

只限教師參閱

FOR TEACHERS' USE ONLY

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

2000年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2000

數學 試卷一

MATHEMATICS PAPER 1

本評卷參考乃考試局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.

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2000-CE-MATH 1-1

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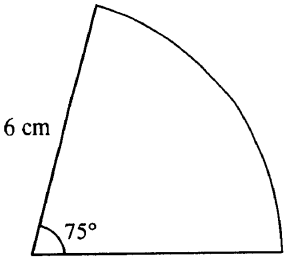
Hong Kong Certificate of Education Examination
Mathematics Paper 1

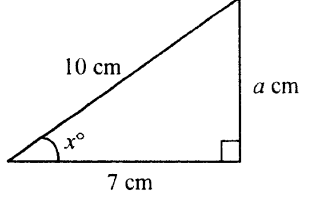
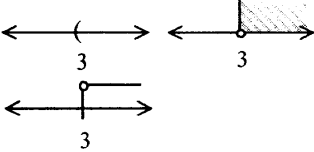
General Marking Instructions

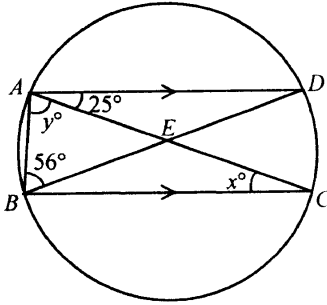
1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits **all the marks** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

| | |
|--------------------------|--|
| ‘M’ marks | awarded for correct methods being used; |
| ‘A’ marks | awarded for the accuracy of the answers; |
| Marks without ‘M’ or ‘A’ | awarded for correctly completing a proof or arriving at an answer given in a question. |

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
 - a. The symbol $\textcircled{u-1}$ should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* for the whole paper.
 - b. The symbol $\textcircled{pp-1}$ should be used to denote 1 mark deducted for *pp*. At most deduct **2 marks** for *pp* for the whole paper. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
 - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
 - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. Marks entered in the Page Total Box should be the NET total scored on that page.
8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’, ‘f.t.’ stands for ‘follow through’ and ‘or equivalent’ means ‘accepting equivalent forms of the equation which may have not been simplified but without uncollected like terms’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles or (brackets). All fractional answers must be simplified.

| Solution | Marks | Remarks |
|---|------------------------------|--|
| 1. When $C = 30$, $30 = \frac{5}{9}(F - 32)$ $\frac{30 \times 9}{5} = F - 32 \quad (\text{or } 270 = 5F - 160)$ $F = 86$ | 1M 1A 1A | substituting $C = 30$ removing brackets |
| $\therefore C = \frac{5}{9}(F - 32)$ $\therefore F = \frac{9}{5}C + 32 \quad (\text{or } 9C = 5F - 160)$ When $C = 30$, $F = \frac{9}{5} \times 30 + 32$ $F = 86$ | 1A 1M 1A | removing brackets substituting $C = 30$ |
| 2. $\frac{x^{-3}y}{x^2} = \frac{y}{x^2 x^3} \quad (\text{or } x^{-2} x^{-3} y)$ $= \frac{y}{x^{2+3}} \quad (\text{or } x^{-2-3} y)$ $= \frac{y}{x^5}$ | (3) 1M 1M 1A (3) | applying $a^{-m} = \frac{1}{a^m}$ applying $a^m a^n = a^{m+n}$ |
| 3. Area of the sector = $\frac{75}{360}(6^2 \pi) \text{ cm}^2$ $\approx 23.6 \text{ cm}^2 \quad (\text{or } 7\frac{1}{2}\pi \text{ cm}^2)$ | 1M+1A 1A | 1M for ratio or area of circle r.t. 23.6 or $\frac{15}{2}\pi, 7.5\pi$ |
| Area of the sector = $\frac{1}{2} \times 6^2 \times \frac{75}{180} \pi \text{ cm}^2$ $\approx 23.6 \text{ cm}^2 \quad (\text{or } 7\frac{1}{2}\pi \text{ cm}^2)$ | 1M+1A 1A | 1M for $\frac{1}{2}r^2\theta$ or correct value of θ r.t. 23.6 or $\frac{15}{2}\pi, 7.5\pi$ |
| | (3) |  |

