FORMULAS FOR REFERENCE

SPHERE	Surface area	$= 4\pi r^2$
	Volume	$= \frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	$= 2\pi rh$
	Volume	$= \pi r^2 h$
CONE	Area of curved surface	$= \pi rl$
	Volume	$= \frac{1}{3}\pi r^2 h$
PRISM	Volume	= base area × height
PYRAMID	Volume	$= \frac{1}{3}$ × base area × height

SECTION A (39 marks)
Answer ALL questions in this section.
There is no need to start each question on a fresh page.

In questions 1 and 2, working need not be shown and you are required to give the answers only.

1. (a) If $y = 2x + 3$, express x in terms of y.

(b) Factorize $ax + 2ay + bx + 2by$.

(c) Simplify $9\sqrt{3} - \sqrt{75}$.

(d) The marks scored by eleven students in a mathematics quiz are as follows:

10 20 30 45 50 60 65 65 65 70 70.

Find

(i) the mean,
(ii) the mode
and (iii) the median

of the above marks.

(6 marks)

2. (a) Express $135°$ in radians. (Give your answer in terms of π.)

(b) If $\sin x° = \sin 36°$ and $90 < x < 270$, find the value of x.

(c) If $\cos y° = -\cos 36°$ and $180 < y < 360$, find the value of y.

(d) In Figure 1, find the area of the sector.

(e) The ratio of the radii of two spheres is 2 : 3. Find the ratio of their volumes.

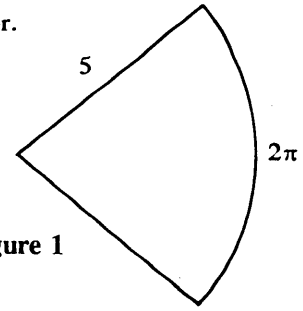

Figure 1

(5 marks)

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3. When $(x + 3)(x - 2) + 2$ is divided by $x - k$, the remainder is k^2. Find the value(s) of k.

(3 marks)

4. Suppose x varies directly as y^2 and inversely as z. When $y = 3$ and $z = 10$, $x = 54$.

(a) Express x in terms of y and z.

(b) Find x when $y = 5$ and $z = 12$.

(3 marks)

5. In Figure 2, calculate

(a) the length of BE,

(b) the values of x and y.

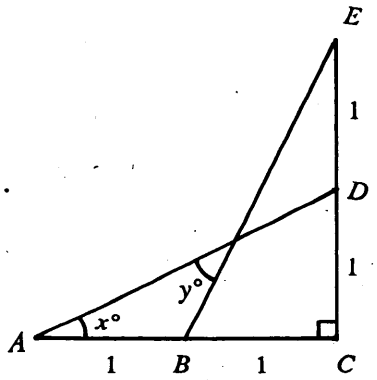

Figure 2

(4 marks)

6. A merchant bought an article for $\$x$. He put it in his shop for sale at a marked price 70% higher than its cost. The article was then sold to a customer at a discount of 5%.

(a) What was the percentage gain for the merchant by selling the article?

(b) If the customer paid \$2 907 for the article, find the value of x.

(5 marks)

7. (a) Simplify $\frac{(a^4b^{-2})^2}{ab}$ and express your answer with positive indices.

(b) If $\log 2 = x$ and $\log 3 = y$, express $\log\sqrt{12}$ in terms of x and y.

(6 marks)

8.

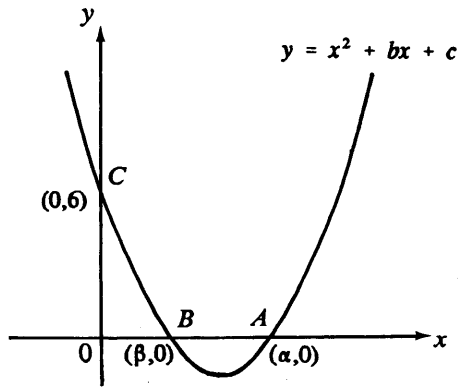

Figure 3

In Figure 3, the curve $y = x^2 + bx + c$ meets the y-axis at $C(0, 6)$ and the x-axis at $A(\alpha, 0)$ and $B(\beta, 0)$, where $\alpha > \beta$.

(a) Find c and hence find the value of $\alpha\beta$.

(b) Express $\alpha + \beta$ in terms of b.

