SECTION A(1) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

1. Simplify $\dfrac{(a^{-3})^2}{a}$ and express your answer with positive indices. (3 marks)

2. Make x the subject of the formula $a = b + \dfrac{c}{x}$. (3 marks)

3. Find the range of values of x which satisfy both $3x - 4 > 2(x - 1)$ and $x < 6$. (3 marks)

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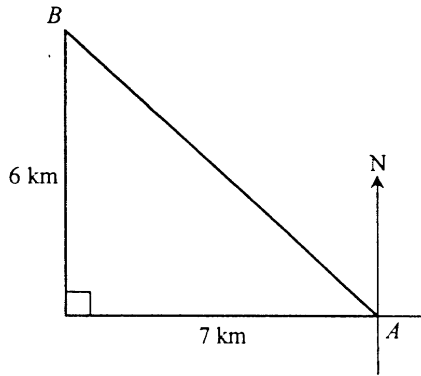4. In Figure 1, find the bearing of B from A. (3 marks)

Figure 1

5. In Figure 2, A, B, C, D are points on a circle and AC is a diameter. Find x and y. (4 marks)
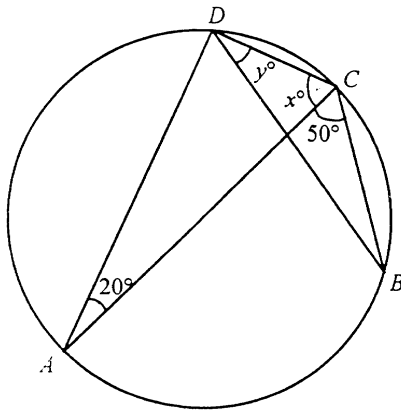

Figure 2

6. y varies partly as x and partly as x^2. When $x = 2$, $y = 20$ and when $x = 3$, $y = 39$. Express y in terms of x. (4 marks)

7. The graph of $y = x^2 - x - 6$ cuts the x-axis at $A(a, 0)$, $B(b, 0)$ and the y-axis at $C(0, c)$ as shown in Figure 3. Find a, b and c.

(4 marks)

$y = x^2 - x - 6$
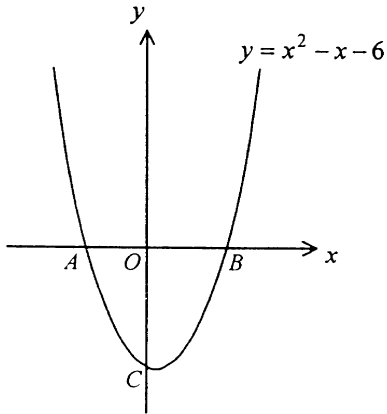
Figure 3

8. The heights of 6 students are x cm, 161 cm, 168 cm, 159 cm, 161 cm and 152 cm. The mean height of these students is 158 cm.

(4 marks)

(a) Find x.

(b) Find the median of the heights of these students.

9.   Figure 4 shows a sector.

(a)   Find r.

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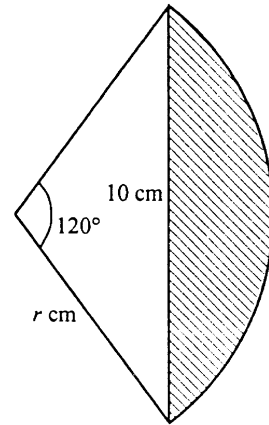
Figure 4

(b)   Find the area of the shaded region.

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_____

_____

_____

_____

10 cm

120°

r cm

SECTION A(2) (33 marks)
Answer ALL questions in this section and write your answers in the spaces provided.

10. In Figure 5, $A(-8, 8)$ and $B(16, -4)$ are two points. The perpendicular bisector ℓ of the line segment AB cuts AB at M and the x-axis at P.
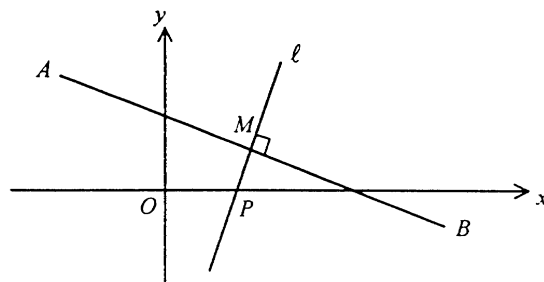
Figure 5

(a) Find the equation of ℓ .

(4 marks)

(b) Find the length of BP .

(2 marks)

(c) If N is the mid-point of AP , find the length of MN .

