**FORMULAS FOR REFERENCE**

| | | | |
|---|---|---|---|
| SPHERE | Surface area | = | $4\pi r^2$ |
| | Volume | = | $\frac{4}{3}\pi r^3$ |
| CYLINDER | Area of curved surface | = | $2\pi rh$ |
| | Volume | = | $\pi r^2 h$ |
| CONE | Area of curved surface | = | $\pi r l$ |
| | Volume | = | $\frac{1}{3}\pi r^2 h$ |
| PRISM | Volume | = | base area × height |
| PYRAMID | Volume | = | $\frac{1}{3}$ × base area × height |



**SECTION A (51 marks)**
**Answer ALL questions in this section.**
**There is no need to start each question on a fresh page.**

1. Make $r$ the subject of the formula $h = a + r(1 + p^2)$ .

   If $h = 8$ , $a = 6$ and $p = -4$ , find the value of $r$ .

   (3 marks)

2. Simplify $\dfrac{a^{\frac{5}{4}} \sqrt[4]{a^3}}{a^{-2}}$ .

   (3 marks)

3. The $n$-th term $T_n$ of a sequence $T_1$ , $T_2$ , $T_3$ , ... is $7 - 3n$ .

   (a) Write down the first 4 terms of the sequence.

   (b) Find the sum of the first 100 terms of the sequence.

   (4 marks)

4. Show that $x + 1$ is a factor of $x^3 - x^2 - 3x - 1$ .

   Hence solve $x^3 - x^2 - 3x - 1 = 0$ .
   (Leave your answers in surd form.)

   (5 marks)



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5. Solve (i) $\frac{x+5}{2} > 4$ ;

(ii) $x^2 - 6x + 8 < 0$ .

Hence write down the range of values of $x$ which satisfy both the inequalities in (i) and (ii).

(5 marks)

6. In Figure 1, $A$, $B$, $C$, $D$ are points on a circle. $CB$ and $DA$ are produced to meet at $P$. If $AB // DC$, prove that $AP = BP$.

(5 marks)

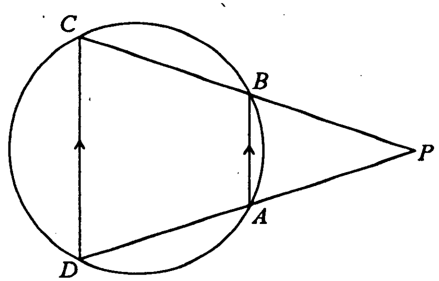


**Figure 1**

7. Figure 2 shows a circular dartboard. Its surface consists of two concentric circles of radii 12 cm and 2 cm respectively.

(a) Find the area of the shaded region on the dartboard.

(b) Two darts are thrown and hit the dartboard. Find the probability that

(i) both darts hit the shaded region ;

(ii) only one dart hits the shaded region.

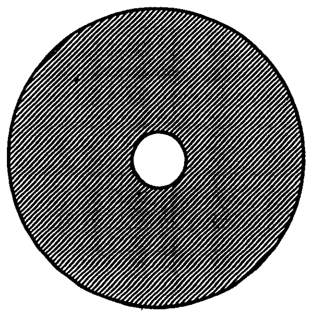

**Figure 2**

(7 marks)



8. Figure 3.1 shows a paper cup in the form of a right circular cone of base radius 5 cm and height 12 cm .

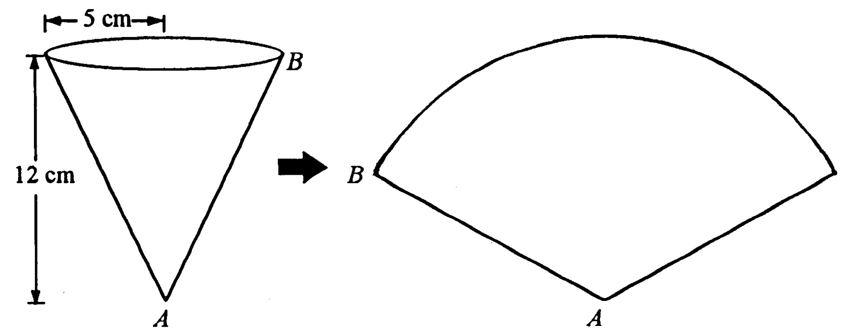


**Figure 3.1** **Figure 3.2**

(a) Find the capacity of the paper cup.