(c) Using the results in (a) and (b), express $(\alpha - \beta)^2$ in terms of b. Hence find the area of ΔABC in terms of b.

(7 marks)

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SECTION B (60 marks)
Answer any FIVE questions in this section.
Each question carries 12 marks.

9. Siu Ming lives in Tuen Mun. He travels to school either by LRT (Light Railway Transit) or on foot. The probability of being late for school is $\frac{1}{7}$ if he travels by LRT and $\frac{1}{10}$ if he travels on foot.

(a) In a certain week, Siu Ming travels to school by LRT on Monday, Tuesday and Wednesday. Find the probability that

(i) he will be late on all these three days;

(ii) he will not be late on all these three days.

(4 marks)

(b) In the same week, Siu Ming travels to school on foot on Thursday, Friday and Saturday. Find the probability that

(i) he will be late on Thursday and Friday only in these three days;

(ii) he will be late on any two of these three days.

(4 marks)

(c) On Sunday, Siu Ming goes to school to take part in a basketball match. If he is equally likely to travel by LRT or on foot, find the probability that he will be late on that day.

(4 marks)

10. Figure 4a shows the longitudinal section of a right cylindrical water tank of base radius 2 metres and height 3 metres. The tank is filled with water to a depth of 1.5 metres.

(a) Express the volume of water in the tank in terms of π . (1 mark)

(b) If a solid sphere of radius 0.6 metre is put into the tank and is completely submerged in water, the water level rises by h metres. Find h (see Figure 4b) . (3 marks)

(c) A solid sphere of radius r metres is put into the tank and is just submerged in water (see Figure 4c) .

(i) Show that $2r^3 - 12r + 9 = 0$.

(ii) Show that the equation in (i) has a root between 0.6 and 1, and find this root, correct to 2 decimal places, by the method of bisection.

(8 marks)

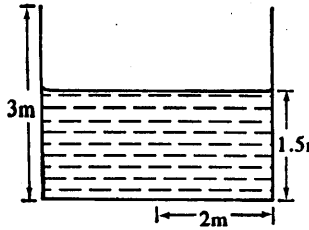

Figure 4a

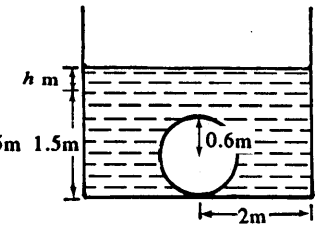

Figure 4b

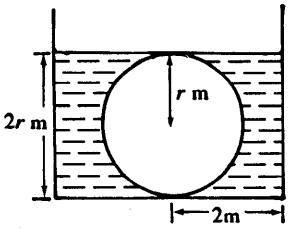

Figure 4c

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11. (a) Draw the following straight lines on the graph paper provided on Page 8 :

$$x + y = 10 ,$$
$$x + 2y = 12 ,$$
$$2x = 3y .$$

(3 marks)

(b) Mr. Chan intends to employ a contractor to build a rectangular flower bed $ABCD$ with length AB equal to x metres and width BC equal to y metres. This project includes building a wall of length x metres along the side AB and fences along the other three sides as shown in Figure 5.

Mr. Chan wishes to have the total length of the four sides of the flower bed not less than 20 metres, and he also adds the condition that twice the length of the flower bed should not less than three times its width. However, no contractor will build the fences if their total length is less than 12 metres.

(i) Write down all the above constraints for x and y.

(ii) Mr. Chan has to pay the contractor \$500 per metre for building the wall and \$300 per metre for building the fences. Find the length and width of the flower bed so that the total payment for building the wall and fences is the minimum. Find also the minimum total payment.

(9 marks)

