88-CE
MATHS
PAPER I

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1988

數學 試卷一
MATHEMATICS PAPER I

8.30 am–10.30 am (2 hours)
This paper must be answered in English

Attempt **ALL** questions in Section A and any **FIVE** questions in Section B.

Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area $\times$ height
PYRAMID	Volume	=	$\frac{1}{3}$ $\times$ base area $\times$ height

SECTION A Answer ALL questions in this section.
There is no need to start each question on a fresh page.
Geometry theorems need not be quoted when used.

1. Factorize $a^2 - a - 6$ and $a^3 + 8$.

   Hence find their L.C.M.

(5 marks)

2. Simplify

   (a) $\dfrac{\sin(180^\circ - \theta)}{\sin(90^\circ + \theta)}$,

   (b) $\sin^2(\pi - \phi) + \sin^2\left(\dfrac{3\pi}{2} + \phi\right)$.

(5 marks)

3. Solve the inequality $2x^2 \geqslant 5x$.

(5 marks)

4. The quadratic equation

   $9x^2 - (k+1)x + 1 = 0$ ....(*)

   has equal roots.

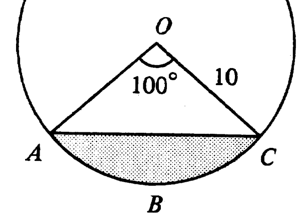
   (a) Find the two possible values of the constant k.

   (b) If k takes the negative value obtained, solve equation (*).

(6 marks)

5.

Figure 1



In Figure 1, ABC is a circle with centre O and radius 10. $\angle AOC = 100^\circ$. Calculate, correct to 2 decimal places,

(a) the area of sector $OABC$,

(b) the area of $\triangle OAC$,

(c) the area of segment ABC.

(6 marks)

6. Given that $\log 2 = r$ and $\log 3 = s$, express the following in terms of r and s:

   (a) $\log 18$,

   (b) $\log 15$.

   [Note: In this question, all logarithms are to the base 10.]

(6 marks)

7.

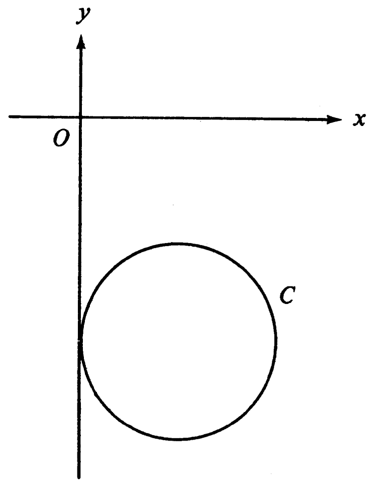


Figure 2

In Figure 2, the circle C has equation

$$x^2 + y^2 - 4x + 10y + k = 0 ,$$

where k is a constant.

(a) Find the coordinates of the centre of C.

(b) If C touches the y-axis, find the radius of C and the value of k.

(6 marks)

SECTION B **Answer any FIVE questions from this section. Each question carries 12 marks.**

8. (a) P is a point inside a square $ABCD$ such that PBC is an equilateral triangle. AP is produced to meet CD at Q.

(i) Draw a diagram to represent the above information.

(ii) Calculate $\angle PAB$ and $\angle PQC$.

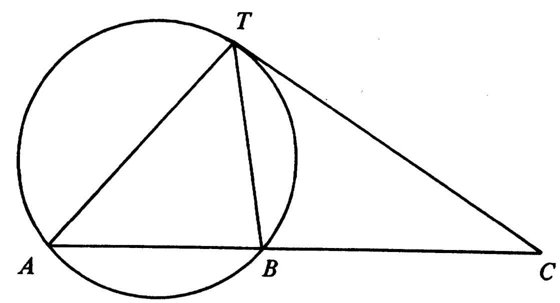
(7 marks)

(b) In Figure 3, CT is tangent to the circle ABT.

(i) Find a triangle similar to ΔACT and give reasons.

(ii) If $CT = 6$ and $BC = 5$, find AB.

Figure 3



(5 marks)

9. (a) Write down the smallest and the largest multiples of 7 between 100 and 999.

(2 marks)

(b) How many multiples of 7 are there between 100 and 999?

Find the sum of these multiples.

(6 marks)

(c) Find the sum of all positive three-digit integers which are **NOT** divisible by 7.

(4 marks)

10. A variable quantity y is the sum of two parts. The first part varies directly as another variable x, while the second part varies directly as x^2. When $x = 1$, $y = -5$; when $x = 2$, $y = -8$.

(a) Express y in terms of x.

Hence find the value of y when $x = 6$.

(8 marks)

(b) Express y in the form $(x - p)^2 - q$, where p and q are constants.

Hence find the least possible value of y when x varies.

(4 marks)

11. Figure 4 shows the cumulative frequency curve of the marks of 600 students in a mathematics contest.

(a) From the curve, find

(i) the median, and

(ii) the interquartile range of the distribution of marks.

(4 marks)

(b) A student with marks greater than or equal to 100 will be awarded a prize.

(i) Find the number of students who will be awarded prizes.

(ii) If one student is chosen at random from the 600 students, find the probability that the student is a prize-winner.

(iii) If two students are chosen at random, find the probability that

(1) both of them are prize-winners,

(2) at least one of them is a prize-winner.

(8 marks)

11. (cont'd)

Candidates need NOT hand in this graph.

