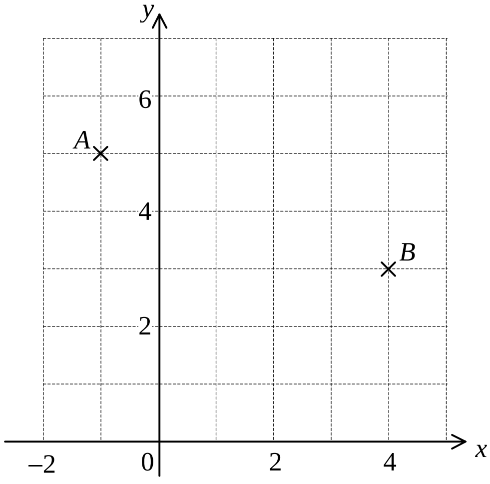

Figure 4

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8. The price of a textbook was $\$80$ last year. The price is increased by 20% this year. (4 marks)

(a) Find the new price.

(b) Peter is given a 20% discount when buying the textbook from a bookstore this year. How much does he pay for this book?

9. In Figure 5, find AB and the area of ΔABC. (5 marks)

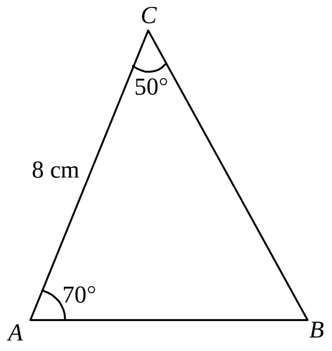

Figure 5

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Section A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. The histogram in Figure 6 shows the distribution of scores of a class of 40 students in a test.

Distribution of scores of 40 students

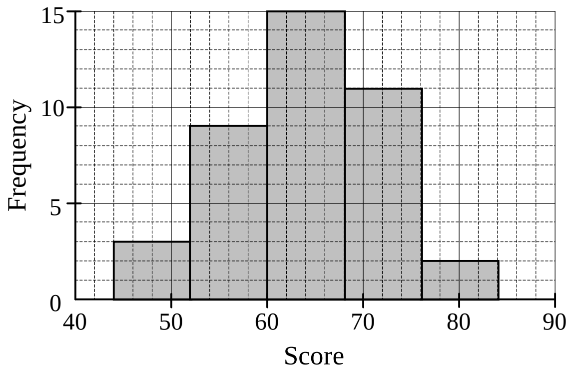

Figure 6

Table 1 Frequency distribution table for the scores of 40 students

Score (x)	Class mid-value (Class mark)	Frequency
$44 \le x < 52$		3
$52 \le x < 60$		
	64	15
$68 \le x < 76$		11
	80	

(a) Complete Table 1. (3 marks)

(b) Estimate the mean and standard deviation of the distribution. (2 marks)

(c) Susan scores 76 in this test. Find her standard score. (2 marks)

(d) Another test is given to the same class of students. It is found that the mean and standard deviation of the scores in this second test are 58 and 10 respectively. Relative to her classmates, if Susan performs equally well in these two tests, estimate her score in the second test. (2 marks)

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11. As shown in Figure 7, a piece of square paper $ABCD$ of side 12 cm is folded along a line segment PQ so that the vertex A coincides with the mid-point of the side BC. Let the new positions of A and D be A' and D' respectively, and denote by R the intersection of $A'D'$ and CD.

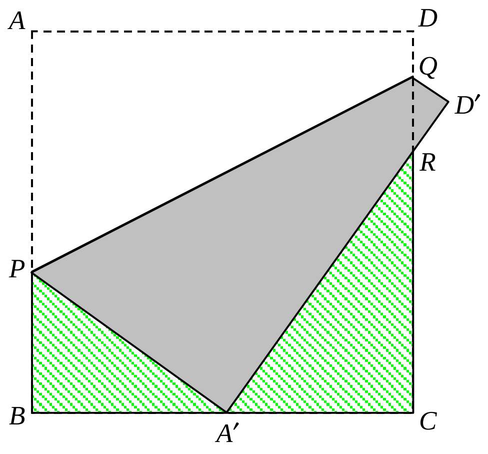

Figure 7

(a) Let the length of AP be x cm. By considering the triangle PBA', find x. (3 marks)

