 93 Math. I

Solution	Marks	Remarks
1. (a) The simple interest = \$1.5	1A	
(b) $h = 64.3$	1A	Any figure roundable to 64.3
(c) $x = \frac{21}{5}$ (= 4.2)	1A	
(d) (i) $x + 2y$ is greatest at (1, 4)	1A	Accept answers showing in reasonable order
(ii) $x + 2y$ is least at (0, −3)	1A	
The greatest value is 9.	1A	
The least value is −6.	1A	
	7	
2. (a) $f(3) = 5$	1A	
(b) $y = \frac{6x-3}{2x}$ $\left(= 3 - \frac{3}{2x}\right)$	1A	
(c) $\frac{1}{x-1} - \frac{1}{x+1} = \frac{2}{x^2-1}$ $\left(= \frac{2}{(x-1)(x+1)}\right)$	1A	
(d) The remainder is 2.	1A	
(e) H.C.F. $= 2xy^2$	1A+1A	
L.C.M. $= 12x^2y^3z$ (or $2^2 \cdot 3x^2y^3z$)		
(f) $r = 1$, $s = -2$	1A+1A	
(g) $\frac{1}{\sqrt{3}-1} = \frac{\sqrt{3}+1}{2}$	1A	
	9	
3. $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{3}{2}$		
$2\sin\theta + 2\cos\theta = 3\sin\theta - 3\cos\theta$		
$\sin\theta = 5\cos\theta$	1A	
∴ $\tan\theta = 5$	1M	
$\theta = 78.7°$ or $259°$	1A+1A	roundable to 78.7°, 259° deduct 1A for each excess answer
	4	

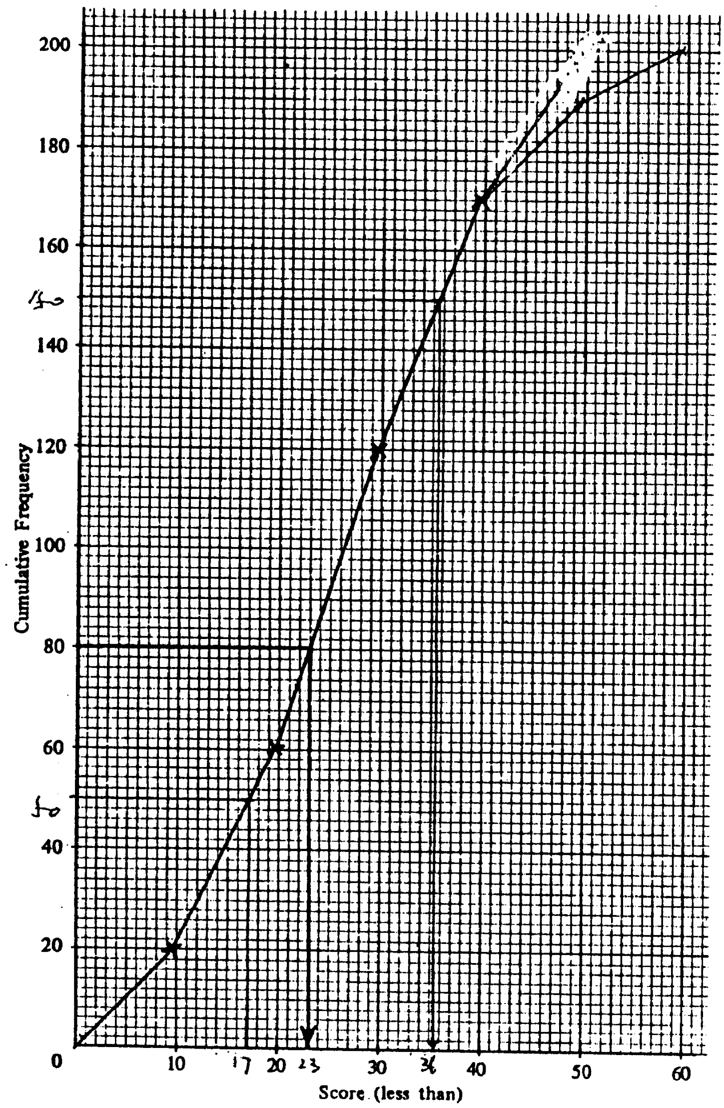
(1,4) … 9; (4,0) … 4; (−3,0) … 3; (0,−3) … −6 — 0分; greatest value = 9, least value = −6. 4分. $\frac{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\cos\theta}} = \frac{3}{2}$ 1M; $\tan\theta = 5$ 1A. 用 radian 表示亦可. [illegible]