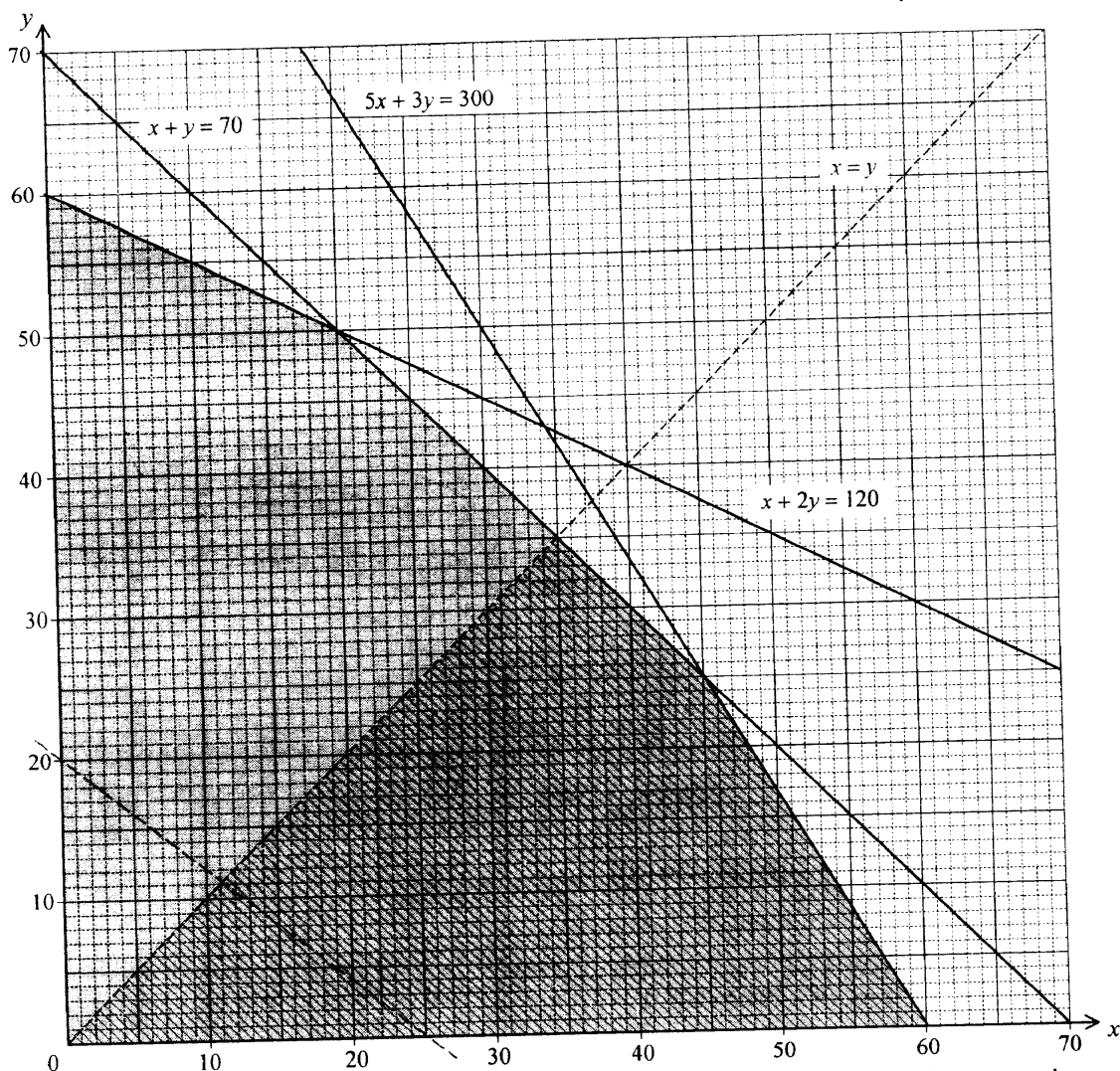
| Solution | Marks | Remarks |
|--|---|---|
| 4. $a^2 + 7^2 = 10^2$ (or $a = \sqrt{10^2 - 7^2}$) $a = \sqrt{51}$ (or 7.14) $\cos x^\circ = \frac{7}{10}$ (or $\sin x^\circ = \frac{\sqrt{51}}{10}$, $\tan x^\circ = \frac{\sqrt{51}}{7}$) $x \approx 45.6$ | 1A 1A 1M 1A | r.t. 7.14 r.t. 45.6, $u-1$ for $x \approx 45.6^\circ$, $x \approx 45^\circ 34'$, $x^\circ \approx 45.6^\circ$, $x^\circ \approx 45.6$ |
| $\cos x^\circ = \frac{7}{10}$ $x \approx 45.6$ $a \approx 10 \sin 45.6^\circ$ (or $a \approx 7 \tan 45.6^\circ$) $a \approx 7.14$ | 1A 1A 1M 1A |  |
| 5. $\frac{11-2x}{5} < 1$ $11-2x < 5$ (or $\frac{11}{5} - \frac{2}{5}x < 1$) $-2x < -6$ $2x > 6$ (or $6 < 2x$, $\frac{2}{5}x > \frac{6}{5}$) $x > 3$ | (4) 1A+1A 1A 1M (4) | For any 2 of these 3 steps, 1A for each. 2 of these 3 steps can be omitted. or  |
| 6. $f(-3)$ (or $2(-3)^3 + 6(-3)^2 - 2(-3) - 7$) $= -1$ \therefore The remainder is -1 . | 2A 1A | |
| $ \begin{array}{r} 2+0-2 \\ 1+3 \overline{)2+6-2-7} \\ \underline{2+6} \\ -2-7 \\ \underline{-2-6} \\ -1 \end{array} $ \therefore Remainder = -1 | 2A 1A | |
| | (3) | |

