FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3}$ $\times$ base area $\times$ height

SECTION A (39 marks)
Answer ALL questions in this section.
There is no need to start each question on a fresh page.

1. A person bought 10 gold coins at \$3000 each and later sold them all at \$2700 each.

   (a) Find the total loss.

   (b) Find the percentage loss.

   (4 marks)

2. (a) Simplify $\frac{a}{\sqrt{a}}$, expressing your answer in index form.

   (b) Simplify $\frac{\log(a^2) + \log(b^4)}{\log(ab^2)}$, where $a, b > 0$.

   (5 marks)

3. Rewrite $\sin^2\theta : \cos\theta = -3 : 2$ in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a , b and c are integers.

   Hence solve for θ , where $0° \leqslant \theta < 360°$.

   (6 marks)

4. (a) Solve the following inequalities:

(i) $6x + 1 \geqslant 2x - 3$,

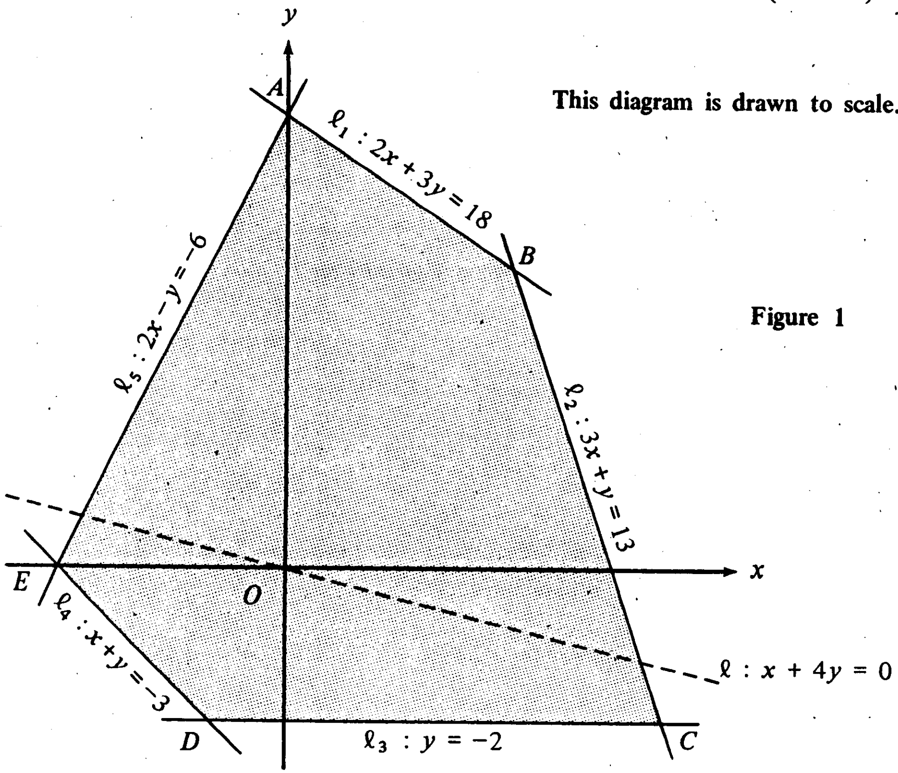
(ii) $(2 - x)(x + 3) > 0$.

(b) Using (a), find the values of x which satisfy

both $6x + 1 \geqslant 2x - 3$ and $(2 - x)(x + 3) > 0$.

(6 marks)

5.



This diagram is drawn to scale.

Figure 1

In Figure 1, the shaded region *ABCDE* is bounded by the five given lines ℓ_1 , ℓ_2 , ℓ_3 , ℓ_4 and ℓ_5 . The line $\ell : x + 4y = 0$ passes through the origin O .

Let $P = x + 4y - 2$, where (x , y) is any point in the shaded region including the boundary. Find the greatest and the least values of P .

(6 marks)

6.