Remark

Solution	Marks	Remarks
$\sin\theta = 5\cos\theta$ (same as above)	1A	
$\sin^2\theta = 25\cos^2\theta$		
$\sin^2\theta = 25(1 - \sin^2\theta)$	1M	
$\sin^2\theta = \frac{25}{26}$		
(i) If $\sin\theta = \sqrt{\frac{25}{26}}$, $\theta = 78.7°$ or $101°$ (rej.)	1A	
(ii) If $\sin\theta = -\sqrt{\frac{25}{26}}$, $\theta = 259°$ or $281°$ (rej.)	1A	

寫成全部 sine 或全部 cosine. [illegible] rej.

Solution	Marks	Remarks
4. $x^2 - x - 2 < 0$		
$(x+1)(x-2) < 0$	1A	for factorization or accept as $x=-1,2$
$\therefore \quad -1 < x < 2$	2A	deduct 1A for any equal sign, accept graphical solution [illegible]
Putting $x = y - 100$, we have or $(y-100+1)(y-100-2) < 0$	1M	
$-1 < y - 100 < 2$		
$\therefore \quad 99 < y < 102$	2A	
	6	
5. (a) $9^x = \sqrt{3}$		
$9^x = 3^{\frac{1}{2}}$ (or $3^{2x} = \sqrt{3}$, $9^{2x} = 3$ etc.) $9^{\frac{1}{4}} = \sqrt{3}$	1A	
$3^{2x} = 3^{\frac{1}{2}}$ $\therefore x = \frac{1}{4}$ (全對)		
$2x = \frac{1}{2}$	1M	equating index with the same base
$\therefore \quad x = \frac{1}{4}$	1A 3	
OR Taking logarithms		
$x \log 9 = \log\sqrt{3}$	1M	
$x = \dfrac{\log\sqrt{3}}{\log 9}$	1A	
$= 0.25$	1A	
(b) $x\left(\dfrac{x^{-1}}{y^2}\right)^{-3} = x\left(\dfrac{1}{xy^2}\right)^{-3}$		
$= x(xy^2)^3$		
$= x(x^3y^6)$		
$= x^4y^6$	2M+1A	1M for correct use of the formula $a^{-n} = \dfrac{1}{a^n}$ 1M for correct use of the formula $(a^m)^n = a^{mn}$
	3 6	

Solution	Marks	Remarks
6. (a) $\alpha + \beta = \frac{m}{2}$ (can be anywhere)	1A	
$\alpha\beta = \frac{500}{2}$ $(= 250)$	1A	
The area of the picture $= \alpha\beta = 250$	1A	
(b) (i) The perimeter $= 2(\alpha + \beta)$		
$= 2(\frac{m}{2}) = m$	1A	
(ii) The area of the border		
$= (\alpha + 4)(\beta + 4) - \alpha\beta$	1A+ 1M	$(\alpha + 4)(\beta + 4)$ subtracting answer in (a)
$= \alpha\beta + 4(\alpha + \beta) + 16 - \alpha\beta$		
$= 4(\frac{m}{2}) + 16$		
$= 2m + 16$	1A	
	7	
OR		
$= 2[2(\beta + 4) + 2\alpha]$	1M+1A	1M for summation of areas
$= 4(\alpha + \beta) + 16$		
$= 2m + 16$	1A	
7.		
		accept plotting the points with error ±0.5 line segment from score 0 to score 9.5 is optional