| Solution | Marks | Remarks |
|---|--|--|
| <p>7. $x = 25$</p> <p>$\therefore \angle ADB = x^\circ$</p> <p>$\therefore y = 180 - 56 - 25 - x$ $= 74$</p> | <p>1A</p> <p>1M</p> <p>1M</p> <hr/> <p>1A</p> <p>(4)</p> | <p>$u-1$ for $x = 25^\circ$, $x^\circ = 25^\circ$</p> <p>applying \angles in same segment</p> <p>$u-1$ for $y = 74^\circ$, $y^\circ = 74^\circ$</p>  |
| <p>8. Actual area = 220×5000^2 cm²</p> <p>$= \frac{220 \times 5000^2}{100^2}$ m²</p> <p>$= 550\,000$ m² (or area in m² = 550 000)</p> | <p>2M</p> <p>1M</p> <hr/> <p>1A</p> <p>(4)</p> | <p>for $\times 5000^2$, ignore unit</p> <p>for $\div 100^2$, pp-1 for not handling units properly</p> |
| <p>9. (a) Slope of $L = \frac{4-0}{-4-6}$</p> <p>$= -\frac{2}{5}$ (or -0.4)</p> | <p>1A</p> | |
| <p>(b) Equation of L:</p> <p>$y = -\frac{2}{5}(x-6)$ (or $\frac{y-4}{x+4} = -\frac{2}{5}$)</p> | <p>1M</p> | |
| <p>$y = -\frac{2}{5}x + \frac{12}{5}$ (or $2x + 5y - 12 = 0$)</p> | <p>1A</p> | <p>or equivalent</p> |
| <p>(c) When $x = 0$,</p> <p>$y = \frac{12}{5}$ (or $y = 2.4$)</p> <p>$\therefore C = (0, \frac{12}{5})$</p> | <p>1M</p> <p>1A</p> | |
| | <hr/> <p>(5)</p> | |