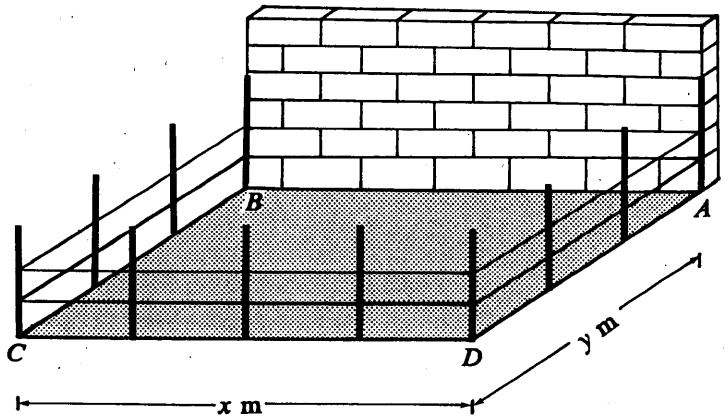

Figure 5

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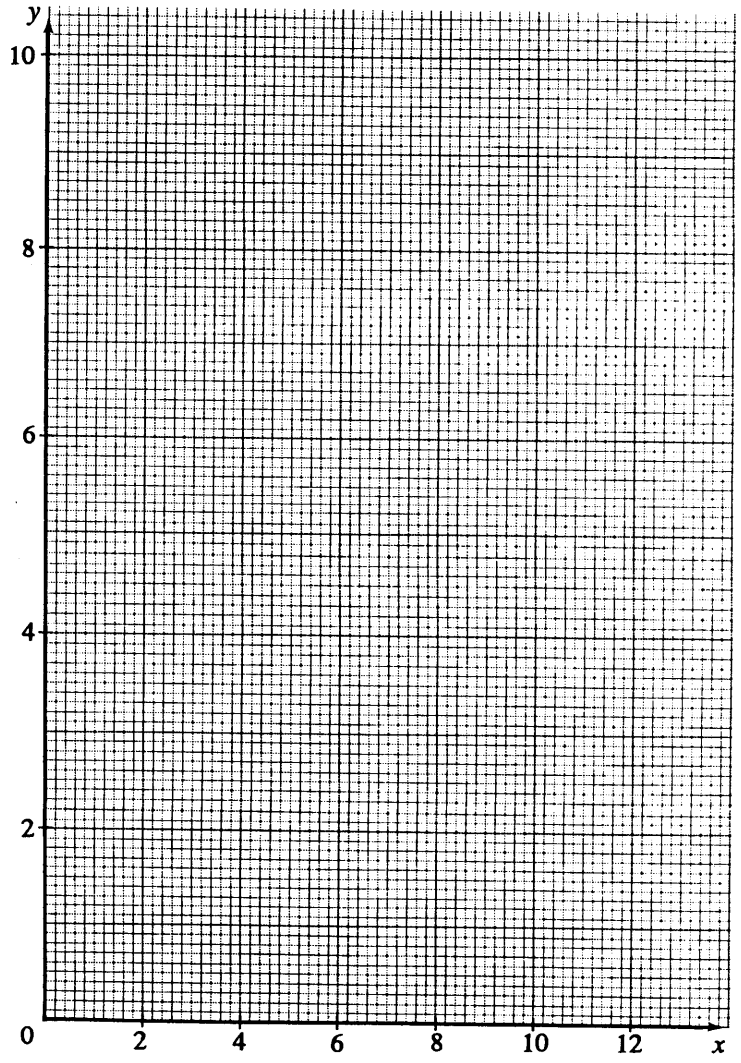
11.(Cont'd) **If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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12. Figure 6 shows two circles

C_1 : $x^2+y^2 = 1$,

C_2 : $(x-10)^2+y^2 = 49$.

O is the origin and A is the centre of C_2. QP is an external common tangent to C_1 and C_2 with points of contact Q and P respectively. The slope of QP is positive.

(a) Write down the coordinates of A and the radius of C_2. (2 marks)

(b) PQ is produced to cut the x-axis at R. Find the x-coordinate of R by considering similar triangles. (3 marks)

(c) Using the result in (b), find the slope of QP. (2 marks)

(d) Using the results of (b) and (c), find the equation of the external common tangent QP. (3 marks)

(e) Find the equation of the other external common tangent to C_1 and C_2. (2 marks)