(b) Prove that the triangles PBA' and $A'CR$ are similar. (3 marks)

(c) Find the length of $A'R$. (2 marks)

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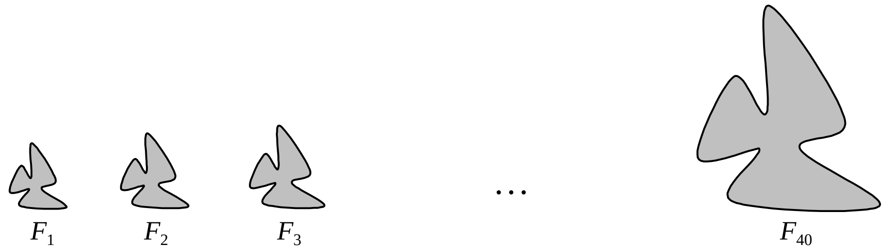
12. $F_1, F_2, F_3, \cdots, F_{40}$ as shown below are 40 similar figures. The perimeter of F_1 is 10 cm. The perimeter of each succeeding figure is 1 cm longer than that of the previous one.



(a) (i) Find the perimeter of F_{40}.

(ii) Find the sum of the perimeters of the 40 figures.

(4 marks)

(b) It is known that the area of F_1 is $4\ \text{cm}^2$.

(i) Find the area of F_2.

(ii) Determine with justification whether the areas of $F_1, F_2, F_3, \cdots, F_{40}$ form an arithmetic sequence?

(4 marks)

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13. S is the sum of two parts. One part varies as t and the other part varies as the square of t. The table below shows certain pairs of the values of S and t.

S	0	33	56	69	72	65	48	21
t	0	1	2	3	4	5	6	7

(a) Express S in terms of t. (3 marks)

(b) Find the value(s) of t when $S = 40$. (2 marks)

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(c) Using the data given in the table, plot the graph of S against t for $0 \leq t \leq 7$ in Figure 8 .

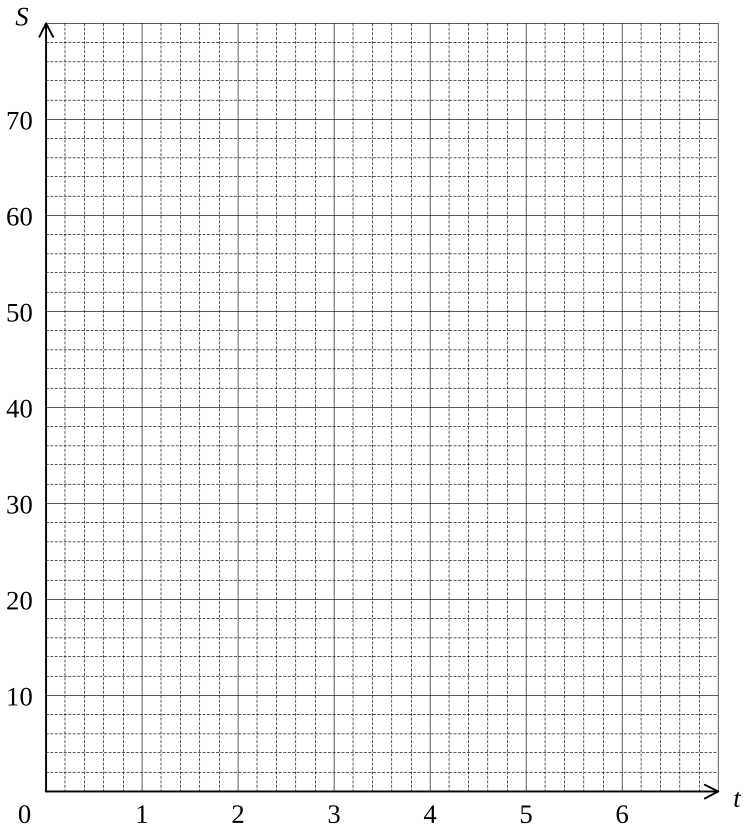

Figure 8

Read from the graph the value of t when the value of S is greatest.

(3 marks)

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SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

14. (a) Let $f(x) = x^5 - 6x + 5$.

(i) Complete Table 2.