(b) If the paper cup is cut along the slant side $AB$ and unfolded to become a sector as shown in Figure 3.2, find

(i) the area of the sector ;

(ii) the angle of the sector.

(6 marks)

9. In Figure 4, $\mathcal{R}$ is the region (including the boundary) bounded by the three straight lines

$L_1$: $3x+2y-7=0$ ,
$L_2$: $3x-5y+7=0$ and
$L_3$: $2x-y-7=0$ .

$L_1$ and $L_2$ intersect at $A(1, 2)$ .
$L_2$ and $L_3$ intersect at $B(6, 5)$ .

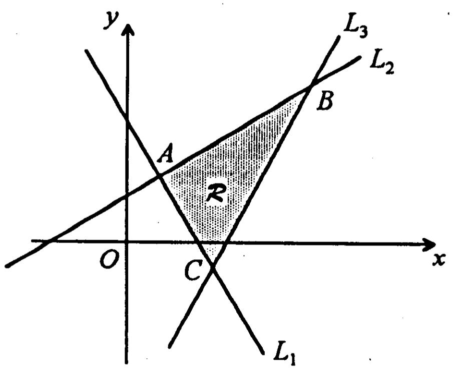

**Figure 4**

(a) Find the coordinates of $C$ at which $L_1$ and $L_3$ intersect.

(b) Write down the three inequalities which define the region $\mathcal{R}$.

(c) Find the maximum value of $2x-2y-7$ where $(x, y)$ is any point in the region $\mathcal{R}$.

(6 marks)

10. In Figure 5, $AB=CD$ and $AE=BC$ .

(a) Find $x$ .

(b) Which two triangles in the figure are congruent?

(c) Find $\theta$, $y$ and $z$ .
(7 marks)

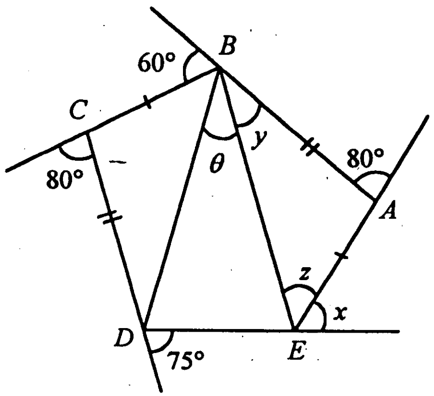

**Figure 5**



**SECTION B (48 marks)**
**Answer any FOUR questions in this section.**
**Each question carries 12 marks.**

11. $\mathcal{C}_1$ is the circle with centre $A(0, 2)$ and radius 2 . It cuts the $y$-axis at the origin $O$ and the point $B$ . $\mathcal{C}_2$ is another circle with equation $x^2+(y-2)^2=25$ . The line $L$ passing through $B$ with slope 2 cuts $\mathcal{C}_2$ at the points $Q$ and $R$ as shown in Figure 6.

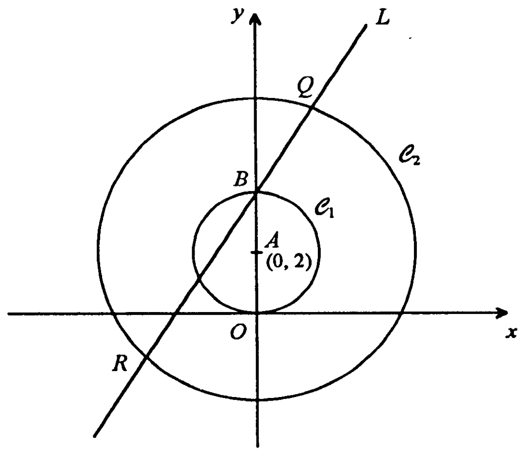

**Figure 6**

(a) Find
(i) the equation of $\mathcal{C}_1$ ;
(ii) the equation of $L$ .
(3 marks)

(b) Find the coordinates of $Q$ and $R$ . (4 marks)

(c) Find the coordinates of
(i) the point on $L$ which is nearest to $A$ ;
(ii) the point on $\mathcal{C}_1$ which is nearest to $Q$ .
(5 marks)



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12. Bank A offers personal loans at an interest rate of 18% per annum. For each successive month after the day when the loan is taken, loan interest is calculated and an instalment is paid.
(Answers to this question should be corrected to 2 decimal places.)