| Solution | Marks | Remarks |
|--|---|--|
| 10. (a) $10x^2 + 9x - 22 = 0$ $(x+2)(10x-11) = 0$ $x = -2$ or $\frac{11}{10}$ | $(\text{or } x = \frac{-9 \pm \sqrt{9^2 + 4 \times 10 \times 22}}{2 \times 10})$ $(\text{or } x = -2 \text{ or } 1.1)$ | 1A 1A (2) |
| (b) $10000(1+r\%)^2 + 9000(1+r\%) = 22000$ | 1M+1A | 1M for $10000(1+r\%)^2$ |
| $[10000(1+r\%) + 9000](1+r\%) = 22000$ | 1M+1A | 1M for $10000(1+r\%) + 9000$ |
| $10(1+r\%)^2 + 9(1+r\%) - 22 = 0$ (or $r^2 + 290r - 3000 = 0$, $10(r\%)^2 + 29(r\%) - 3 = 0$) | 1M | $pp-1$ for confusing r with $r\%$ |
| From (a), $1+r\% = 1.1$ $r = 10$ | 1A (4) | for choosing '+ve' value from 1 '+ve' and 1 '-ve' roots, provided that the original equation must be correct |
| 11. (a) Missing value in 1st table = 66 Missing value in 2nd table = 20 | 1A 1A (2) | |
| (b) An estimate of the mean $= \frac{210 \times 3 + 230 \times 13 + 250 \times 30 + 270 \times 20 + 290 \times 9}{75}$ (seconds) ≈ 255 seconds | 1M 1A (2) | r.t. 255 |
| (c) Median ≈ 254 seconds (or 255 seconds) | 1A (1) | r.t. 254 or 255 |
| (d) Number of songs have lengths greater than 220 seconds but not greater than 260 seconds $= 13 + 30$ (or $46 - 3$) $= 43$ Percentage required $= \frac{43}{75} \times 100\%$ $\approx 57.3\%$ (or $57\frac{1}{3}\%$) | 1A 1A (2) | r.t. 57.3 |

| Solution | Marks | Remarks |
|---|--------------|---------------|
| 12. (a) Numbers having two zero digits are 100, 200, ..., 900. Probability required = $\frac{9}{900}$ = $\frac{1}{100}$ (or 0.01) | 1A 1A | for numerator |
| $\text{Probability required} = \frac{1}{10} \times \frac{1}{10}$ $= \frac{1}{100} \quad (\text{ or } 0.01)$ | 1A 1A | |
| <hr style="width: 50%; margin: auto;"/> (2) | | |
| (b) Numbers having no zero digits are 111, 112, ..., 119 911, 912, ..., 919 121, 122, ..., 129 921, 922, ..., 929 ⋮ ⋮ 191, 192, ..., 199 991, 992, ..., 999 Probability required = $\frac{9 \times 9 \times 9}{900}$ = $\frac{81}{100}$ (or 0.81) | 1A 1A | for numerator |
| $\text{Probability required} = \frac{9}{10} \times \frac{9}{10}$ $= \frac{81}{100} \quad (\text{ or } 0.81)$ | 1A 1A | |
| <hr style="width: 50%; margin: auto;"/> (2) | | |
| (c) Numbers having exactly one zero digit are 101, 102, ..., 109, 110, 120, ..., 190 201, 202, ..., 209, 210, 220, ..., 290 ⋮ ⋮ 901, 902, ..., 909, 910, 920, ..., 990 Probability required = $\frac{9 \times 9 + 9 \times 9}{900}$ = $\frac{9}{50}$ (or 0.18) | 1A 1A | for numerator |
| $\text{Probability required} = 1 - \frac{1}{100} - \frac{81}{100}$ $= \frac{9}{50} \quad (\text{ or } 0.18)$ | 1M 1A | |
| $\text{Probability required} = \frac{1}{10} \times \frac{9}{10} \times 2$ $= \frac{9}{50} \quad (\text{ or } 0.18)$ | 1A 1A | |
| <hr style="width: 50%; margin: auto;"/> (2) | | |

