FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height .

SECTION A (39 marks)
Answer ALL questions in this section.
There is no need to start each question on a fresh page.
In questions 1-3, working steps are not required and you need to give the answers only.

1. (a) Express 30° in radians, leaving your answer in terms of π .

   (b) Find x if $\sin x = \frac{1}{2}$ and $90° < x < 180°$.

   (c) Simplify $\frac{1 - \sin^2 A}{\cos A}$.

(3 marks)

2. (a) If $\log x = p$ and $\log y = q$, express $\log xy$ in terms of p and q .

   (b) Find the remainder when $x^3 - 2x^2 + 3x - 4$ is divided by $x - 1$.

   (c) Rationalize $\frac{1}{\sqrt{3} + \sqrt{2}}$. (3 marks)

3. In Figure 1, the shaded region, including the boundary, is determined by three inequalities.

(a) Write down the three inequalities.

(b) How many points (x, y), where x and y are both integers, satisfy the three inequalities in (a)?

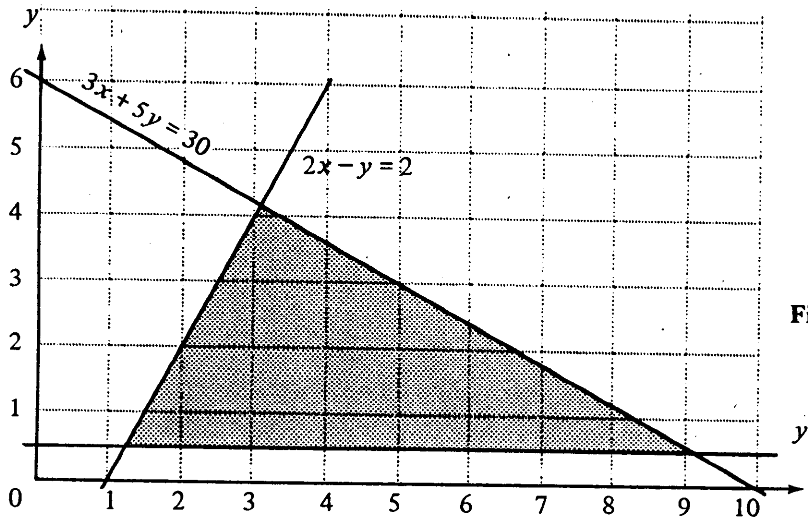

Figure 1

(4 marks)

4. (a) Factorize

(i) $x^2 - 2x$,

(ii) $x^2 - 6x + 8$.

(b) Simplify $\dfrac{1}{x^2 - 2x} + \dfrac{1}{x^2 - 6x + 8}$. (5 marks)

5. L_1 is the line passing through the point $A(10, 5)$ and perpendicular to the line $L_2 : x - 2y + 5 = 0$.

(a) Find the equation of L_1 .

(b) Find the intersection point of L_1 and L_2 . (6 marks)

6. Find the range of values of k so that the quadratic equation $x^2 + 2kx + (k + 6) = 0$ has two distinct real roots.

(6 marks)

7.

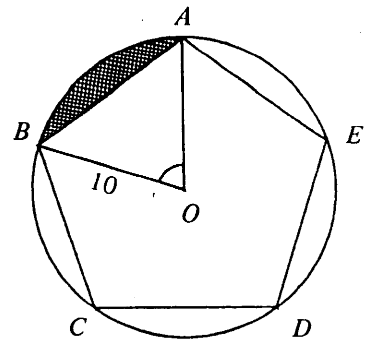

Figure 2

In Figure 2, $ABCDE$ is a regular pentagon inscribed in a circle with centre O and radius 10.

(a) Find $\angle AOB$ and the area of triangle OAB .

(b) Find the area of the shaded part in the figure.

(6 marks)

8. In a sports competition, the mean score of a team of m men and n women is 70.

(a) Find the total score of the team in terms of m and n .

(b) If the mean score of the men is 75 and the mean score of the women is 62, find the ratio $m : n$.

