HKCEE 1989
Mathematics II

89 1. $3^{n-1} \times 3^{n+1}$

A. 3^{n^2-1}
B. 9^{n^2-1}
C. 3^{2n}
D. 6^{2n}
E. 9^{2n}

89 2. $\dfrac{27x^3-8}{3x-2} =$

A. $(3x-2)^2$
B. $9x^2-4$
C. $9x^2+4$
D. $9x^2-6x+4$
E. $9x^2+6x+4$

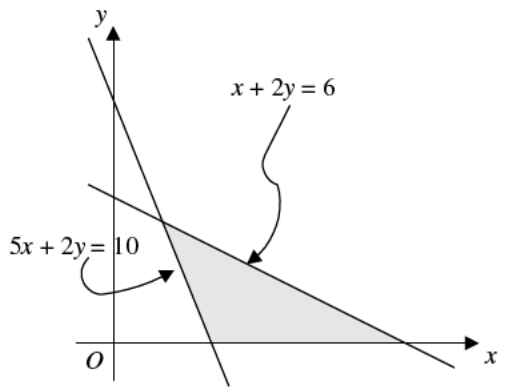
89 3. $\sqrt{\dfrac{x}{\sqrt{x}}} =$

A. $x^{\frac{3}{4}}$
B. $x^{\frac{1}{4}}$
C. $x^{\frac{1}{2}}$
D. $x^{-\frac{1}{4}}$
E. $x^{-\frac{3}{4}}$

89 4. If $f(x) = \dfrac{x}{1-x}$, then $f(\dfrac{1}{x}) =$

A. $\dfrac{1}{x-1}$.
B. $\dfrac{1}{1-x}$.
C. $\dfrac{x}{x-1}$.
D. $\dfrac{x}{1-x}$.
E. $\dfrac{1-x}{x}$.

89 5.



Which of the following systems of inequalities is represented by the shaded region in the figure?

A. $\begin{cases} x+2y \geq 6 \\ 5x+2y \geq 10 \\ y \geq 0 \end{cases}$
B. $\begin{cases} x+2y \leq 6 \\ 5x+2y \leq 10 \\ x \geq 0 \end{cases}$
C. $\begin{cases} x+2y \geq 6 \\ 5x+2y \leq 10 \\ x \geq 0 \end{cases}$
D. $\begin{cases} x+2y \leq 6 \\ 5x+2y \geq 10 \\ y \geq 0 \end{cases}$
E. $\begin{cases} x+2y \geq 6 \\ 5x+2y \leq 10 \\ y \geq 0 \end{cases}$

89 6. Let $f(x) = ax^2 - 5$ and $g(x) = 27x^3 - 18x + 4$. If both expressions leave the same remainder when divided by $3x + 1$, then $a =$

A. -74.
B. 0.
C. 36.
D. 76.
E. 126.

89 7. If $3x > -2y$ and $y < 0$, then

A. $\frac{x}{y} > -\frac{3}{2}$.
B. $\frac{x}{y} > \frac{2}{3}$.
C. $\frac{x}{y} < \frac{2}{3}$.
D. $\frac{x}{y} > -\frac{2}{3}$.
E. $\frac{x}{y} < -\frac{2}{3}$.

89 8. Given that r is the only real root of $x^5 + x - 1 = 0$, which of the following ranges contains r?

A. $-2 < r < -1$
B. $-1 < r < 0$
C. $0 < r < 1$
D. $1 < r < 2$
E. $2 < r < 3$

89 9. If z varies inversely as x and directly as y, then

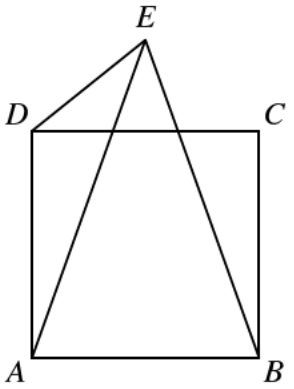
A. xyz is a constant.
B. $\frac{xz}{y}$ is a constant
C. $\frac{yz}{x}$ is a constant
D. $\frac{xz^2}{y}$ is a constant
E. $\frac{z^2}{xy}$ is a constant

89 10. Which of the following is/are true?