Solution	Marks	Remarks
7. (a) Cummulative Frequency Table		

Score (less than)	Cummulative Frequency
9.5	20
19.5	60
29.5	120
39.5	170
49.5	190
59.5	200

Solution	Marks	Remarks
	1A+1A	1A for any 3 correct
	2	
(b) (i) Cummulative frequency polygon.	1M+1A	must be line segments 1M for following the data in (a)
The upper quartile = 36 (or 35)		
The lower quartile = 17		上要 freq polygon 要對才給分
∴ The interquartile range = 36 − 17 = 19	1M+1A	Accept 18–19 1M for using the 25% or $(\frac{N+1}{4})$th value, etc.
(ii) If the pass percentage is set at 60%, the number of students failed would be		
200 × (1 − 60%) = 80 No. of students passed 120	1M	80 or horizontal line through 80 on the graph
The pass score should be 23	1A	
	6	
(c) Mean = 26.5 (exact value).	1A	Working steps are not required
Standard deviation = $\sqrt{166}$ (= 12.9)	1A	r.t. 12.9
	2	
(d) The new mean is increased by 20.	1M	or exact answer
i.e. Mean = 26.5 + 20 = 46.5		
The new standard deviation is unchanged	1M	or exact answer
i.e. Standard deviation = $\sqrt{166}$ (= 12.9)		
	2	

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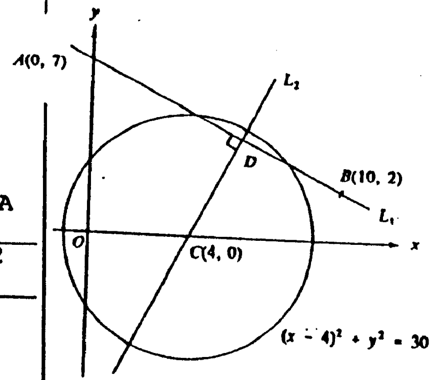
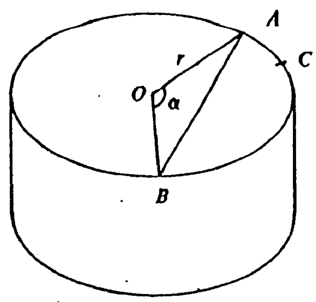
Solution	Marks	Remarks
8. (a) The slope of $L_1 = \frac{2-7}{10-0}\left(=-\frac{1}{2}\right)$	1A	
The equation of L_1 is $y-7=-\frac{1}{2}(x-0)$		
i.e. $y=-\frac{1}{2}x+7$		
(or $x+2y-14=0$, $5x+10y-70=0$, etc.)	1A	
	2	
(b) Slope of $L_1=-\frac{1}{2}$		
As $L_2 \perp L_1$, slope of $L_2=2$	1M	
The equation of L_2 is		
$y-0=2(x-4)$		
i.e. $y=2x-8$ (or $2x-y-8=0$, etc.)	1A	
2		
Solving $\begin{cases} y=-\frac{1}{2}x+7 \\ y=2x-8 \end{cases}$		
$2x-8=-\frac{1}{2}x+7$	1M	"成功消元 eliminate into 1 unknown
The coordinates of D are $x=6$, $y=4$ (or $D=(6, 4)$)	1A	
2	4	
(c) As $AP:PB=k:1$, the coordinates of P are given by		
$x=\frac{10k}{1+k}$, $y=\frac{2k+7}{1+k}$	1A+1A	
Substituting in the equation of the circle,		
$\left(\frac{10k}{1+k}-4\right)^2+\left(\frac{2k+7}{1+k}\right)^2=30$	1M	
$(6k-4)^2+(2k+7)^2=30(k+1)^2$		
$\therefore\ 10k^2-80k+35=0$		
$2k^2-16k+7=0$ .... (*) $4\pm\frac{5}{2}\sqrt{2}$	1	
$k=\frac{16\pm\sqrt{16^2-4\times2\times7}}{4}=\frac{8\pm5\sqrt{2}}{2}$ (7.54 or 0.464)	1A	accept $\frac{16\pm\sqrt{200}}{4}$
As P lies on AD, $\frac{AP}{PB}=\frac{8-5\sqrt{2}}{2}$ (0.464)	1M	choosing the smaller one from 2 positive values
	6	