(c) If there are altogether 39 persons in the team, find the number of men.

(6 marks)

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SECTION B (60 marks)
Answer any FIVE questions from this section.
Each question carries 12 marks.

9.

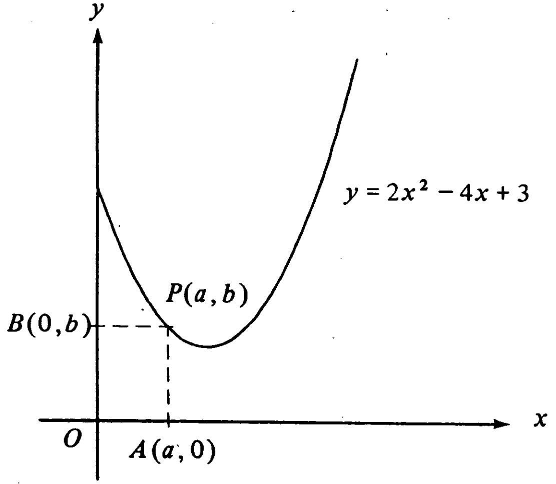

Figure 3

Figure 3 shows the graph of $y = 2x^2 - 4x + 3$, where $x \geq 0$. $P(a, b)$ is a variable point on the graph. A rectangle $OAPB$ is drawn with A and B lying on the x- and y- axes respectively.

(a) (i) Find the area of rectangle $OAPB$ in terms of a.

(ii) Find the two values of a for which $OAPB$ is a square.

(6 marks)

(b) Suppose the area of $OAPB = \frac{3}{2}$.

(i) Show that $4a^3 - 8a^2 + 6a - 3 = 0$ ....(*) .

(ii) Show that there is a root of (*) lying between 1.2 and 1.3 .

Hence use the method of bisection to find this root, correct to 2 decimal places.

(6 marks)

10.

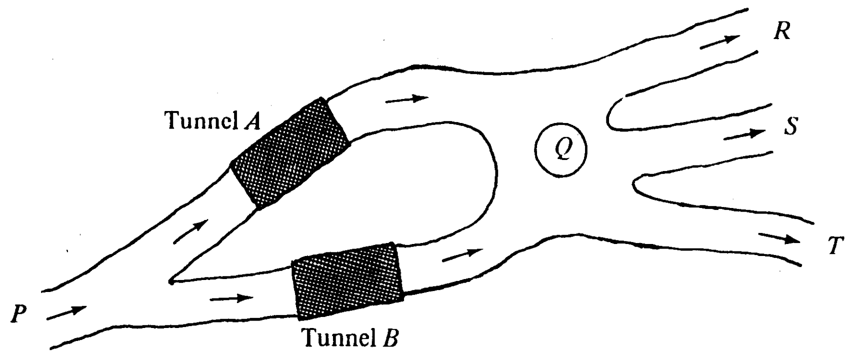

Figure 4

Figure 4 shows a one-way road network system from Town P to Towns R, S and T. Any car leaving Town P will pass through either Tunnel A or Tunnel B and arrive at Towns R, S or T via the roundabout Q. A survey shows that $\frac{2}{5}$ of the cars leaving P will pass through Tunnel A. The survey also shows that $\frac{1}{7}$ of all the cars passing through the roundabout Q will arrive at R, $\frac{2}{7}$ at S, and $\frac{4}{7}$ at T.

(a) Find the probabilities that a car leaving P will

(i) pass through Tunnel B,
(ii) not arrive at T,
(iii) arrive at R through Tunnel B,
(iv) pass through Tunnel A but not arrive at R.

(6 marks)

(b) Two cars leave P. Find the probabilities that

(i) one of them arrives at R and the other one at S,

(ii) both of them arrive at S, one through Tunnel A and the other one through Tunnel B. (6 marks)

11.