| Solution | Marks | Remarks |
|---|----------------|------------------------------|
| 15. (a) x and y satisfy the following conditions: $1000(40x) + 800(30y) \leq 2400000$ or $5x + 3y \leq 300$ $1000(10x) + 800(25y) \leq 1200000$ or $x + 2y \leq 120$ $x + y \leq 70$ x, y are non-negative integers | 1A 1A 1A | Withhold 1 mark for any "<". |



Draw the straight lines $5x + 3y = 300$, $x + 2y = 120$ and $x + y = 70$.

Let $\$P(x, y)$ be the profit generated by x boxes of brand A mixed nuts and y boxes of brand B mixed nuts. Then

$$P(x, y) = 800x + 1000y$$

$$= 200(4x + 5y)$$

By drawing parallel lines of $4x + 5y = 0$,

$$\therefore P(0, 0) = 0, P(0, 60) = 60000, P(20, 50) = 66000,$$

$$P(45, 25) = 61000 \text{ and } P(60, 0) = 48000$$

$P(x, y)$ attains its maximum at $(20, 50)$.

\therefore The profit is the greatest when $x = 20$ and $y = 50$.

1A+1A

1A

1M

1M

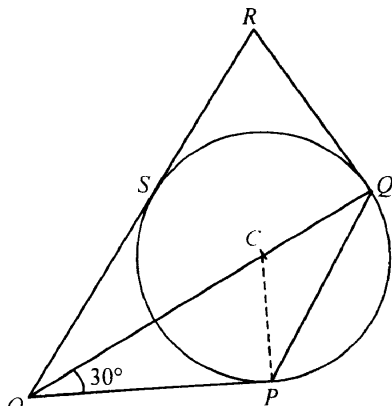
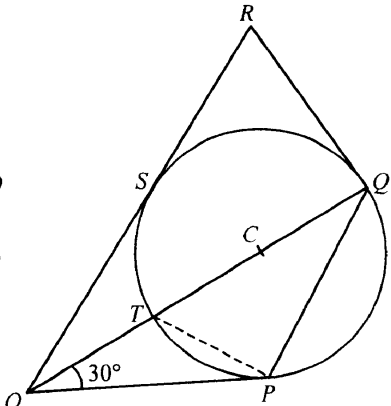
1A

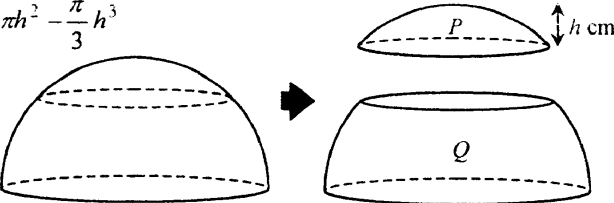
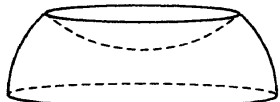
(8)

1A for any correct line. 1A for all. Accept dotted lines or lines without labeling. Position of lines should not lie outside 1 small grid at the edges.

check the line on graph

f.t.