Solution	Marks	Remarks
9. (a) (i) Area of the sector $OACB = \frac{1}{2}r^2\alpha$	1A	
(ii) Area of $\triangle OAB = \frac{1}{2}r^2\sin\alpha$ (or $r^2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$)		
$\therefore$ area of the segment ACB $= \frac{1}{2}r^2\alpha - \frac{1}{2}r^2\sin\alpha$ (or $\frac{1}{2}r^2\alpha - r^2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$)	1M+1A	
(iii) As AB divides the circle in the ratio 4:1		
$\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\sin\alpha = \frac{1}{5}\pi r^2$	1M	correct use of the ratio 4:1
$\therefore$ $\sin\alpha = \alpha - \frac{2\pi}{5}$	1	
Remark		
$\dfrac{\frac{1}{2}r^2(2\pi-\alpha) + \frac{1}{2}r^2\sin\alpha}{\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\sin\alpha} = 4$	1M	
$\pi r^2 - \frac{1}{2}r^2\alpha + \frac{1}{2}r^2\sin\alpha = 2r^2\alpha - 2r^2\sin\alpha$		
$\therefore$ $\sin\alpha = \alpha - \frac{2\pi}{5}$	1	
(iv) Let $f(\alpha) = \sin\alpha - \alpha + \frac{2\pi}{5}$		**OR** $f(\alpha) = \alpha - \sin\alpha - \frac{2\pi}{5}$
$f(2.1)(\approx 0.0198) > 0$ $f(2.2)(\approx -0.1349) < 0$ $\begin{cases} f(2.1) < 0 \\ f(2.2) > 0 \end{cases}$ 亦可		$f(2.1) < 0$ $f(2.2) > 0$
$\therefore$ $f(\alpha) = 0$ has a root between 2.1 and 2.2 .	1	for showing opposite signs

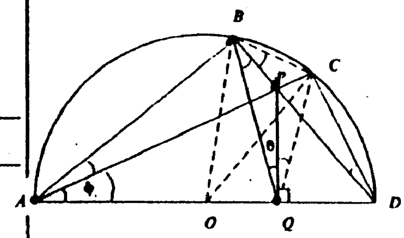
(v)

Interval	Mid-value α_i	$f(\alpha_i)$
$2.1 < \alpha < 2.2$	2.15	−ve (−0.056)
$2.1 < \alpha < 2.15$	2.125	−ve (−0.018)
$2.1 < \alpha < 2.125$	2.1125	+ve (0.00097)
$2.1125 < \alpha < 2.125$	2.11875	−ve (−0.0085)
$2.1125 < \alpha < 2.11875$	2.115625	−ve (−0.0038)
$2.1125 < \alpha < 2.115625$	2.1140625	−ve (−0.0014)
$2.1125 < \alpha < 2.1140625$	2d.p. 亦可	

Solution	Marks	Remarks
(table above)	1M+1A 1M	– for correct sign Testing sign at mid-value Correct choice of next interval Accept using smaller or larger starting intervals
$\therefore$ $\alpha \approx 2.11$ (corr. to 2 d.p.) [illegible] +, − [illegible] 可免寫 interval	1A	Check whether it is bounded by the last interval
	10	
(b) As the curved surface has uniform height, ratio of the curved surface areas of the two parts = ratio of the corresponding arc lengths.		
$= r(2\pi - \alpha) : r\alpha$ OR $(2\pi-\alpha) : \alpha$	1M	OR $r\alpha : r(2\pi-\alpha)$ OR $\alpha : 2\pi - \alpha$
$= 2\pi - 2.11 : 2.11$		
$\approx 1.98 : 1$	1A	1.98 及 1 之位置不能對調 r.t. 1.98
	2	
OR Let the height be h.		
$\therefore$ ratio required $= r(2\pi-\alpha)h : r\alpha h$	1M	
$= \ldots$		
$\approx 1.98 : 1$	1A	