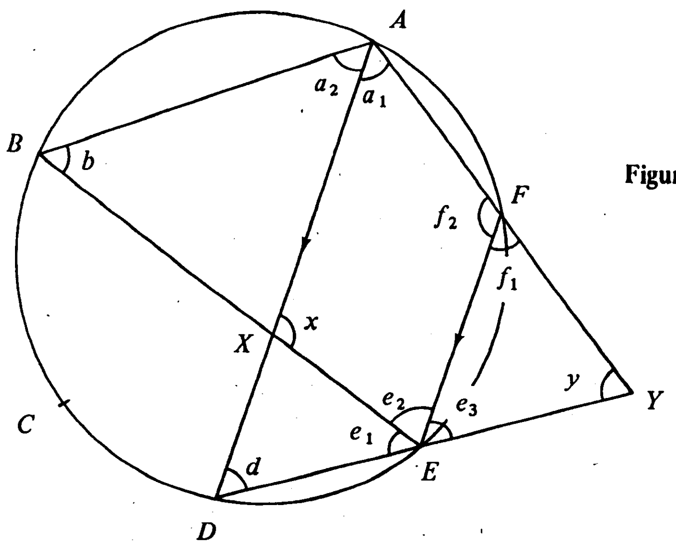

Figure 5

Answers to this question should be written in the blanks provided on p.8 – p.9 .

In Figure 5, A, B, C, D, E and F are points on a circle such that $AD // FE$ and $\overset{\frown}{BCD} = \overset{\frown}{AFE}$. AD intersects BE at X. AF and DE are produced to meet at Y.

(a) Prove that $\triangle EFY$ is isosceles. (3 marks)

(b) Prove that $BA // DE$. (1 mark)

(c) Prove that A, X, E, Y are concyclic. (3 marks)

(d) If $\angle b = 47°$, find $\angle f_1$, $\angle y$ and $\angle x$. (5 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet inside your answer book.

Answers to Question 11

(a) **Proof :**

$\angle f_1 =$ ______ (Corr. $\angle$s , $AD // FE$)

But ______ = ______ (Ext. $\angle$, cyclic quad.)

$\therefore \angle f_1 = \angle e_3$

$\therefore EY =$ ______ (Sides opp. equal $\angle$s)

i.e. $\triangle EFY$ is isosceles

(b) **Proof :**

$\overset{\frown}{BCD} = \overset{\frown}{AFE}$ (Given)

$\therefore \angle a_2 =$ ______ (Equal arcs subtend equal $\angle$s at circumference)

$\therefore BA // DE$ (Alt. $\angle$s equal)

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Answers to Question 11 (Cont'd)

(c) **Proof :**

$\angle a_1$ = ____________ (Corr $\angle$s, $AD // FE$)

But ____________ $= \angle b$ (Ext. $\angle$, cyclic quad.)

and $\angle b = \angle e_1$ (Alt. $\angle$s, $BA // DE$)

$\therefore$ $\angle a_1$ = ____________

$\therefore$ A , X , E , Y are concyclic (Ext. $\angle$ equals int. opp. $\angle$)

(d) **Solution :**

12.

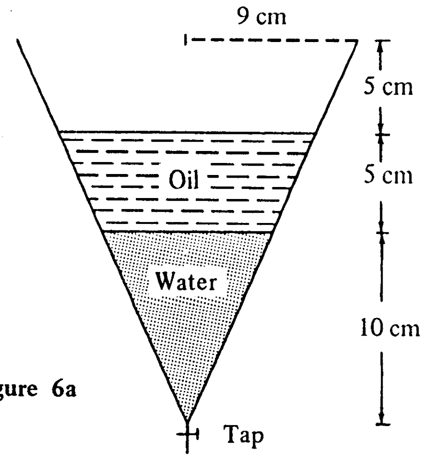

Figure 6a

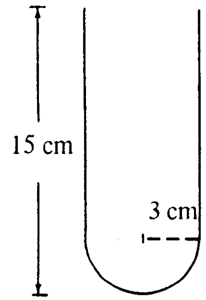

Figure 6b

Figure 6a shows a vertical cross-section of a separating funnel with a small tap at its vertex. The funnel is in the form of a right circular cone of base radius 9 cm and height 20 cm. It contains oil and water (which do not mix) of depths 5 cm and 10 cm respectively, with the water at the bottom.