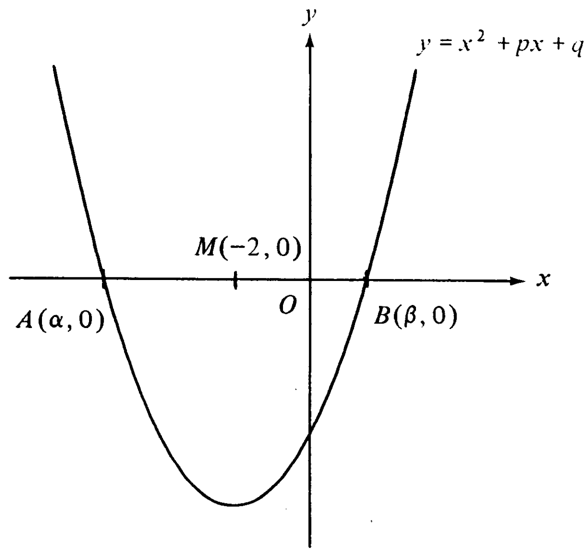


Figure 2

In Figure 2, the curve $y = x^2 + px + q$ cuts the x-axis at the two points $A(\alpha , 0)$ and $B(\beta , 0)$. $M(-2 , 0)$ is the mid-point of AB .

(a) Express $\alpha + \beta$ in terms of p .

Hence find the value of p .

(b) If $\alpha^2 + \beta^2 = 26$, find the value of q .

(6 marks)

7. (a) Find the remainder when $x^{1000} + 6$ is divided by $x + 1$.

(b) (i) Using (a), or otherwise, find the remainder when $8^{1000} + 6$ is divided by 9.

(ii) What is the remainder when 8^{1000} is divided by 9 ?

(6 marks)

SECTION B (60 marks)

Answer any FIVE questions from this section.
Each question carries 12 marks.

8. Let (C_1) be the circle $x^2 + y^2 - 2x + 6y + 1 = 0$ and A be the point $(5, 0)$.

   (a) Find the coordinates of the centre and the radius of (C_1).

   (2 marks)

   (b) Find the distance between the centre of (C_1) and A.

   Hence determine whether A lies inside, outside or on (C_1).

   (3 marks)

   (c) Let s be the shortest distance from A to (C_1).

   (i) Find s.

   (ii) Another circle (C_2) has centre A and radius s. Find its equation.

   (3 marks)

   (d) A line touches the above two circles (C_1) and (C_2) at two distinct points E and F respectively.

   Draw a rough diagram to show this information.

   Find the length of EF.

   (4 marks)

9. In attempting this question, candidates are advised to include a diagram with their answers for reference.

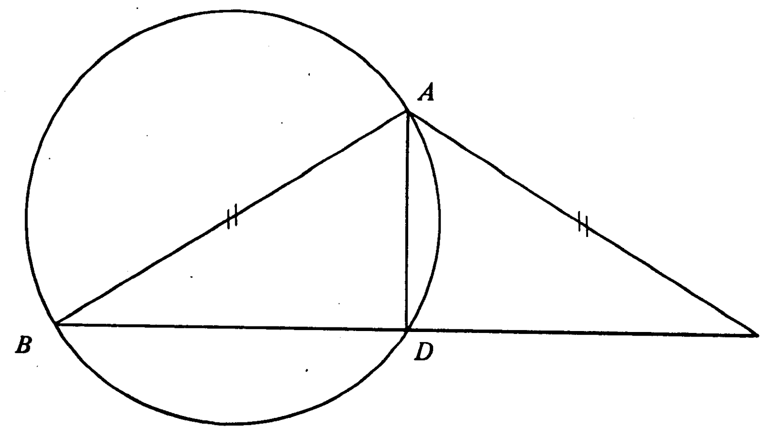


Figure 3

In Figure 3, AB is a diameter of the circle ADB and ABC is an isosceles triangle with $AB = AC$.

(a) Prove that ΔABD and ΔACD are congruent.

(3 marks)

(b) The tangent to the circle at D cuts AC at the point E. Prove that ΔABD and ΔADE are similar.