(a) Mr. Chan took a personal loan of $50 000 from Bank A and agreed to repay the bank in monthly instalments of $9 000 until the loan is fully repaid (the last instalment may be less than $9 000). The outstanding balance of his loan for each of the first three months is shown in Table 1 on Page 8.

(i) Complete Table 1 until the loan is fully repaid.

(ii) Find the amount of his last instalment.

(iii) Calculate the total interest earned by the bank.

(5 marks)

(b) Mrs. Lee also took a personal loan of $50 000 from Bank A. She agreed to pay $9 000 as the first monthly instalment and increase the amount of each instalment by 20% for every successive month until the loan is fully repaid. The outstanding balance of her loan for the first month is shown in Table 2 on Page 8.

Complete Table 2 until the loan is fully repaid.

(4 marks)

(c) Mr. Cheung wants to buy a $50 000 piano for her daughter but he has no savings at hand. He intends to buy the piano by taking a personal loan of $50 000 from Bank A. If he can only save $12 000 from his income every month and uses his savings to repay the loan, can he afford to use the repayment scheme as described in (b)? Explain your answer.

(3 marks)



| Candidate Number | Centre Number | Seat Number | Page Total |
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12.(Cont'd) **If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

**Table 1** The outstanding balance of Mr. Chan's loan for each month

| Month | Loan Interest ($) | Loan Repaid ($) | Outstanding Balance ($) |
|---|---|---|---|
| 1 | 750.00 | 8 250.00 | 41 750.00 |
| 2 | 626.25 | 8 373.75 | 33 376.25 |
| 3 | 500.64 | 8 499.36 | 24 876.89 |
| 4 | | | |
| 5 | | | |
| 6 | | | |

**Table 2** The outstanding balance of Mrs. Lee's loan for each month

| Month | Instalment ($) | Loan Interest ($) | Loan Repaid ($) | Outstanding Balance ($) |
|---|---|---|---|---|
| 1 | 9 000.00 | 750.00 | 8 250.00 | 41 750.00 |
| 2 | | | | |
| 3 | | | | |
| 4 | | | | |
| 5 | | | | |



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13. When other conditions remain the same, the quality of a cup of a particular kind of Chinese tea depends on the amount of time, $t$ seconds, that tea leaves are soaked in water and the temperature, $x$ °C, of the water. It is proposed that the quality of a cup of this kind of tea can be measured by the indicator $Q$ as follows:

$$Q = 20\,000 + F,$$

where $F$ consists of two parts with one part varying jointly as $x$ and $t$, and the other part varying as the square of $t$. The greater the value of $Q$, the better is the quality of the tea.

It is known that $Q = 30\,600$ when $t = 40$, $x = 85$; and $Q = 28\,100$ when $t = 60$, $x = 75$.

(a) Show that $Q = 20\,000 + 5xt - 4t^2$.

(5 marks)

(b) (i) Find the value of $Q$ when the tea leaves are soaked in water for 45 seconds at a temperature of 82 °C.

(ii) When the temperature of water is 78 °C, is it possible to achieve the same value of $Q$ in (b)(i) by changing the amount of time that the tea leaves are soaked in water? Explain your answer briefly.

(4 marks)

(c) Suppose the temperature of water is 80 °C. Using the method of completing the square, find the amount of time the tea leaves need to be soaked in the water in order to achieve the best quality of the tea.

(3 marks)



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14. A youth centre has done a survey on the amount of money $\$x$ teenagers spent on buying clothes for Christmas. The results of the survey are shown in Tables 3 and 4 on Page 12.

(a) A number in Table 3 was accidentally covered in ink. What should this number be?

(1 mark)

(b) Explain why the sum of the percentages in Table 4 is 100.1 instead of 100.

(1 mark)

(c) The cumulative frequency polygon of the distribution of $x$ ($x \le 1\,000$) for girls is drawn in Figure 7 (Page 13).