| | Solution | Marks | Remarks |
|---|--|-------|---|
| <p>16.</p> <div style="display: flex; justify-content: space-around;">   </div> <p style="text-align: center;">Figure 9A Figure 9B</p> <p>(a) Refer to Figure 9A,</p> <p>(L1)... $\angle OPC = 90^\circ$ (tangent \perp radius)</p> <p>(L2)... $\angle PCO = 180^\circ - 90^\circ - 30^\circ = 60^\circ$ (\angle sum of Δ)</p> <p>(L3)... $\angle PQO = \frac{1}{2} \angle PCO = 30^\circ$ (\angle at centre twice \angle at circumference)</p> | | | <p>(tangent properties) [切線\perp半徑]、[切線性質/定理] [Δ內角和] (\angle at centre = $2 \times \angle$ at circumference) [圓心角兩倍於圓周角]、[圓心角是圓周角的兩倍]、[圓心角 = $2 \times$圓周角]</p> |
| | <p>Refer to Figure 9A, and let $\angle CQP = x$.</p> <p>(L4)... $\angle OPC = 90^\circ$ (tangent \perp radius)</p> <p>(L5)... $\angle PCO = 180^\circ - 90^\circ - 30^\circ = 60^\circ$ (\angle sum of Δ)</p> <p>(L6)... $\therefore CP = CQ$ (radius)</p> <p>(L7)... $\therefore \angle CPQ = \angle CQP = x$ (base \angles of isos. Δ)</p> <p>(L8)... $2x = \angle PCO = 60^\circ$ (ext. \angle of Δ)</p> <p>(L9)... $x = 30^\circ$</p> | | <p>(tangent properties) [切線\perp半徑]、[切線性質/定理] [Δ內角和] [等腰Δ底角] [Δ的外角]</p> |
| | <p>Refer to Figure 9B, and let $\angle CQP = x$.</p> <p>(L10)... $\angle TPO = \angle CQP = x$ (\angle in alt. segment)</p> <p>(L11)... $\angle TPQ = 90^\circ$ (\angle in semicircle)</p> <p>(L12)... $\therefore 30^\circ + 90^\circ + 2x = 180^\circ$ (\angle sum of Δ)</p> <p>(L13)... $x = 30^\circ$</p> | | <p>[交錯弓形的圓周角]、[弦切角定理] [半圓上的圓周角] [Δ內角和]</p> |
| <p>Marking Scheme :</p> <p>Case 1 Any correct proof with correct reasons.</p> <p>Case 2 Any correct proof without reasons. In addition, any relevant correct argument with correct reason (at most 1 mark).</p> <p>Case 3 Any relevant correct argument with correct reason.</p> | | | <p>3 1 1 At most 2 marks 1 At most 1 mark</p> |
| | | (3) | |