(2 marks)

(c) In (b), let $AB = 5$ and $BD = 4$.

(i) Find DE.

(ii) CA is produced to meet the circle at the point F. Find AF.

(7 marks)

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10.

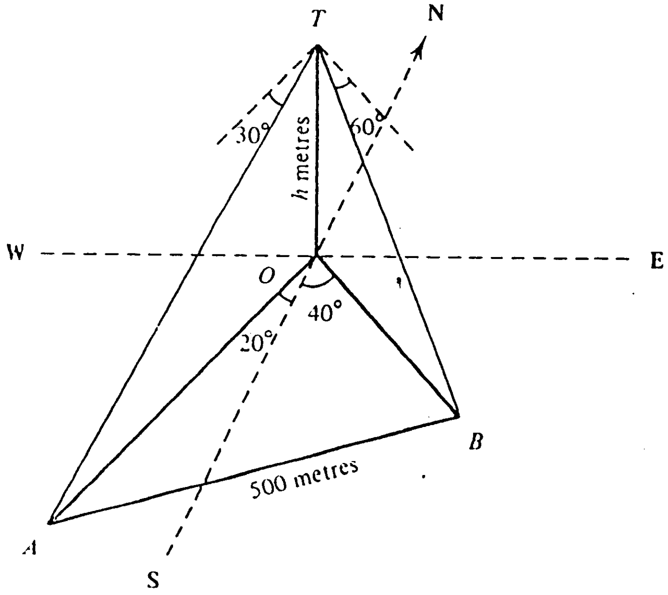

Figure 4

In Figure 4, OT represents a vertical tower of height h metres. From the top T of the tower, two landmarks A and B, 500 metres apart on the same horizontal ground, are observed to have angles of depression 30° and 60° respectively. The bearings of A and B from the tower OT are S20°W and S40°E respectively.

(a) Find the lengths of OA and OB in terms of h.

(3 marks)

(b) Express the length of AB in terms of h. Hence, or otherwise, find the value of h.

(5 marks)

(c) Find $\angle OAB$, correct to the nearest degree.

Hence write down

(i) the bearing of B from A,

(ii) the bearing of A from B.

(4 marks)

11. A solid right circular cylinder has radius r and height h. The volume of the cylinder is V and the total surface area is S.

(a) (i) Express S in terms of r and h.

(ii) Show that $S = 2\pi r^2 + \frac{2V}{r}$.

(3 marks)

(b) Given that $V = 2\pi$ and $S = 6\pi$, show that $r^3 - 3r + 2 = 0$.

Hence find the radius r by factorization.

(4 marks)

(c) Given that $V = 3\pi$ and $S = 10\pi$, find the radius r $(1 < r < 2)$ by the method of bisection, correct to 1 decimal place.

(5 marks)

12. (a) The distribution of the monthly salaries of 100 employees in a firm is shown in the histogram in Figure 5.