I. If both 2 and 3 are factors of m, then 6 is also a factor of m.
II. If 15 is a factor of n, then both 3 and 5 are factors of n.
III. If p is a multiple of both 4 and 6, then p is also a multiple of 24.

A. I only
B. II only
C. I and II only
D. II and III only
E. I, II and III

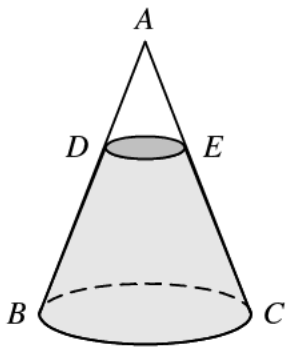
89 11.



In the figure, $ABCD$ is a square and $AE = BE$. $\frac{\text{Area of } AED}{\text{Area of } ABCD} =$

A. $\frac{1}{2}$
B. $\frac{3}{8}$
C. $\frac{1}{3}$
D. $\frac{1}{4}$
E. $\frac{1}{8}$

89 12.



Aright conical vessel placed on horizontal ground contains some water as shown in the figure. If $AD : DB = 2 : 3$, then $\frac{\text{volume of empty space}}{\text{volume of water}} =$

A. $\frac{4}{9}$.
B. $\frac{8}{19}$.
C. $\frac{8}{27}$.
D. $\frac{8}{117}$.
E. $\frac{8}{125}$.

89 13. If A is greater than B by 20% and B is smaller than C by 30%, then

A. A is smaller than C by 16%
B. A is smaller than C by 6%
C. A is greater than C by 6%
D. A is greater than C by 10%
E. A is greater than C by 16%

89 14. At the beginning of a year, a man borrows \$1000 from a bank at 5% per annum, compounded yearly. He promises to repay \$300 at the end of each year. How much will he still owe the bank just after the second repayment?

A. \$402.5
B. \$450
C. \$487.5
D. \$500
E. \$502.5

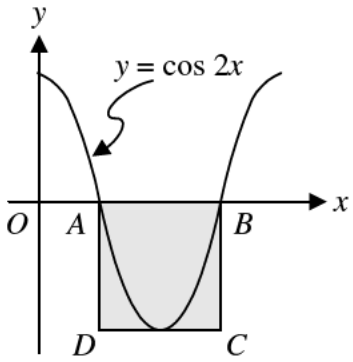
89 15. The least value of $9\cos^2\theta - 6\cos\theta + 1$ is

A. -4.
B. 0.
C. 1.
D. 4.
E. 16.

89 16. $\frac{1}{\frac{1}{\cos\theta}-1} - \frac{1}{\frac{1}{\cos\theta}+1} =$

A. $\frac{2}{\tan^2\theta}$
B. $\frac{2}{\tan\theta}$
C. $2\tan^2\theta$
D. $\frac{2\cos\theta}{\sin^2\theta}$
E. $\frac{2\cos^2\theta}{\sin\theta}$

89 17.



The figure shows the graph of $y = \cos 2x$, where $0 \le x \le \pi$. The area of the rectangle $ABCD$ is

A. $\frac{\pi}{2}$.
B. $\frac{\pi}{4}$.
C. π.

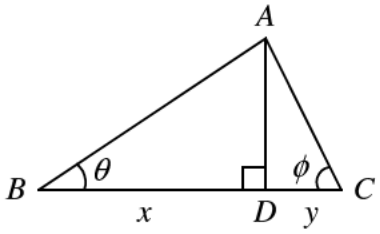
D. $\frac{3\pi}{2}$.

E. 2π.

89 18. Given that $0^o \leq \theta \leq 180^o$, how many roots has the equation $(\sin\theta + 1)(\tan\theta + 3) = 0$?

A. 0
B. 1
C. 2
D. 3
E. 4

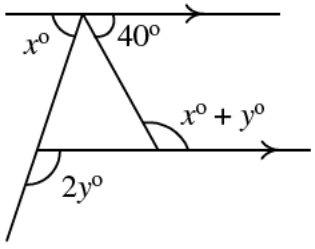
89 19.