| Solution | Marks | Remarks | | | | | | | | | | | | | | | | | | | | | | | | |
|--|--|---|------|-----------------|------|--------------|------------------|-------|---------------|--------------------|-------|---------------|--------------------|-------|---------------|--------------------|-------|--------------|---------------------|-------|---------------|---------------------|-------|--------------|-------------------------------|--|
| <p>18. (a) Let $V = ah^2 + bh^3$ where a, b are non-zero constants. Then</p> $\begin{cases} \frac{29}{3}\pi = a + b \\ 81\pi = 9a + 27b \end{cases} \quad \begin{cases} a + b = \frac{29}{3}\pi \dots\dots\dots(1) \\ a + 3b = 9\pi \dots\dots\dots(2) \end{cases}$ <p>(2) - (1) gives $2b = -\frac{2}{3}\pi$</p> <p>Hence $b = -\frac{\pi}{3}$ and $a = 10\pi$</p> <p>$\therefore V = 10\pi h^2 - \frac{\pi}{3}h^3$</p> | <p>1A</p> <p>1M</p> <p>1A</p> | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | <p>(3)</p> |  | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>(b) (i) Surface area = $2\pi \times 10^2$ (cm²) ≈ 628 cm² (or 200π cm²)</p> <p>(ii) \therefore Volume of hemisphere = $\frac{2}{3}\pi \times 10^3$ (cm³)</p> $\therefore \frac{2}{3}\pi \times 10^3 - 2V = \frac{1400}{3}\pi$ $\frac{2}{3}\pi \times 10^3 - 2(10\pi h^2 - \frac{\pi}{3}h^3) = \frac{1400}{3}\pi$ $\frac{2}{3}\pi(1000 - 30h^2 + h^3 - 700) = 0$ $h^3 - 30h^2 + 300 = 0$ <p>From the graph in Figure 11.3, $3.3 < h < 3.4$ (or $3.35 < h < 3.4$ etc.)</p> | <p>1A</p> <p>1A</p> <p>1M</p> <p>1</p> <p>1M</p> | <p>r.t. 628</p> <p>or claiming to draw $y = -300$, writing $h \approx 3.35, h \approx 3.4$ etc.</p> | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Let $f(h) = h^3 - 30h^2 + 300$, then $f(3.3) > 0$ and $f(3.4) < 0$. Using the method of bisection,</p> <table border="1" data-bbox="290 1361 981 1684"> <thead> <tr> <th>Interval</th> <th>"mid-value"</th> <th>f(h)</th> </tr> </thead> <tbody> <tr> <td>$3.3 < h < 3.4$</td> <td>3.35</td> <td>+ve (0.9204)</td> </tr> <tr> <td>$3.35 < h < 3.4$</td> <td>3.375</td> <td>-ve (-3.2754)</td> </tr> <tr> <td>$3.35 < h < 3.375$</td> <td>3.363</td> <td>-ve (-1.2583)</td> </tr> <tr> <td>$3.35 < h < 3.363$</td> <td>3.357</td> <td>-ve (-0.2519)</td> </tr> <tr> <td>$3.35 < h < 3.357$</td> <td>3.354</td> <td>+ve (0.2507)</td> </tr> <tr> <td>$3.354 < h < 3.357$</td> <td>3.356</td> <td>-ve (-0.0843)</td> </tr> <tr> <td>$3.354 < h < 3.356$</td> <td>3.355</td> <td>+ve (0.0832)</td> </tr> </tbody> </table> <p>$\therefore 3.355 < h < 3.356$ $h \approx 3.36$ (correct to 2 decimal places)</p> | Interval | "mid-value" | f(h) | $3.3 < h < 3.4$ | 3.35 | +ve (0.9204) | $3.35 < h < 3.4$ | 3.375 | -ve (-3.2754) | $3.35 < h < 3.375$ | 3.363 | -ve (-1.2583) | $3.35 < h < 3.363$ | 3.357 | -ve (-0.2519) | $3.35 < h < 3.357$ | 3.354 | +ve (0.2507) | $3.354 < h < 3.357$ | 3.356 | -ve (-0.0843) | $3.354 < h < 3.356$ | 3.355 | +ve (0.0832) | <p>1M</p> <p>1M</p> <p>1A</p> | <p>use interval $\subseteq [0, 5]$ containing the root as the starting interval testing sign of "mid-value" or any intermediate value choosing the correct interval</p> <p>f.t.</p> |
| Interval | "mid-value" | f(h) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.3 < h < 3.4$ | 3.35 | +ve (0.9204) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.35 < h < 3.4$ | 3.375 | -ve (-3.2754) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.35 < h < 3.375$ | 3.363 | -ve (-1.2583) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.35 < h < 3.363$ | 3.357 | -ve (-0.2519) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.35 < h < 3.357$ | 3.354 | +ve (0.2507) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.354 < h < 3.357$ | 3.356 | -ve (-0.0843) | | | | | | | | | | | | | | | | | | | | | | | | |
| $3.354 < h < 3.356$ | 3.355 | +ve (0.0832) | | | | | | | | | | | | | | | | | | | | | | | | |
| <p>Let $f(h) = h^3 - 30h^2 + 300$.</p> <p>$\therefore f(3.34) \approx 2.5917$ $f(3.35) \approx 0.9203$ $f(3.355) \approx 0.0832$ $f(3.36) \approx -0.7549$ $f(3.37) \approx -2.4342$</p> <p>$\therefore h \approx 3.36$ (correct to 2 decimal places)</p> | <p>1M+1M</p> <p>1A</p> | <p>f.t.</p> | | | | | | | | | | | | | | | | | | | | | | | | |
| | <p>(8)</p> | | | | | | | | | | | | | | | | | | | | | | | | | |