(ii) It is known that the equation $f(x) = 0$ has only one root greater than 1 . Using (i) and the method of bisection, find this root correct to 3 decimal places.

(5 marks)

Table 2

x	$f(x)$
1	0
1.05	
1.1	
1.15	0.111

(b) From 1997 to 2000, Mr. Chan deposited \$1 000 in a bank at the beginning of each year at an interest rate of $r\%$ per annum, compounded yearly. For the money deposited, the amount accumulated at the beginning of 2001 was \$5 000 . Using (a), find r correct to 1 decimal place.

(6 marks)

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15. (a) In Figure 9, shade the region that represents the solution to the following constraints:

$$\begin{cases} 1 \le x \le 9, \\ 0 \le y \le 9, \\ 5x - 2y > 15. \end{cases}$$

(4 marks)

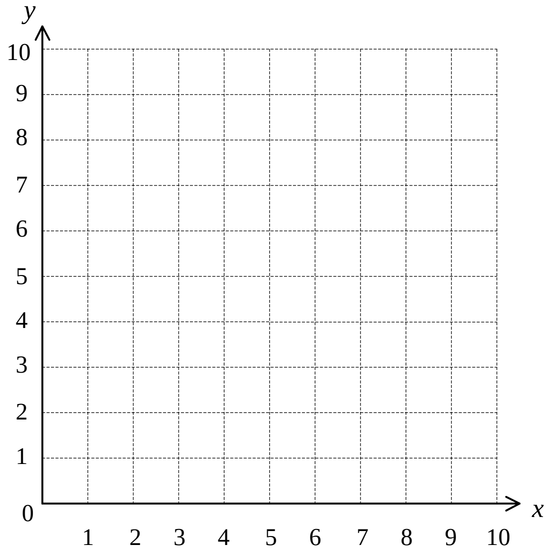

Figure 9

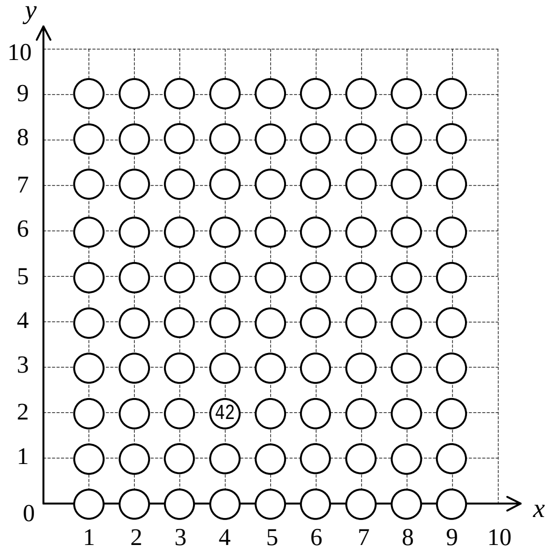

Figure 10

(b) A restaurant has 90 tables. Figure 10 shows its floor plan where a circle represents a table. Each table is assigned a 2-digit number from 10 to 99. A rectangular coordinate system is introduced to the floor plan such that the table numbered $10x + y$ is located at (x, y) where x is the tens digit and y is the units digit of the table number. The table numbered 42 has been marked in the figure as an illustration.

The restaurant is partitioned into two areas, one smoking and one non-smoking. Only those tables with the digits of their table numbers satisfying the constraints in (a) are in the smoking area.

(i) In Figure 10, shade all the circles which represent the tables in the smoking area.

(ii) Two tables are randomly selected, one after another and without replacement from the 90 tables. Find the probability that

(I) the first selected table is in the smoking area;

(II) of the two selected tables, one is in the smoking area, and the other is in the non-smoking area and its number is a multiple of 3.

(7 marks)

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16. Figure 11 shows a piece of pentagonal cardboard $ABCDE$. It is formed by cutting off two equilateral triangular parts, each of side x cm, from an equilateral triangular cardboard AFG. AB is 6 cm long and the area of $BCDE$ is $5\sqrt{3}$ cm^2.