(i) Construct the cumulative frequency table of the distribution of $x$ ($x \le 1\,000$) for boys.

(ii) On the same graph (Figure 7), draw the cumulative frequency polygon of the distribution in (i).

(iii) Find the medians of $x$ for boys and girls respectively in this survey.

(iv) Estimate the total number of teenagers in this survey spending not more than \$700 on buying clothes for Christmas.

(8 marks)

(d) By considering the percentages in Tables 3 and 4, find evidence to support the statement

"In this survey, more boys did not spend any money on buying clothes for Christmas."

Explain briefly why we have to consider the percentages instead of the frequencies.

(2 marks)



**Table 3** The amount of money spent by *boys* on buying clothes for Christmas

| $x$ | Frequency | Percentage (%) |
|---|---|---|
| 0 | 70 | 20.0 |
| $0 < x \le 200$ | 17 | 4.9 |
| $200 < x \le 400$ | 48 | 13.7 |
| $400 < x \le 600$ | 83 | [redacted] |
| $600 < x \le 800$ | 92 | 26.3 |
| $800 < x \le 1\,000$ | 36 | 10.3 |
| $x > 1\,000$ | 4 | 1.1 |
| Total frequency = | 350 | |

**Table 4** The amount of money spent by *girls* on buying clothes for Christmas

| $x$ | Frequency | Percentage (%) |
|---|---|---|
| 0 | 81 | 15.0 |
| $0 < x \le 200$ | 51 | 9.4 |
| $200 < x \le 400$ | 135 | 25.0 |
| $400 < x \le 600$ | 87 | 16.1 |
| $600 < x \le 800$ | 74 | 13.7 |
| $800 < x \le 1\,000$ | 56 | 10.4 |
| $x > 1\,000$ | 57 | 10.5 |
| Total frequency = | 541 | |

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14.(Cont'd) **If you attempt Question 14, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