(2 marks)

11. A school conducted a survey on the placement of her S.5 graduates last year. There were 200 graduates, of which 120 were boys and 80 were girls. The placement of the boys was shown in Figure 6.

Placement of the 120 boys
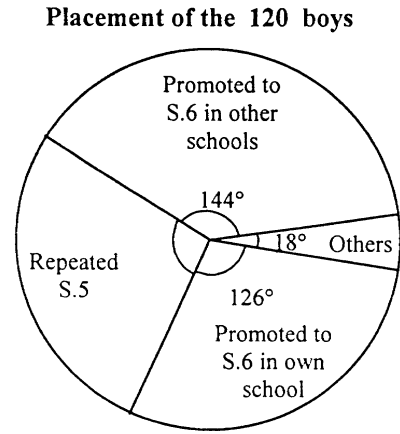
Figure 6

(a) Find the number of boys who repeated S.5 . (2 marks)

(b) Among all the boys promoted to S.6 , what percentage of them was promoted in their own school? (2 marks)

(c) The result of the survey also showed that 22.5% of the girls were promoted to S.6 in their own school. Find the percentage of graduates promoted to S.6 in their own school. (2 marks)

12. Mr. Sun is waiting for a bus at a bus stop. It is known that 75% of the buses are air-conditioned, of which 20% have Octopus machines installed. No Octopus machines have been installed on buses without air-conditioning.

(a) Find the probability that the next bus has an Octopus machine installed. (2 marks)

(b) The bus fare is $3.00. Mr. Sun does not have an Octopus card but has two 1-dollar coins and three 2-dollar coins in his pocket. If he randomly takes out two coins, what is the probability that the total value of these coins is exactly $3.00 ? (4 marks)

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13. In Figure 7.1, a piece of wood in the form of an inverted right circular cone is cut into two portions by a plane parallel to its base. The upper portion is a frustum with height 10 cm , and the radii of the two parallel faces are 9 cm and 4 cm respectively. The pen-stand shown in Figure 7.2 is made from the frustum by drilling a hole in the middle. The hole consists of a cylindrical upper part of radius 5 cm and a hemispherical lower part of the same radius. The depth of the hole is 9 cm .
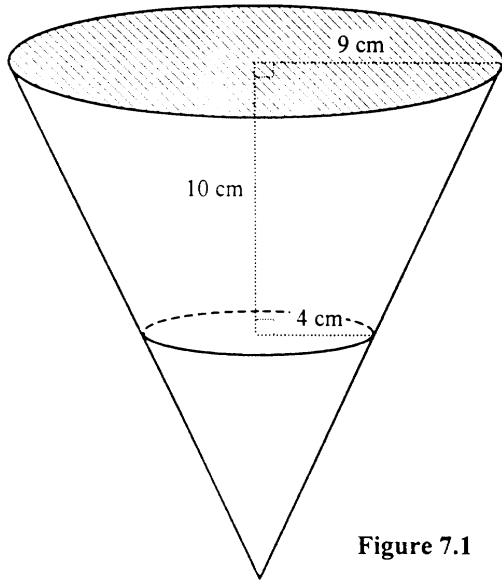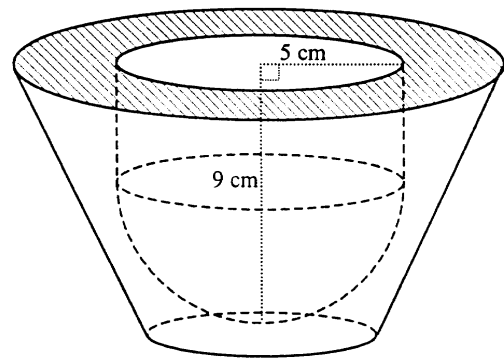
Figure 7.1   **Figure 7.2**

(a)   Find, in terms of π, the capacity of the hole.   (3 marks)

(b)   Find, in terms of π, the volume of wood in the pen-stand.   (4 marks)