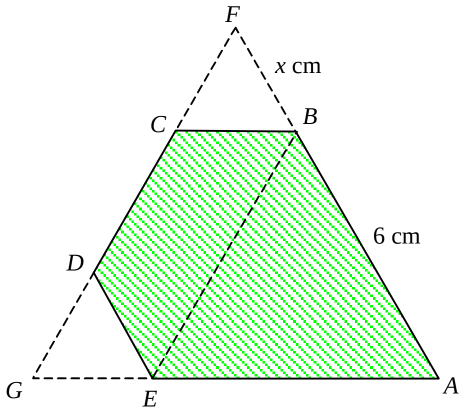

Figure 11

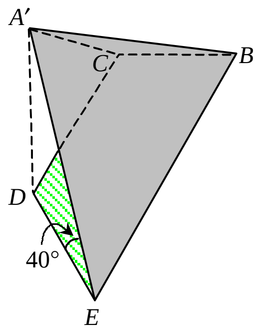

Figure 12

(a) Show that $x^2 - 12x + 20 = 0$.

Hence find x.

(4 marks)

(b) The triangular part ABE in Figure 11 is folded up along the line BE until the vertex A comes to the position A' (as shown in Figure 12) such that $\angle A'ED = 40°$.

(i) Find the length of $A'D$.

(ii) Find the angle between the planes $BCDE$ and $A'BE$.

(iii) If A', B, C, D, E are the vertices of a pyramid with base $BCDE$, find the volume of the pyramid.

(7 marks)

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17. (a) In Figure 13, OP is a diameter of the circle. The altitude QR of the acute-angled triangle OPQ cuts the circle at S. Let the coordinates of P and S be $(p, 0)$ and (a, b) respectively.

(i) Find the equation of the circle OPS.

(ii) Using (i) or otherwise, show that $OS^2 = OP \cdot OQ \cos \angle POQ$.

(7 marks)

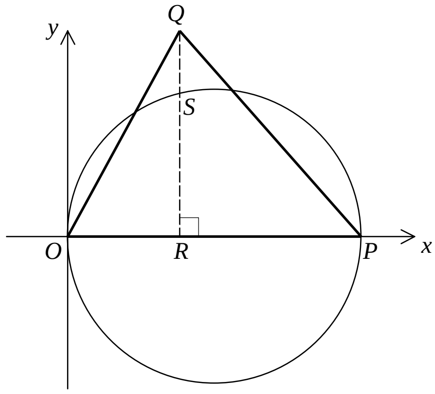

Figure 13

(b) In Figure 14, ABC is an acute-angled triangle. AC and BC are diameters of the circles $AGDC$ and $BCEF$ respectively.

(i) Show that BE is an altitude of ΔABC.

(ii) Using (a) or otherwise, compare the length of CF with that of CG. Justify your answer.

(4 marks)

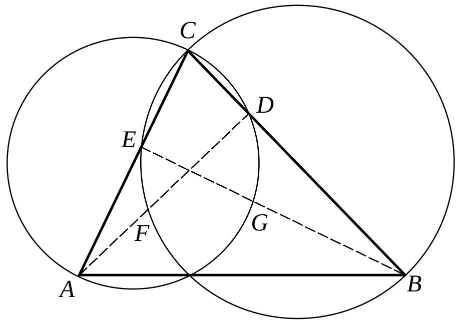

Figure 14

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END OF PAPER

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2001

Mathematics 1
Section A(1)

1. $\dfrac{m}{n^2}$

2. 5

3. 8.62 cm

4. $x < -3$ or $x > 2$

5. 60°

6. $x = 2y - 3$
   x will be increased by 2 if y is increased by 1 .

7. (a) $(-1, 5)$, $(4, 3)$

   (b) $2x + 5y - 23 = 0$

8. (a) $96

   (b) $76.8

9. 7.08 cm , 26.6 cm^2

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Section A(2)

10. (a)

Score (x)	Class mid-value (Class mark)	Frequency
$44 \le x < 52$	48	3
$52 \le x < 60$	56	9
$60 \le x < 68$	64	15
$68 \le x < 76$	72	11
$76 \le x < 84$	80	2