In the figure, $AD \perp BC$. Find $\frac{x}{y}$

A. $\frac{\sin\phi}{\sin\theta}$
B. $\frac{\cos\phi}{\cos\theta}$
C. $\frac{\tan\phi}{\tan\theta}$
D. $\frac{\cos\theta}{\cos\phi}$
E. $\frac{\tan\theta}{\tan\phi}$

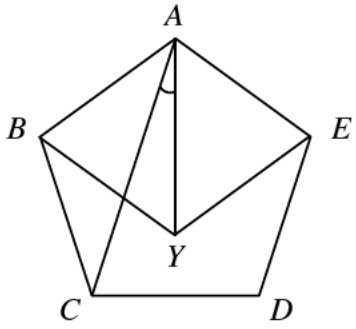
89 20.



Referring to the figure, find y.

A. 20
B. 30
C. 40
D. 50
E. 80

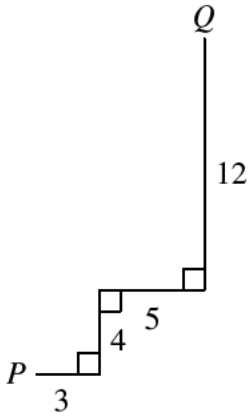
89 21.



In the figure, $ABCDE$ is a regular pentagon and $ABYE$ is a rhombus. Find $\angle CAY$.

A. 27^o
B. 24^o
C. 21^o
D. 18^o
E. 15^o

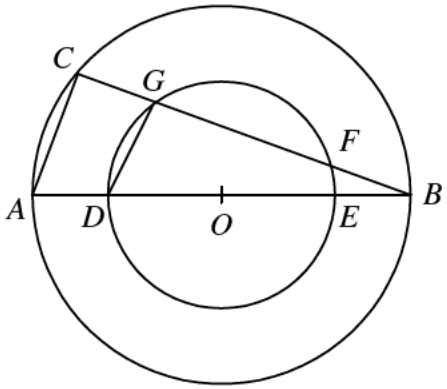
89 22.



Referring to the figure, find the length of the line segment joining P and Q.

A. 25
B. $10\sqrt{5}$
C. 18
D. $8\sqrt{5}$
E. $\sqrt{194}$

89
23.

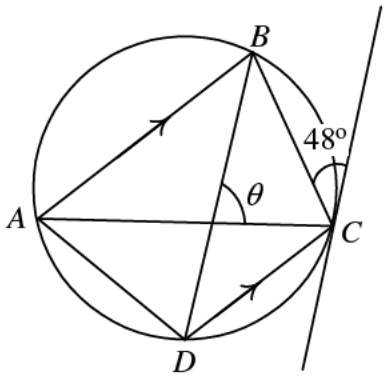


In the figure O is the centre of two Concentric circles. $ADOEB$ and $CGFB$ are straight lines. Which of the following is/are true?

I. $AC \parallel DG$
II. $BF = CG$
III. A, E, F and C are concyclic

A. I only
B. II only
C. I and II only
D. I and III only
E. I, II and III

89
24.



In the figure, TC is a tangent to the circle at C and $AB \parallel DC$. If $\angle BCT = 48^{\circ}$, then $\theta =$

A. 48°
B. 72°
C. 84°
D. 90°
E. 96°

89
25.

Referring to the data 1, 1, 1, 1, 1, 2, 2, 2, 3, which of the following is/are true?

I. median < mean
II. range = 3
III. mode = 3

A. I only
B. II only
C. III only
D. I and II only
E. I, II and III

89
26.

A **BIASED** die is thrown. Suppose the probabilities of getting 1, 2, 3, 4, and 4 are respectively $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ and $\frac{1}{32}$. What is the probability of getting 6?

A. $\frac{1}{64}$
B. $\frac{1}{36}$
C. $\frac{1}{32}$
D. $\frac{1}{12}$
E. $\frac{1}{6}$

89
27.