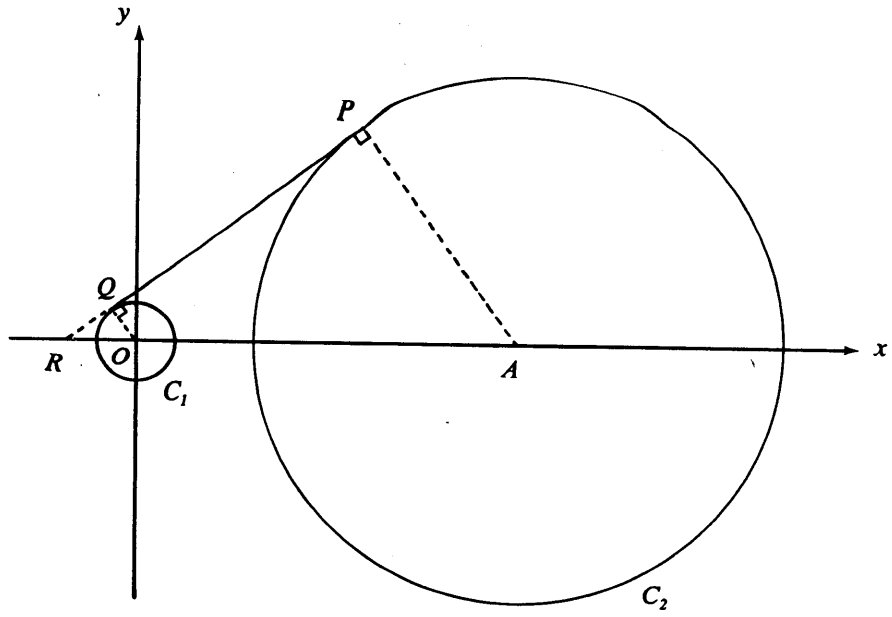

Figure 6

13. **Answers to this question should be written in the blanks provided on p.12 - p.13 .**

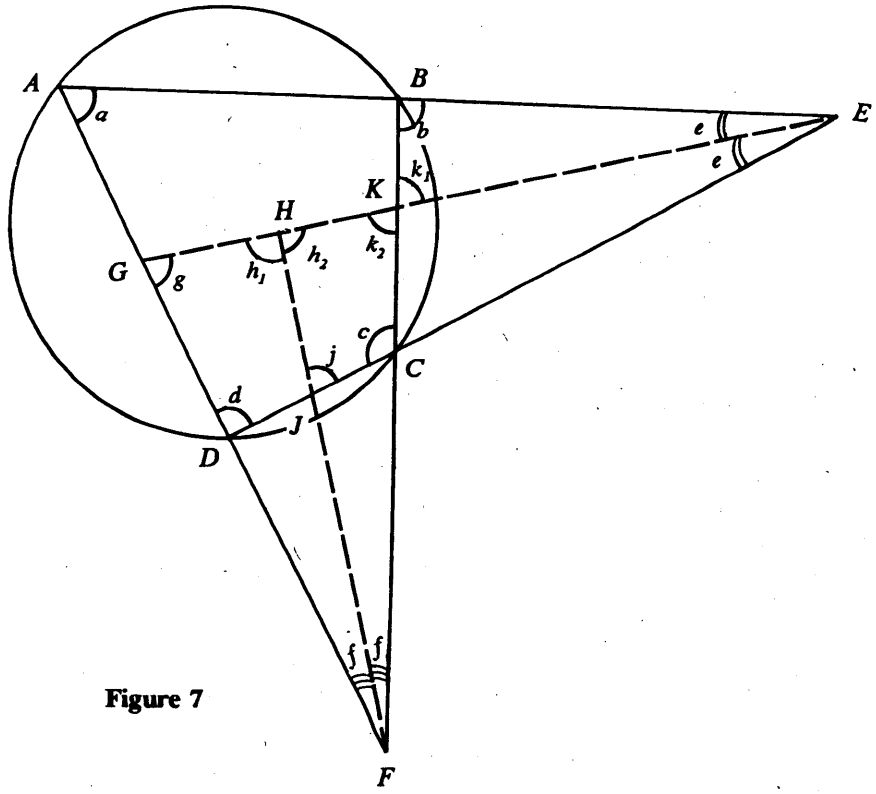

Figure 7

In Figure 7, A, B, C, D are points on a circle and ABE, $GHKE$, $DJCE$, $AGDF$, HJF, $BKCF$ are straight lines. FH bisects $\angle AFB$ and GE bisects $\angle AED$.

(a) Prove that $\angle FGH = \angle FKH$. (3 marks)

(b) Prove that $FH \perp GK$. (3 marks)

(c) (i) If $\angle AED = \angle AFB$, prove that D, J, H, G are concyclic.

(ii) If $\angle AED = 28°$ and $\angle AFB = 46°$, find $\angle BCD$. (6 marks)

Candidate Number	Centre Number	Seat Number	Page Total

If you attempt Question 13, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.