(b) Mean = 64
Standard deviation = 8

(c) Standard score $= \dfrac{76-64}{8}$

$= 1.5$

(d) Let her score in the second test be y, then

$$\frac{y-58}{10} = 1.5$$

$$y = 73$$

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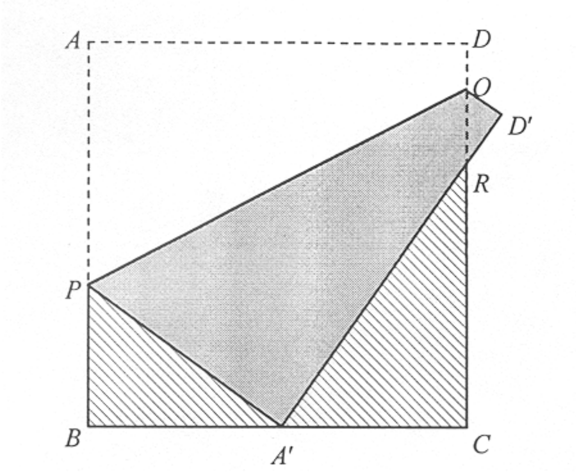
11.



(a) Since $A'P = x$ cm ,

$\therefore \quad (12-x)^2 + 6^2 = x^2$

$144 - 24x + x^2 + 36 = x^2$

$x = 7.5$

(b) In Δs PBA' and $A'CR$,

(i) $\angle PBA' = \angle A'CR = 90°$

Since $\angle A'PB + 90° + \angle BA'P = 180°$ ($\angle$ sum of Δ)

and $\angle RA'C + 90° + \angle BA'P = 180°$ (adj. $\angle$s on st. line)

$\therefore$ (ii) $\angle A'PB = \angle RA'C$

Hence $\Delta PBA' \sim \Delta A'CR$ (AAA)

(c) Let $A'R = y$ cm and use the result of (b),

$$\frac{A'R}{A'C} = \frac{PA'}{PB}$$

$$\frac{y}{6} = \frac{7.5}{12-7.5}$$

$y = 10$

i.e. $A'R = 10$ cm

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12. (a) (i) Perimeter of F_{40} $= [10 + (40 - 1) \times 1]$ cm
$= 49$ cm

(ii) The sum of the perimeters of the 40 figures
$= [\, 40 \times \dfrac{10 + 49}{2} \,]$ cm
$= 1180$ cm

(b) (i) Area of F_2 $= [\, 4 \times \left(\dfrac{11}{10}\right)^2 \,]$ cm^2
$= 4.84$ cm^2

(ii) Area of F_3 $= 4 \times \left(\dfrac{12}{10}\right)^2$ $\text{cm}^2 = 5.76$ cm^2

$\because$ Area of F_2 $-$ Area of F_1 $\neq$ Area of F_3 $-$ Area of F_2
($0.84 \text{ cm}^2 \neq 0.92 \text{ cm}^2$)

$\therefore$ the areas of figures $F_1, F_2, \cdots, F_{40}$ do not form an arithmetic sequence.

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13. (a) Let $S = at + bt^2$ for some non-zero constants a and b.