14. In Figure 8, *ABCD* is a parallelogram. *EBDF* is a straight line and *EB = DF* .
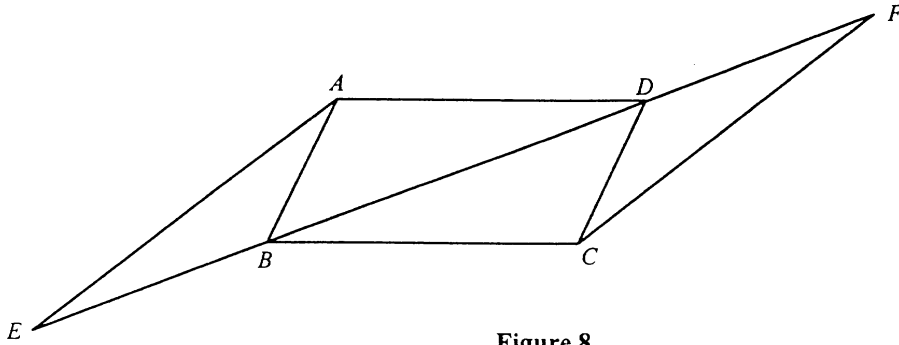
Figure 8

(a)   Prove that $\angle ABE = \angle CDF$.   (2 marks)

(b)   Prove that *EA // CF* .   (4 marks)

SECTION B (33 marks)
Answer any THREE questions in this section and write your answers in the spaces provided.
Each question carries 11 marks.

15. The Water Supply Department is planning to replace an old service reservoir X with a new service reservoir Y.

(a) Service reservoir X contains water to a depth of 10 m . Water leaks at a certain rate so that the depth, D m , of water t days later is given by $D = 10 - 0.01t - 0.006t^3$. Using the method of bisection and starting with interval $[11, 12]$, determine the time required for the reservoir to become empty. Give your answer correct to the nearest 0.1 day. (4 marks)

(b) Service reservoir Y is at first empty. Three water pipes A, B and C with constant filling rates can be used to fill this reservoir completely. If the pipes are used individually, using pipe A to fill up the reservoir takes 3 days more than pipe B while using pipe C takes 2 days less than pipe B. If the three pipes are used simultaneously, filling up the reservoir takes 4 days. Suppose pipe B alone takes x days to fill up this reservoir.

Show that $x^3 - 11x^2 - 14x + 24 = 0$.

Hence find x by factorizing $x^3 - 11x^2 - 14x + 24$. (7 marks)

16. (a) In Figure 9.1, ABC is a triangle right-angled at B. D is a point on AB. A circle is drawn with DB as a diameter. The line through D and parallel to AC cuts the circle at E. CE is produced to cut the circle at F.

(i) Prove that A, F, B and C are concyclic.

(ii) If M is the mid-point of AC, explain why $MB = MF$.

(5 marks)

Figure 9.1

(b) In Figure 9.2, the equation of circle RST is $x^2 + y^2 + 10x - 6y + 9 = 0$. QST is a straight line. The coordinates of P, Q, R, S are $(-17, 0)$, $(0, 17)$, $(-9, 0)$ and $(-2, 7)$ respectively.

(i) Prove that $PQ \parallel RS$.

(ii) Find the coordinates of T.

(iii) Are the points P, Q, O and T concyclic? Explain your answer.

(6 marks)

Figure 9.2

17. The manager of a factory estimated that in year 2000, the income of the factory will drop by $r\%$ each month from $\$500\,000$ in January to $\$284\,400$ in December.

(a) Find r correct to the nearest integer. (3 marks)

(b) Suppose the factory's production cost is $\$400\,000$ in January 2000. The manager proposed to cut the cost by $\$20\,000$ every month (i.e., the cost will be $\$380\,000$ in February and $\$360\,000$ in March etc.) and claimed that it would not affect the monthly income.

   (i) Using the value of r obtained in (a), show that the factory will still make a profit for the whole year.

   (ii) The factory will start a research project at the beginning of year 2000 on improving its production method. The cost of running the research project is $\$300\,000$ per month. The project will be stopped at the end of the k-th month if the total cost spent in these k months on running the project exceeds the total production cost for the remaining months of the year.

   Show that $k^2 - 71k + 348 < 0$.

   Hence determine how long the research project will last.

   (8 marks)

18. In Figure 10, a paper card ABC in the shape of an equilateral triangle of side 24 cm is folded to form a paper aeroplane. D, E and F are points on edge BC so that $BD = DE = EF = FC$. The aeroplane is formed by folding the paper card along the lines AD, AE and AF so that AD and AF coincide. It is supported by two vertical sticks BM and CN of equal length so that A, B, D, F, C lie on the same plane and A, E, M, N lie on the same horizontal ground.

Figure 10

(a) Find the distance between the tips, B and C, of the wings of the aeroplane. (6 marks)

(b) Find the inclination of the wings of the aeroplane to the horizontal ground. (2 marks)

(c) Find the length of the stick CN. (3 marks)