Solution	Marks	Remarks
10. (a) The annual food production in		
(i) the 3rd year $= 8 + 1 \times 2$		
$= 10$ (or 10 (million tonnes))	1A	$u-1$ for 10×10^6 or 10×10^6 million tonnes
(ii) the nth year $= 8 + (n-1)$ — or 加上單位 (million tonnes)	1A	
$= 7 + n$		
	2	
(b) The total food production in the first 25 years		
$= \frac{25}{2}[2\times 8 + (25-1)\times 1]$ or $\frac{25}{2}[8 + 8+(25-1)\times 1]$	1A	
$= 500$	1A	
	2	
(c) The population of the country at the end of		
(i) the 3rd year $= 2 \times (1 + 6\%)^2$		
$= 2.25$ million	1A	r.t. 2.25
(ii) the nth year $= 2 \times (1 + 6\%)^{n-1}$ million (or $2 \times 1.06^{n-1}$ million)	1A	
	2	
(d) For the population to be doubled,		
$2 \times (1 + 6\%)^n = 4$ (≥ 亦可)	1M	accept "answer of c(ii) = 4"
Taking logarithm $n \log 1.06 = \log 2$	1M	for taking lograithm
$n = \frac{\log 2}{\log 1.06} = 11.9$		
The minimum number of years for the population to be doubled is 12 years.	1A	accept values r.t. 11.9
	3	
(e) The annual food production per capita of the 100th year		
$= \frac{7 + 100}{2 \times 1.06^{99}}$	1M+1A	1M for substituting $n=100$ to $\frac{\text{ans.of(a)(ii)}}{\text{ans.of(c)(ii)}}$
$= 0.167$		
< 0.2		
$\therefore$ the country will face a food shortage problem.	1M	corresponding logical conclusion
	3	

Solution	Marks	Remarks
11. (a) Join AB.		
$\angle ABD = 90°$ ($\boxed{\angle \text{ in a semicircle}}$)	1	accept "semicircle" or "diameter"
and $\angle AQP = \boxed{90°}$ (Given) 缺 degree 扣1	1	
$\therefore$ $\angle ABD + \angle AQP = \boxed{180°}$ 缺∠符號 扣1	1	
$\therefore$ $AQPB$ is a cyclic quadrilateral. (Opp. $\angle$s supp.)		
i.e. A, Q, P, B are concyclic.	3	
(b) (i) Join CD.		
Using the same argument as in (a), it can be shown that $PQDC$ is a cyclic quadrilateral.		
$\therefore$ $\angle PQC = \boxed{\angle PDC \text{ (or } \angle BDC)}$ ($\angle$s in the same segment)	1	
Now consider the cyclic quadrilateral $ADCB$.		
$\angle BDC = \boxed{\angle BAC \text{ (or } \angle BAP)}$ ($\angle$s in the same segment)	1	
As $AQPB$ is a cyclic quadrilateral,		
$\angle BAP = \boxed{\angle BQP \text{ (or } \theta)}$ ($\angle$s in the same segment)	1	
$\therefore$ $\angle BQC = \angle BQP + \angle PQC$		
In terms of θ,		
$\angle BQC = \boxed{2\theta}$	1	
(ii) Consider the given semi-circle.		
$\angle BOC = 2 \times \angle BAC$ $\boxed{\angle \text{ at centre} = \text{twice } \angle \text{ at } \odot^{ce}}$	1	accept "$\angle$ at centre" or "O is the centre"
But $\angle BAC = \theta$ (Proved) (edge, side 亦可)		
$\therefore$ $\angle BOC = \boxed{2\theta}$	1	
	6	
(c) Solution :		
Consider the quadrilateral $AQPB$.		
$\angle PBQ$ $(= \angle PAQ) = \phi$	1	
But in the given semi-circle,		
$\angle CBD$ $(= \angle CAD) = \phi$	1	
$\therefore$ $\angle CBQ = \angle CBD + \angle PBQ$		
$= 2\phi$ 只得答案有一分	1	
	3	
OR		
$\because$ $\angle BQC = \angle BOC$		
$\therefore$ B, O, Q, C are concyclic	1	
Hence $\angle CBQ = \angle COQ$	1	
$= 2\angle CAD$		
$= 2\phi$	1	