A bag contains 4 red, 3 green and 2 white balls. Three men A, B and C each draw one ball in turn from the bag at random without replacement. If A draws first, B second and C third, what is the probability that the balls drawn by B and C are both white?

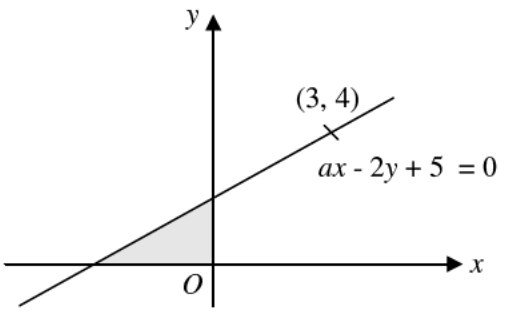
A. $\frac{1}{36}$
B. $\frac{1}{28}$
C. $\frac{4}{81}$
D. $\frac{25}{72}$

E. $\frac{11}{28}$

89 28. The equation of the straight line perpendicular to $2x + y - 3 = 0$ and passing through $(1, -1)$ is

A. $x + 2y + 1 = 0$.
B. $x - 2y - 3 = 0$.
C. $-x + 2y - 1 = 0$.
D. $2x + y - 1 = 0$.
E. $2x - y - 3 = 0$.

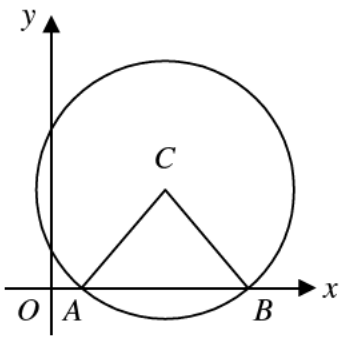
89 29.



In the figure, the line $ax - 2y + 5 = 0$ passes through the point $(3, 4)$. What is the area of the shaded part?

A. 6
B. $\frac{25}{4}$
C. 10
D. 12
E. $\frac{25}{2}$

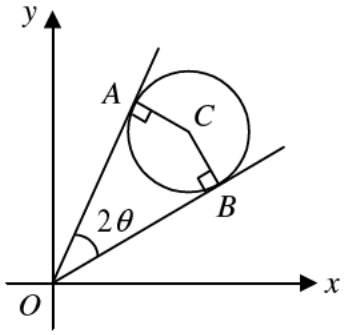
89 30.



In the figure, C is the centre of the circle $x^2 + y^2 - 8x - 7y + 12 = 0$. If the circle cuts the x-axis at A and B, find the area of ΔCAB.

A. $\frac{7}{4}$
B. $\frac{7}{2}$
C. 7
D. 8
E. 14

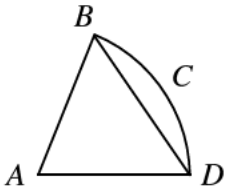
89 31.

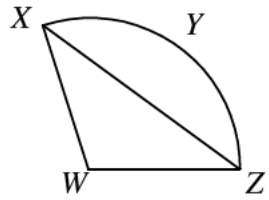


In the figure, C is the centre of the circle $x^2 + y^2 - 6x - 8y + 21 = 0$. OA and OB are tangents. If $\angle AOB = 2\theta$, find $\sin\theta$.

A. $\frac{\sqrt{21}}{5}$
B. $\frac{4}{5}$
C. $\frac{3}{5}$
D. $\frac{2}{\sqrt{21}}$
E. $\frac{2}{5}$

89 32.





In the figure, $ABCD$ and $WXYZ$ are sectors of equal radii. If
arc BCD : arc $XYZ = s : t$, then which of the following is/are true?

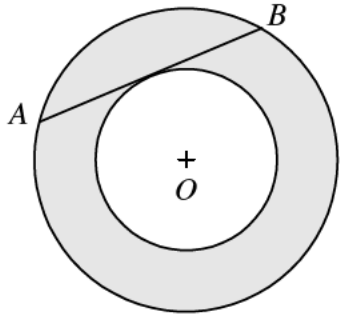
I. $\dfrac{BD}{XZ} = \dfrac{s}{t}$

II. $\dfrac{\text{area of sector } ABCD}{\text{area of sector } WXYZ} = \dfrac{s}{t}$

III. $\dfrac{\angle BAD}{\angle XWZ} = \dfrac{s}{t}$

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

89
33.