(a) (i) Find the capacity of the separating funnel in terms of π .

(ii) Find the ratios

volume of water : total volume of oil and water : capacity of the funnel.

Hence, or otherwise, find the ratios

volume of water : volume of oil : capacity of the funnel.

(6 marks)

(b) All the water in the funnel is drained through the tap into a glass tube of height 15 cm. The glass tube consists of a hollow cylindrical upper part of radius 3 cm and a hollow hemispherical lower part of the same radius, as shown in Figure 6b.

Find the depth of the water in the glass tube. (3 marks)

(c) After all the water has been drained into the glass tube, find the depth of the oil remaining in the funnel. (3 marks)

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13.

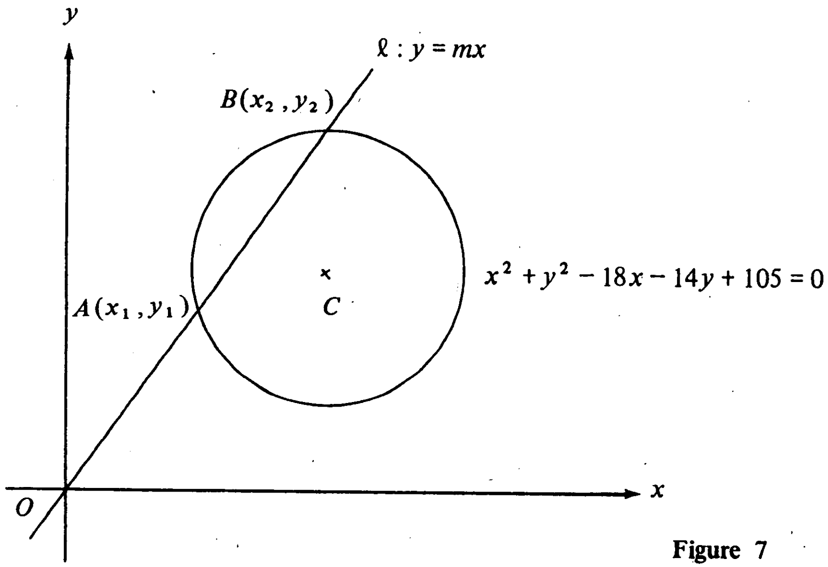

Figure 7

In Figure 7, the line $\ell : y = mx$ passes through the origin and intersects the circle $x^2 + y^2 - 18x - 14y + 105 = 0$ at two distinct points $A(x_1, y_1)$ and $B(x_2, y_2)$.

(a) Find the coordinates of the centre C and the radius of the circle.

(2 marks)

(b) By substituting $y = mx$ into $x^2 + y^2 - 18x - 14y + 105 = 0$, show that $x_1 x_2 = \dfrac{105}{1 + m^2}$.

(2 marks)

(c) Express the length of OA in terms of m and x_1 and the length of OB in terms of m and x_2 .

Hence find the value of the product of OA and OB .

(4 marks)

(d) If the perpendicular distance between the line ℓ and the centre C is 3, find the lengths of AB and OA .

(4 marks)

14. (a) Given the G.P. a^n , $a^{n-1}b$, $a^{n-2}b^2$, $\cdots$, a^2b^{n-2} , ab^{n-1} , where a and b are unequal and non-zero real numbers, find the common ratio and the sum to n terms of the G.P.

(3 marks)

(b) A man joins a saving plan by depositing in his bank account a sum of money at the beginning of every year. At the beginning of the first year, he puts an initial deposit of $\$P$. Every year afterwards, he deposits 10% more than he does in the previous year. The bank pays interest at a rate of 8% p.a., compounded yearly.

(i) Find, in terms of P , an expression for the amount in his account at the end of

(1) the first year,
(2) the second year,
(3) the third year.