Solution	Marks	Remarks
12. (a) (i) As PQ is perpendicular to the plane ABQ		
$\tan 45° = \frac{PQ}{AQ}$ and $\tan 60° = \frac{PQ}{BQ}$ (或 $PQ=AQ$)	1A	for either
$\therefore \quad AQ = \frac{h}{\tan 45°} = h$ metres	1A	
$BQ = \frac{h}{\tan 60°} = \frac{h}{\sqrt{3}}$ metres (or $0.577h$)	1A	r.t. 0.577
(ii) Consider $\triangle ABQ$.		
By the cosine rule,		
$AB^2 = AQ^2 + BQ^2 - 2AQ \cdot BQ \cos \angle AQB$		
$100^2 = h^2 + (\frac{h}{\sqrt{3}})^2 - 2(h)(\frac{h}{\sqrt{3}}) \cos 80°$	1M+1A	
$= 1.13282h^2$		
$\therefore \quad h = 94.0 \quad (93.9549)$	1A	r.t. 94.0
Consider $\triangle ABQ$ again.		
By the sine rule, $\frac{BQ}{\sin \angle QAB} = \frac{AB}{\sin \angle AQB}$,		
$\frac{\frac{93.9549}{\sqrt{3}}}{\sin \angle QAB} = \frac{100}{\sin 80°}$	1M	
$\therefore \quad \sin \angle QAB = \frac{93.9549 \times \frac{1}{\sqrt{3}} \sin 80°}{100} = 0.5342$		
$\angle QAB = 32.3° \quad (32.2902°)$	1A	r.t. 32.3 accept 32°15′-32°21′
	8	

Solution	Marks	Remarks
OR		
...		
By the cosine rule,		
$QB^2 = AQ^2 + AB^2 - 2(AQ)(AB) \cos \angle QAB$		
$\left(\frac{93.9549}{\sqrt{3}}\right)^2 = 93.9549^2 + 100^2 - 2 \times 93.9549 \times 100 \cos \angle QAB$	1M	
$\cos \angle QAB = 0.8454$		
$\angle QAB = 32.3°$	1A	

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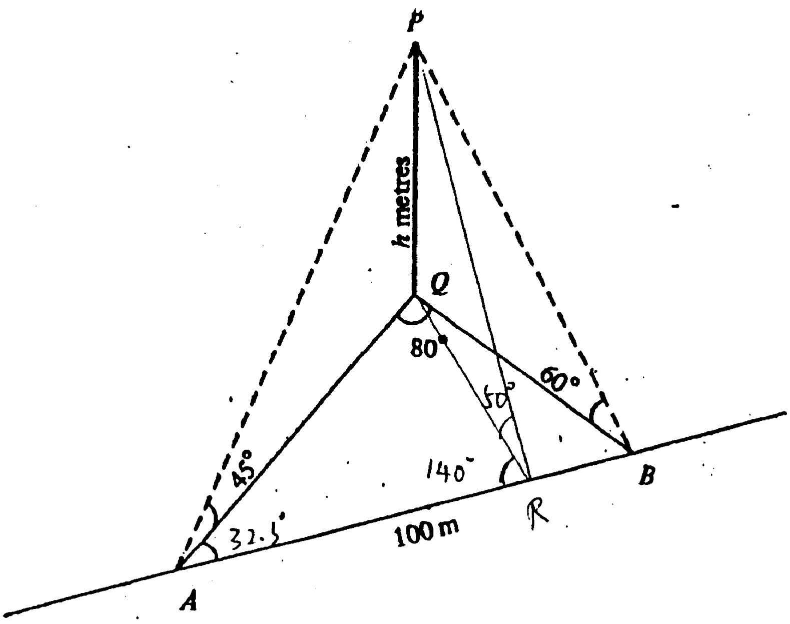
Solution	Marks	Remarks
(b) Consider $\triangle PQR$.		
$\tan 50° = \dfrac{h}{QR}$	1M	
$\therefore \quad QR = \dfrac{h}{\tan 50°}$ metres		
Consider $\triangle AQR$.		
By the sine rule, $\dfrac{QR}{\sin\angle QAR} = \dfrac{AQ}{\sin\angle ARQ}$		
$\dfrac{\frac{h}{\tan 50°}}{\sin 32.2902°} = \dfrac{h}{\sin\angle ARQ}$	1M	
$\angle ARQ = \sin^{-1}(\sin 32.2902° \tan 50°)$		
$= \sin^{-1} 0.636644$		
$= 140.45796°$ (as it is obtuse)	1A	r.t. 140-141
$\therefore \quad \angle AQR = 180° - 140.45796° - 32.2902°$		
$= 7.2518°$		
Using the sine rule again.		
$AR = \dfrac{AQ \sin 7.2518°}{\sin 140.45796°}$		
$= 18.6$ m	1A	r.t. 18.4-18.7
	4	