In the figure, O is the centre of two concentric circles. AB is tangent to the smaller circle. If $AB = 2$, find the area of the shaded part.

A. $\dfrac{\pi}{2}$
B. π
C. 2π
D. 4π
E. It cannot be found.

89
34. If 10 arithmetic means are inserted between a and b, then the last one is

A. $\dfrac{10a+b}{11}$.
B. $\dfrac{9a+b}{10}$.
C. $\dfrac{10(b-a)}{11}$.
D. $\dfrac{a+9b}{10}$.
E. $\dfrac{a+10b}{11}$.

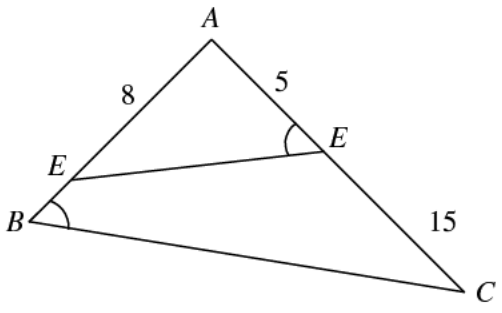
89
35. Given that $y \propto \dfrac{1}{x}$, if x increased by 25%, find the percentage change in y.

A. Decreased by 20%
B. Decreased by 25%
C. Decreased by 80%
D. Increased by 20%
E. Increased by 25%

89
36. The costs of two kinds of coffee A and B are \$12/kg and \$20/kg respectively. In what ratio by weight should A and B be mixed so that the mixture will cost \$15/kg?

A. 4 : 3
B. 5 : 2
C. 5 : 3
D. 3 : 2
E. 5 : 4

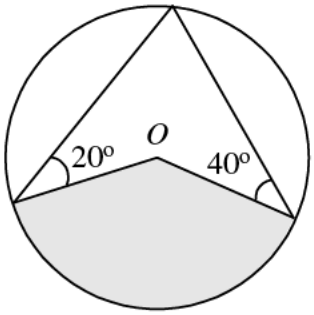
89
37.



In the figure, D and E are points on AB and AC respectively such that $\angle ABC = \angle AED$, $AD = 8$, $AE = 5$ and $EC = 15$. If the area of ΔADE is 16, then the area of the quadrilateral $BCED$ is

A. 200 .
B. 100 .
C. 96 .
D. 84 .
E. 40 .

89
38.



In the figure, O is the centre of the circle of radius 6 cm. The area of the shaded part is

A. $2\pi\,\text{cm}^2$.
B. $4\pi\,\text{cm}^2$.
C. $6\pi\,\text{cm}^2$.
D. $9\pi\,\text{cm}^2$.
E. $12\pi\,\text{cm}^2$.

89
39. If the sum to infinity of the G.P. 1, $-t$, t^2, $-t^3$, … is $\frac{2}{3}$, find the fourth term.

A. $-\frac{1}{16}$
B. $-\frac{1}{8}$
C. $\frac{1}{16}$
D. $\frac{1}{8}$
E. $\frac{5}{8}$

89
40. If $\frac{x+3y}{2x+y} = 2$, find $\frac{3x+y}{x+2y}$

A. 2
B. 3
C. $\frac{1}{2}$
D. $\frac{1}{3}$
E. $\frac{6}{7}$

89
41. $\frac{(1-x^2)^n + (1-x)^n}{(1-x)^{2n}} =$

A. $\frac{(1+x)^n + 1}{(1-x)^n}$
B. $\frac{2-x-x^2}{(1-x)^2}$
C. $\frac{(1+x)^n + 1}{(1-x)^2}$
D. $\frac{(1-x)^n + 1}{(1+x)^n}$
E. $\frac{2-x^n+x^{2n}}{1-x^{2n}}$