Solving $\begin{cases} 33 = a + b \\ 56 = 2a + 4b \end{cases}$, we have

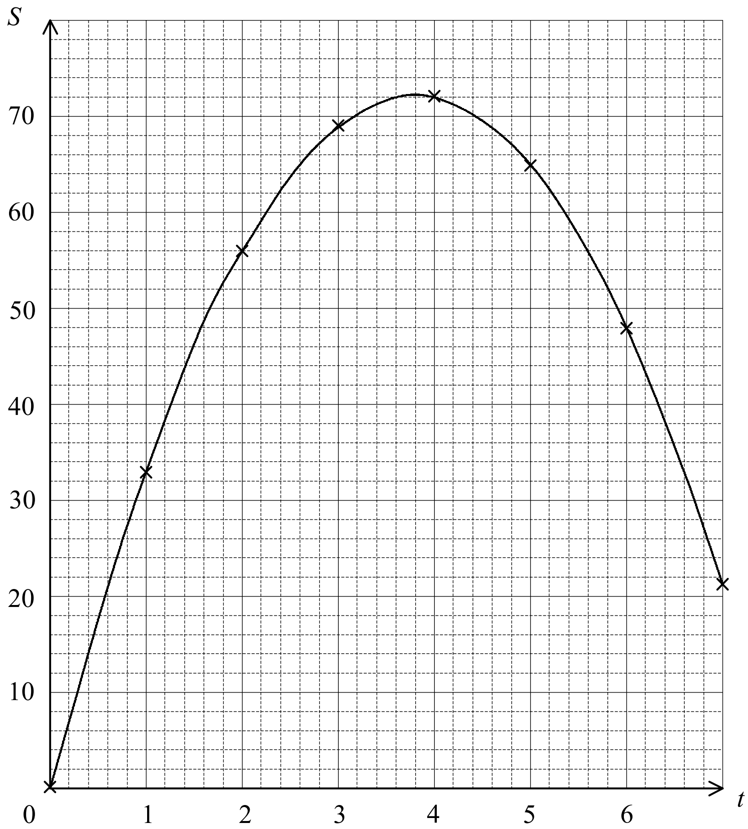
$a = 38$ and $b = -5$

$\therefore \quad S = 38t - 5t^2$

(b) When $S = 40$, $5t^2 - 38t + 40 = 0$

$t = 1.26$ or 6.34

(c)



From the graph, S is greatest when $t \approx 3.8$.

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Section B

14. (a) (i) -0.0237 , 0.0105

(ii) From (i) , the root lies in the interval [1.05, 1.1].
Using the method of bisection,

a [f(a) < 0]	b [f(b) > 0]	$m = \frac{a+b}{2}$	f(m)
1.0500	1.1000	1.0750	–0.0144
1.0750	1.1000	1.0875	–0.0039
1.0875	1.1000	1.0938	0.0028
1.0875	1.0938	1.0907	–0.0006
1.0907	1.0938	1.0923	0.0011
1.0907	1.0923	1.0915	0.0002
1.0907	1.0915		

$\therefore \quad 1.0907 < h < 1.0915$

$x \approx 1.091$ (correct to 3 decimal places)

(b) The given conditions lead to the equation

$1000(1+r\%)^4 + 1000(1+r\%)^3 + 1000(1+r\%)^2 + 1000(1+r\%) = 5000$

Let $x = 1+r\%$, then

$1000x^4 + 1000x^3 + 1000x^2 + 1000x = 5000$

$x^4 + x^3 + x^2 + x = 5$

$\frac{x(x^4-1)}{x-1} = 5$

$x^5 - x = 5x - 5$

$x^5 - 6x + 5 = 0$

From (a), $x \approx 1.091$

i.e. $r \approx 9.1$

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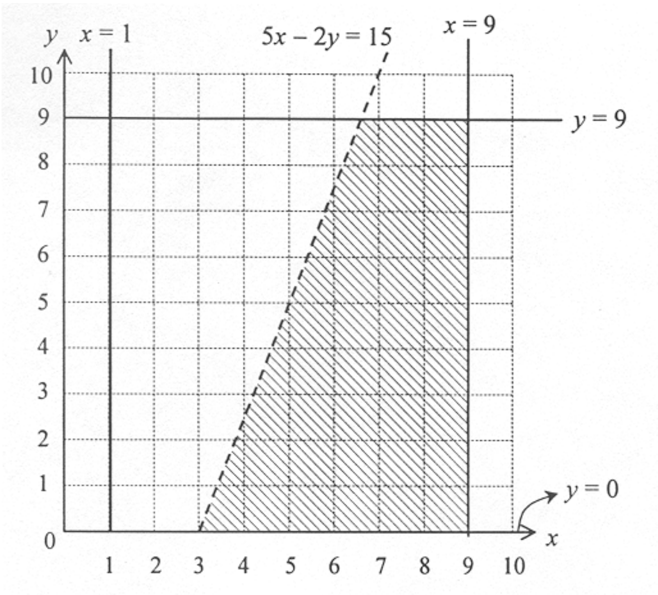
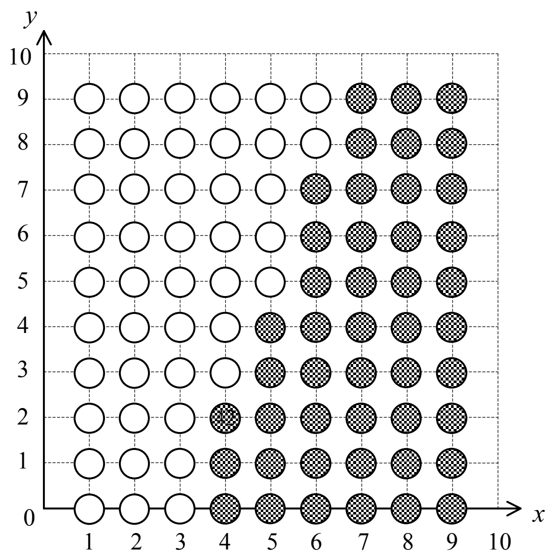
15. (a)