$QR^2 = AQ^2 + AR^2 - 2AQ \cdot AR \cos 32.29°$ (1M)

$(78.875)^2 = (85.854)^2 + AR^2 - 2\times(85.854) AR \cos 32.29°$

$AR^2 - 158.8 AR + 2611.17 = 0$ (1A)

$AR = 18.64$ or $AR = 140.8$ (rejected)

$AR = 18.6$ (1A)

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Solution	Marks	Remarks
13. (a) (i) The prob. that the elections in the Tuen Mun and Yuen Long constituencies would both be won by the Democrats		
$= 0.65 \times 0.45$	1A	
$= 0.2925$	1A	r.t. 0.293 accept $\frac{117}{400}$ or $\frac{2925}{10000}$
(ii) The prob. that the elections in the two constituencies would both be won by the Liberals		
$= (0.25 + 0.1) \times 0.55$ $= 0.1925$	1A	
∴ the probability that the elections in the two constituencies would both be won by the same party		
$= 0.2925 + 0.1925$	1M	
$= 0.485$	1A	r.t. 0.485-0.486 accept $\frac{97}{200}$ or $\frac{485}{1000}$
	5	
(b) (i) The probability that a vote came from the Tuen Mun constituency and was for 'The Democrats'		
$= \frac{40000 \times 70\%}{60000}$ (or $\frac{28000}{60000}$)	1A	
$= \frac{7}{15}$ (or 0.467)		
The probability that both votes came from the Tuen Mun constituency and were for 'The Democrats'		
$= \left(\frac{7}{15}\right)^2$	1M	
$= \frac{49}{225}$ (or 0.218)	1A	r.t. 0.218
(ii) The probability that a vote was for 'The Democrats'		
$= \frac{40000 \times 70\% + 20000 \times 40\%}{60000}$ $\left(\text{or } \frac{28000 + 8000}{60000}\right)$	1A	
$= \frac{3}{5}$ (or 0.6)		$\frac{28000}{60000}$ $\frac{8000}{60000}$
The probability that both votes were for 'The Democrats'.		
$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$ (or 0.36)	1A	$\frac{28000}{60000}$ $\frac{8000}{60000}$
(iii) The probability that a vote was for 'The Liberals'		
$= \frac{8000 + 4000 + 12000}{60000}$ (or $1 - \frac{3}{5}$)		
$= \frac{24000}{60000} = \frac{2}{5}$		
The probability that both votes were for different parties		
$= 1 - (\frac{3}{5})^2 - (\frac{2}{5})^2$ or $2 \times \frac{3}{5} \times \frac{2}{5}$	1A	
$= \frac{12}{25}$ (or 0.48)	1A	
	7	

Candidate	No. of votes
A	28000
B	8000
C	4000
P	8000
Q	12000