89
42. $\log_4 2\sqrt{2} =$

A. $\frac{3}{8}$
B. $\frac{3}{4}$
C. $\frac{1}{4}$
D. $2^{\frac{3}{4}}$
E. $2^{\frac{3}{8}}$

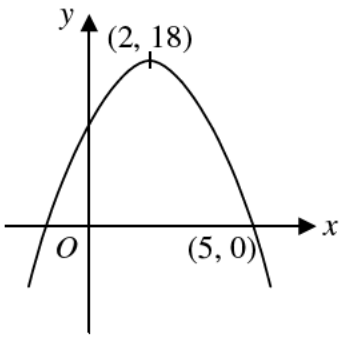
89
43. If $x = \sqrt{a+1} - \sqrt{a}$, where $a > 0$, then $x + \frac{1}{x}$

A. 2.
B. $2\sqrt{a}$.
C. $2\sqrt{a+1}$.
D. $2\sqrt{a+1}-\sqrt{a}$.
E. $2(\sqrt{a+1}+\sqrt{a})$.

89 44. If p is a root of $ax^2 + bx + c = 0$, which of the following is a root of $a(\frac{x-3}{2})^2 + b(\frac{x-3}{2}) + c = 0$?

A. $2p + 3$
B. $2p - 3$
C. $3 - 2p$
D. $\frac{p+3}{2}$
E. $\frac{p-3}{2}$

89 45.



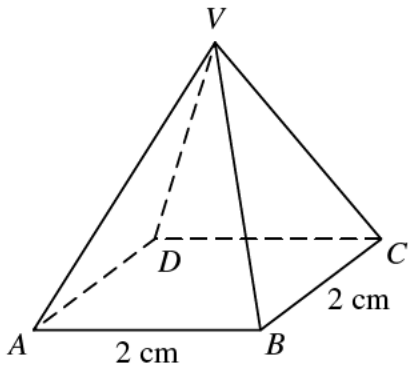
In figure shows the graph of a quadratic function $y = f(x)$. Given that the graph has vertex (2, 18) and it cuts the x-axis at
(5, 0), find the quadratic function.

A. $y = (x - 2)^2 + 18$
B. $y = -(x - 2)^2 + 18$
C. $y = (x + 1)(x - 5)$
D. $y = -2(x + 1)(x - 5)$
E. $y = 2(x - 1)(x + 5)$

89 46. If $2\sin 2\theta - \sin\theta\cos\theta - \cos^2\theta = 0$, the $\tan\theta$

A. 1 or $\frac{1}{2}$.
B. -1 or $\frac{1}{2}$.
C. 1 or $-\frac{1}{2}$.
D. -1 or $-\frac{1}{2}$.
E. 1 or -2 .

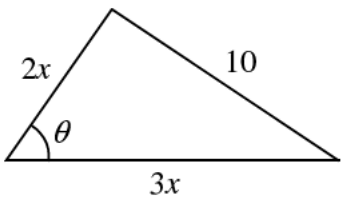
89 47.



In the figure, $VABCD$ is a right pyramid of height 3 cm. The base $ABCD$ is a square of side 2 cm. Let θ be the angle between the face VBC and the base. Find $\tan\theta$

A. $\frac{1}{3}$
B. $\frac{\sqrt{2}}{3}$
C. $\frac{3}{2}$
D. $\frac{3\sqrt{2}}{2}$
E. 3

89 48.

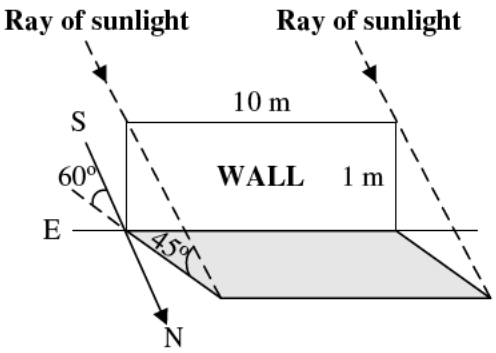


In the figure, if $\cos\theta = \frac{3}{4}$, find the value of x.