**The cumulative frequency polygon of the distribution of $x$ $(x \leq 1000)$ for girls**

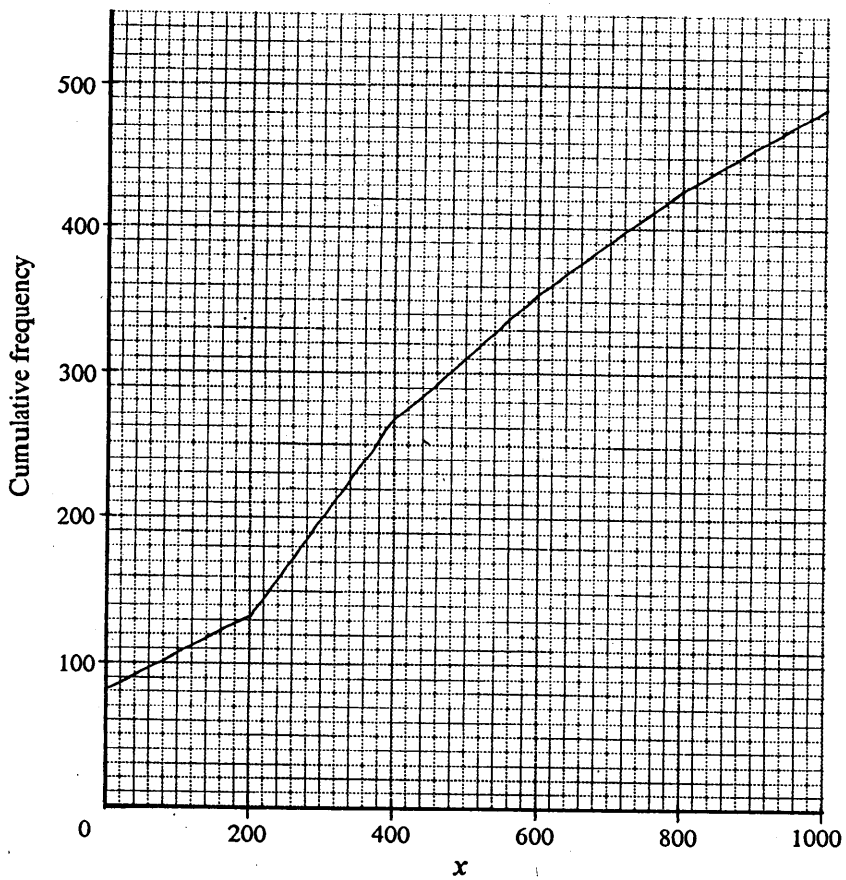


**Figure 7**



15. In Figure 8, the rectangular plane $ABCD$ is a hillside with inclination $30°$. $C'$ and $O'$ are vertically below $C$ and $O$ respectively so that $A$, $B$, $C'$, $O'$ are on the same horizontal plane. $BO$ is a straight path on the hillside which makes an angle $60°$ with $BC$, and $OT$ is a vertical tower. $AB = 2\,000$ m, $BO = 1\,000$ m and $OT = 50$ m.

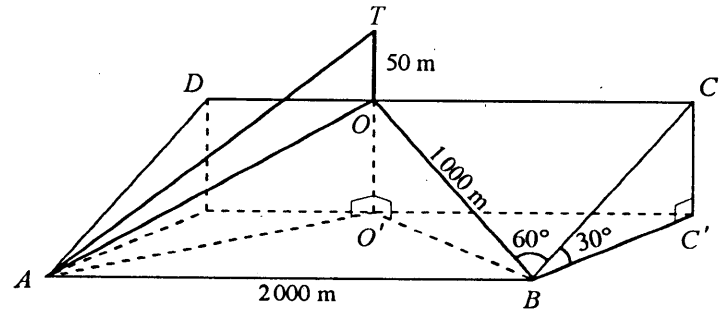


**Figure 8**

(a) Find $BC$ and $CC'$. (2 marks)

(b) Find the inclination of $BO$ with the horizontal. (2 marks)

(c) Find $AT$. (5 marks)

(d) There are cable cars going directly from $A$ to $T$. A man wants to go to $T$ from $B$ and he can do this by taking either one of the following two routes:

Route I: Walking uphill along $BO$ at an average speed of 0.3 m/s and taking a lift in the tower for 1 minute from $O$ to $T$.

Route II: Walking along $BA$ at an average speed of 0.8 m/s and taking a cable car from $A$ to $T$ at an average speed of 3.2 m/s.

Determine which route takes a shorter time.

(3 marks)

16. Figure 9.1 shows an open rectangular box made of thin metal plates. Its length, width and height are 30 cm, 20 cm and 15 cm respectively.

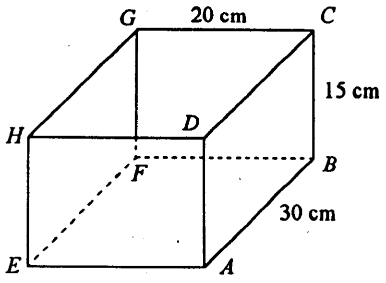


Figure 9.1

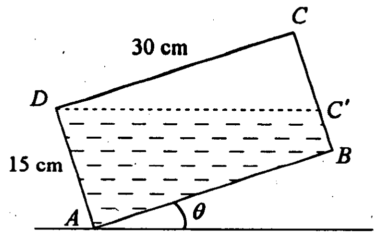


Figure 9.2

(a) (i) Find the area of the metal plates used in making the box.

(ii) Find the capacity of the box.

(3 marks)

(b) Initially, the box is placed on a horizontal table and is full of water. Water is poured out by gradually tilting the box about the edge $AE$. When $V\text{ cm}^3$ of water has been poured out, the inclination of the edge $AB$ with the horizontal is $\theta$ ($0° < \theta < 90°$). Figure 9.2 shows the side view of the box when the water level $DC'$ is above $B$.

(i) When $V\text{ cm}^3$ is half the capacity of the box, find $\theta$.

(ii) If $\tan\theta = \frac{1}{3}$, find $V$.

(iii) If $V = 6750$, find $\theta$.

(7 marks)

(c) Suppose water is poured out by gradually tilting the box about the edge $AB$ instead of $AE$. When half of the water has been poured out of the box, state with reasons whether the inclination of $AE$ with the horizontal is larger than, smaller than, or equal to the value of $\theta$ in (b)(i).

(2 marks)

**END OF PAPER**