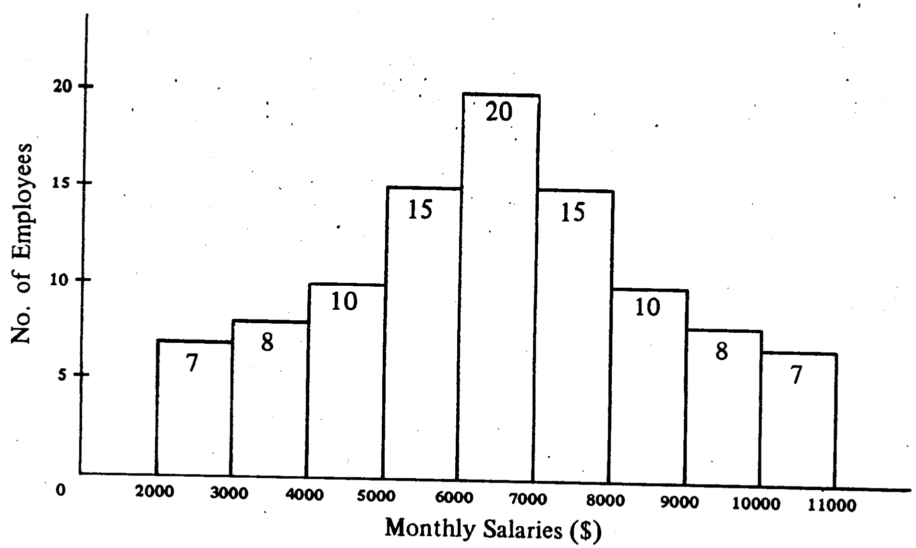

Figure 5 : Distribution of monthly salaries of 100 employees

(i) Find the modal class, median, mean, interquartile range and mean deviation of the monthly salaries of the 100 employees.

(7 marks)

(ii) Now the firm employs 10 more employees whose monthly salaries are all $6500. Will the standard deviation of the monthly salaries of all the employees in the firm become greater, smaller or remain unchanged? Explain briefly.

(2 marks)

(b) The mean of 7 numbers $x_1, x_2, \ldots, x_7$ is $\bar{x}$ and the *squares* of their deviations from $\bar{x}$ are 9, 4, 1, 0, 1, 4, 9 respectively. Find the standard deviation of the 7 numbers.

(3 marks)

13.

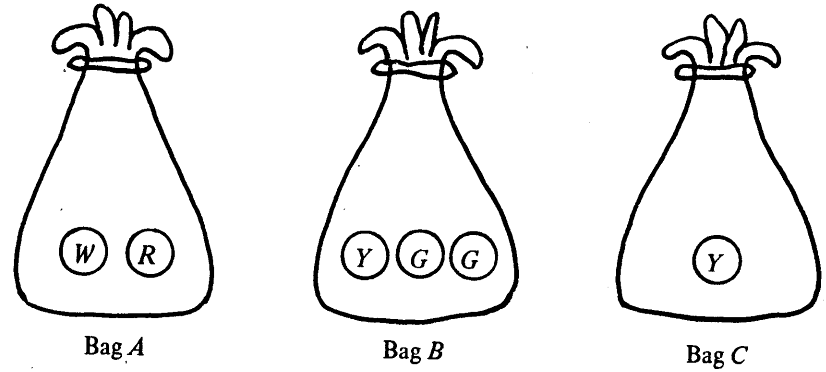

Figure 6

Figure 6 shows 3 bags A, B and C.

Bag A contains 1 white ball (W) and 1 red ball (R).
Bag B contains 1 yellow ball (Y) and 2 green balls (G).
Bag C contains only 1 yellow ball (Y).

(a) Peter chooses one bag at random and then randomly draws one ball from the bag. Find the probability that

(i) the ball drawn is green;

(ii) the ball drawn is yellow.

(6 marks)

(b) After Peter has drawn a ball in the way described in (a), he puts it back into the original bag. Next, Alice chooses one bag at random and then randomly draws one ball from the bag. Find the probability that

(i) the balls drawn by Peter and Alice are both green;

(ii) the balls drawn by Peter and Alice are both yellow and from the same bag.

(6 marks)

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14. The positive integers 1, 2, 3, ... are divided into groups G_1, G_2, G_3, ..., so that the k^{th} group G_k consists of k consecutive integers as follows:

$$G_1 : 1$$
$$G_2 : 2 , 3$$
$$G_3 : 4 , 5 , 6$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$G_{k-1} : u_1 , u_2 , \ldots , u_{k-1}$$
$$G_k : v_1 , v_2 , \ldots , v_{k-1} , v_k$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$
$$\cdots\cdots\cdots$$

(a) (i) Write down all the integers in the 6^{th} group G_6.

(ii) What is the total number of integers in the first 6 groups G_1, G_2, ..., G_6 ?

(4 marks)

(b) Find, in terms of k,

(i) the last integer u_{k-1} in G_{k-1} and the first integer v_1 in G_k,

(ii) the sum of all the integers in G_k.

(8 marks)

END OF PAPER