A. 2

B. 3
C. 4
D. 5
E. 6

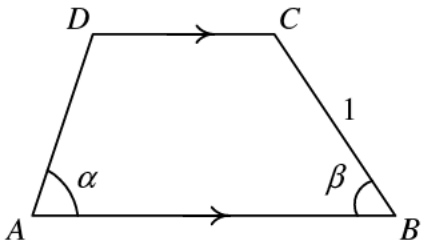
89
49.



A vertical rectangular wall on the horizontal ground, 1 m high and 10 m long, runs east and west as shown in the figure. If the sun bears S60°E at an elevation of 45°, find the area of the shadow of the wall on the ground.

A. $\frac{5}{2}$ m^2
B. 5 m^2
C. $5\sqrt{2}$ m^2
D. $5\sqrt{3}$ m^2
E. 10 m^2

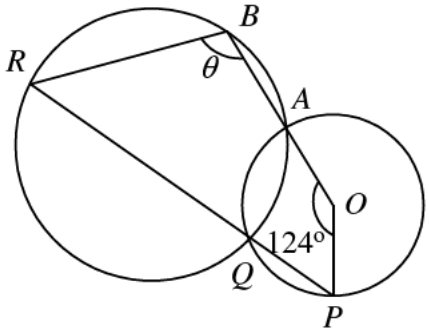
89
50.



In the figure, $ABCD$ is a trapezium with AB // DC. If $BC = 1$, then $AD =$

A. $\frac{\sin\beta}{\sin\alpha}$.
B. $\frac{\sin\alpha}{\sin\beta}$.
C. $\sin\alpha\sin\beta$.
D. $\frac{\cos\beta}{\cos\alpha}$.
E. $\frac{\cos\alpha}{\cos\beta}$.

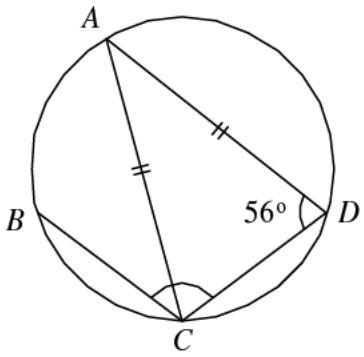
89
51.



In the figure, O is the centre of the smaller circle. OAB and PQR are straight lines. Find θ.

A. 56°
B. 108°
C. 112°
D. 118°
E. 124°

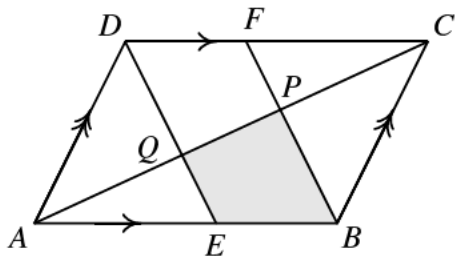
89
52.



In the figure, B is the mid-point of arc AC. $AC = AD$. If $\angle ADC = 56°$, then $\angle BCD =$

A. 84°.
B. 90°.
C. 96°.
D. 112°.
E. 124°.

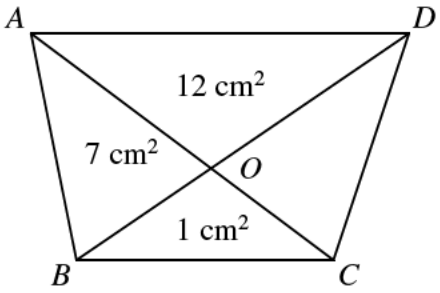
89
53.



In the figure, $ABCD$ is a parallelogram. E and F are the mid-points of AB and DC respectively. BF and ED cut AC at P and Q respectively. If the area of $ABCD$ is 48, find the area of the shaded part.

A. 6
B. 8
C. 9.6
D. 12
E. 16

89
54.



In the figure, AC cuts BD at O. The areas of ΔAOB, ΔAOD and ΔBOC are 7 cm^2, 12 cm^2 and 10.5 cm^2 respectively. Find the area of ΔOCD.

A. 5.5 cm^2
B. 8 cm^2
C. 8.5 cm^2
D. 15.5 cm^2
E. 18 cm^2