Answers to Question 13

Notations as shown in Figure 7 are used in the following proofs.

(a) In ΔBKE, $b + e + k_1 = 180°$ ([ ])

$k_1 = 180° - b - e$

Similarly, in ΔGDE,

$g = 180° - d - e$

$\because$ $b = d$ ([ ])

$\therefore$ $k_1 = g$

$\because$ $k_1 = k_2$ ([ ])

$\therefore$ $g = k_2$

i.e. $\angle FGH = \angle FKH$

(b) In ΔFHG, $h_1 + f + g = 180°$ ($\angle$ sum of Δ)

$h_1 = 180° - f - g$

Similarly, in ΔFHK,

$h_2 =$ [ ]

$\because$ $g = k_2$ (proved)

$\therefore$ $h_1 =$ [ ]

$\because$ $h_1 + h_2 = 180°$ ([ ])

$\therefore$ $2h_1 = 180°$

$h_1 = 90°$

i.e. $FH \perp GK$

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Answers to Question 13 (Cont'd)

Page Total

(c)(i) In ΔEHJ, $h_1 = j + e$ ( )

$j = h_1 - e$

$= 90° - e$

In ΔFHG, $g + h_1 + f = 180°$ ($\angle$ sum of Δ)

$g = 180° - h_1 - f$

$= 180° - 90° - f$

$= 90° - f$

$\because$ $\angle AED = \angle AFB$ (Given)

$2e = 2f$

$e = f$

$\therefore$ [ ] $= g$

Hence, D,J,H,G are concyclic. ( )

(c)(ii) Solution :

14. In Figure 8, OT is a vertical tower of height h metres and O, P and Q are points on the same horizontal plane. When a man is at P, he finds that the tower is due north and that the angle of elevation of the top T of the tower is 30°. When he walks a distance of 500 metres in the direction N50°E to Q, he finds that the bearing of the tower is N70°W.

(a) Find OQ and OP. (3 marks)

(b) Find h. (2 marks)

(c) Find the angle of elevation of T from Q, giving your answer correct to the nearest degree. (2 marks)

(d) (i) If he walks a further distance of 400 metres from Q in a direction Nθ°E to a point R (not shown in Figure 8) on the same horizontal plane, he finds that the angle of elevation of T is 20°. Find $\angle OQR$ and hence write down the value of θ to the nearest integer.

(ii) If he starts from Q again and walks the same distance of 400 metres in another direction to a point S on the same horizontal plane, he finds that the angle of elevation of T is again 20°. Find the bearing of S from Q, giving your answer correct to the nearest degree.

(5 marks)

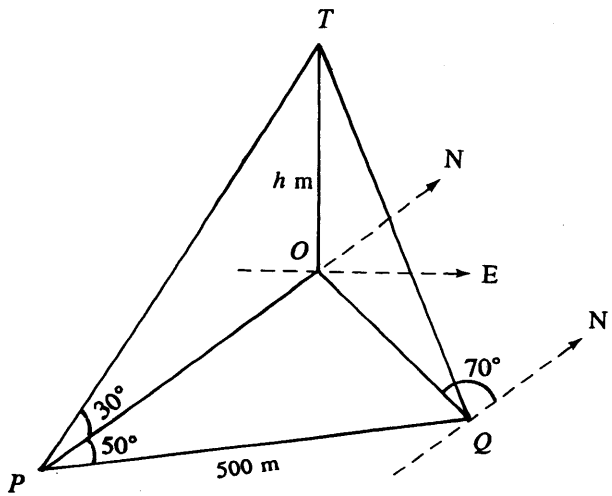

Figure 8

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15. Suppose the number of babies born in Hong Kong in 1994 is 70000 and in subsequent years, the number of babies born each year increases by 2% of that of the previous year.

(a) Find the number of babies born in Hong Kong

(i) in the first year after 1994;

(ii) in the nth year after 1994.

(2 marks)

(b) In which year will the number of babies born in Hong Kong first exceed 90000? (3 marks)

(c) Find the total number of babies born in Hong Kong from 1997 to 2046 inclusive. (3 marks)

(d) It is known that from 1901 to 2099, a year is a leap year if its number is divisible by 4.

(i) Find the number of leap years between 1997 and 2046.

(ii) Find the total number of babies born in Hong Kong in the leap years between 1997 and 2046.

(4 marks)

END OF PAPER