(Note: You need not simplify your expressions)

(ii) Using (a), or otherwise, show that the amount in his account at the end of the nth year is $\$54P(1.1^n - 1.08^n)$.

(7 marks)

(c) A flat is worth \$1 080 000 at the beginning of a certain year and at the same time, a man joins the saving plan in (b) with an initial deposit $\$P$ = \$20 000. Suppose the value of the flat grows by 15% every year. Show that at the end of the nth year , the value of the flat is greater than the amount in the man's account.

(2 marks)

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15.

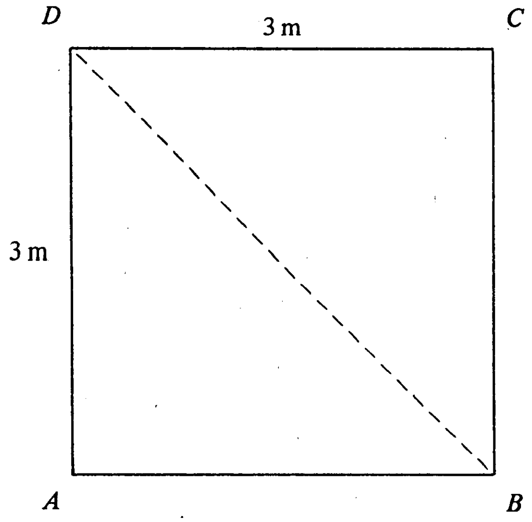

Figure 8a

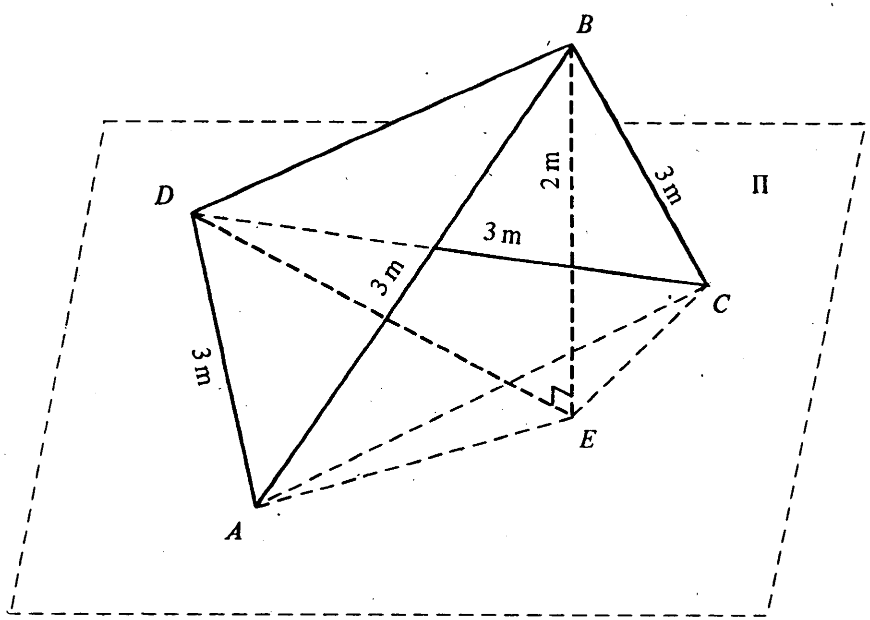

Figure 8b

15.(Cont'd)

In Figure 8a, $ABCD$ is a thin square metal sheet of side three metres. The metal sheet is folded along BD and the edges AD and CD of the folded metal sheet are placed on a horizontal plane Π with B two metres vertically above the plane Π . E is the foot of the perpendicular from B to the plane Π . (See Figure 8b)

(a) Find the lengths of BD , ED and AE , leaving your answers in surd form.

(3 marks)

(b) Find $\angle ADE$.

(3 marks)

(c) Find the angle between BD and the plane Π .

(2 marks)

(d) Find the angle between the planes ABD and CBD .

(4 marks)

END OF PAPER