(b) (i)



(ii) (I) Required probability $= \frac{46}{90} = \frac{23}{45}$

(II) Required probability $= \frac{46}{90} \times \frac{14}{89} + \frac{14}{90} \times \frac{46}{89} = \frac{644}{4005}$

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16. (a) In the trapezium $BCDE$,

$$\text{height} = x\sin 60^\circ \text{ cm} = \frac{\sqrt{3}}{2}x \text{ cm}$$

$$CD = (6-x) \text{ cm}$$

$$\therefore \quad \frac{6+(6-x)}{2}\times\frac{\sqrt{3}}{2}x = 5\sqrt{3}$$

$$\frac{\sqrt{3}(12-x)x}{4} = 5\sqrt{3}$$

$$x^2 - 12x + 20 = 0$$

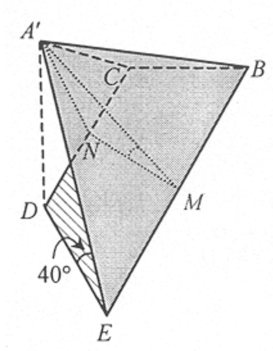
$$(x-2)(x-10) = 0$$

$$x = 2 \text{ or } x = 10 \text{ (rejected)}$$

(b) (i) $A'D^2 = [6^2 + 2^2 - 2(6)(2)\cos 40^\circ] \text{ cm}^2 \approx 21.6149 \text{ cm}^2$

$A'D \approx 4.65$ cm

(ii) Let M, N be the mid-points of EB and DC respectively, then



$A'M = 6\sin 60^\circ \text{ cm} = 3\sqrt{3}$ cm ,

$MN = 2\sin 60^\circ \text{ cm} = \sqrt{3}$ cm , and

$$A'N = \sqrt{A'D^2 - DN^2} \approx \sqrt{21.6149 - 2^2} \text{ cm} \approx \sqrt{17.6149} \text{ cm}$$

The angle between the planes $BCDE$ and $A'BE$ is $\angle A'MN$.

$$\cos\angle A'MN \approx \frac{(3\sqrt{3})^2 + (\sqrt{3})^2 - 17.6149}{2(3\sqrt{3})(\sqrt{3})} \approx 0.6881$$

$\angle A'MN \approx 46.5^\circ$

(iii) Required volume $= \frac{1}{3}$(area of trapezium $CDEB$)($A'M \sin\angle A'MN$)

$$\approx \frac{1}{3}(5\sqrt{3})(3\sqrt{3}\sin 46.5^\circ) \text{ cm}^3$$

$$\approx 10.9 \text{ cm}^3$$

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17. (a) (i) Centre = $\left(\frac{p}{2}, 0\right)$, radius = $\frac{p}{2}$

Equation of the circle OPS :

$$\left(x-\frac{p}{2}\right)^2 + y^2 = \left(\frac{p}{2}\right)^2$$

$$x^2 + y^2 - px = 0$$

(ii) $\because$ S lies on the circle OPS.,

$\therefore$ $a^2 + b^2 - pa = 0$

Using Pythagoras' Theorem,

$$\begin{aligned} OS^2 &= a^2 + b^2 \\ &= pa \\ &= OP \cdot OR \\ &= OP \cdot OQ \cos \angle POQ \end{aligned}$$

(b) (i) $\because$ BC is a diameter of the circle $BCEF$,

$\therefore$ $\angle BEC = 90°$ ($\angle$ in semicircle)

i.e. BE is an altitude of ΔABC.

(ii) Since the points C, A, B, G and E are defined analogously as the points O, P, Q, S and R in (a),

$\therefore$ $CG^2 = CA \cdot CB \cos \angle ACB$.

Similarly, AD is also an altitude of ΔABC and

$CF^2 = CB \cdot CA \cos \angle ACB$.

Hence $CG = CF$.

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