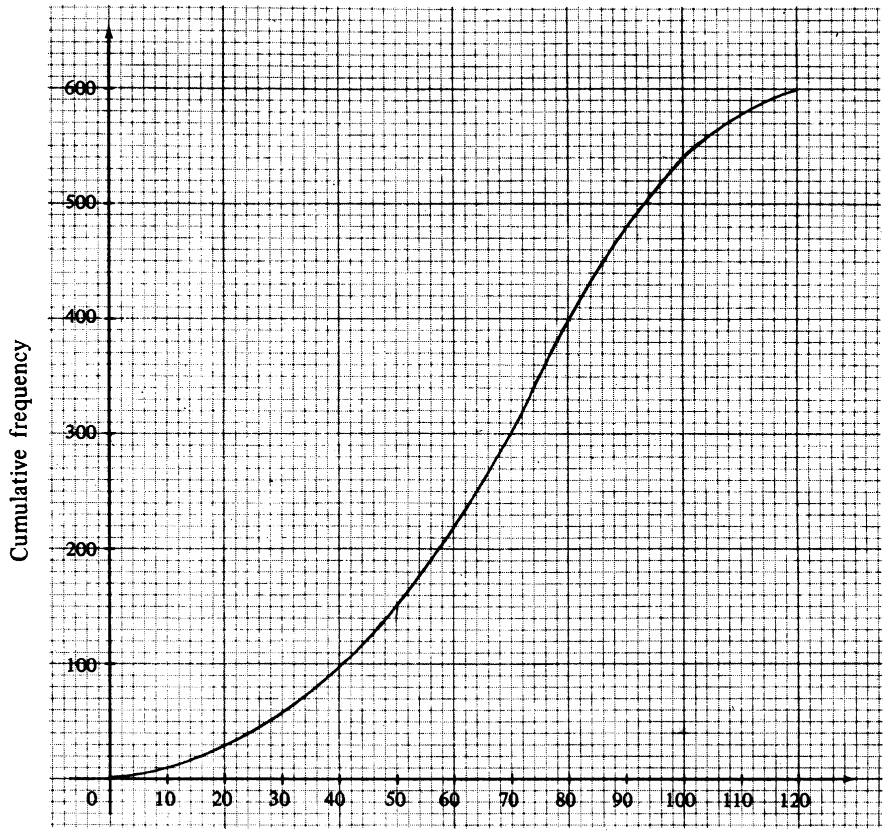


Figure 4

12. In Figure 5, L_1 is the line $x = 3$ and L_2 is the line $y = 4$. L_3 is the line passing through the points (3, 0) and (0, 4).

(a) Find the equation of L_3 in the form $ax + by = c$, where a, b and c are integers.

(2 marks)

(b) Write down the three constraints which determine the shaded region, including the boundary.

(3 marks)

(c) Let $P = x + 4y$. If (x, y) is any point satisfying all the constraints in (b), find the greatest and the least values of P.

(4 marks)

(d) If one more constraint $2x - 3y + 3 \leqslant 0$ is added, shade in Figure 5 the new region satisfying all the four constraints.

For any point (x, y) lying in the new region, find the least value of P defined in (c).

(3 marks)

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12. (cont'd)

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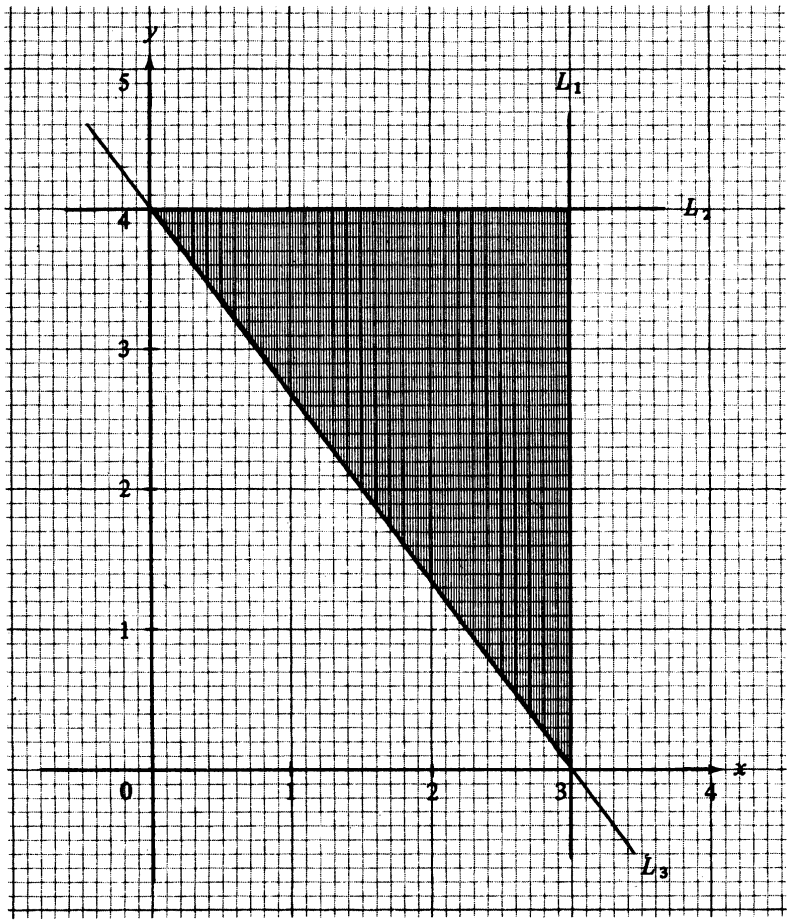

Figure 5

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13.

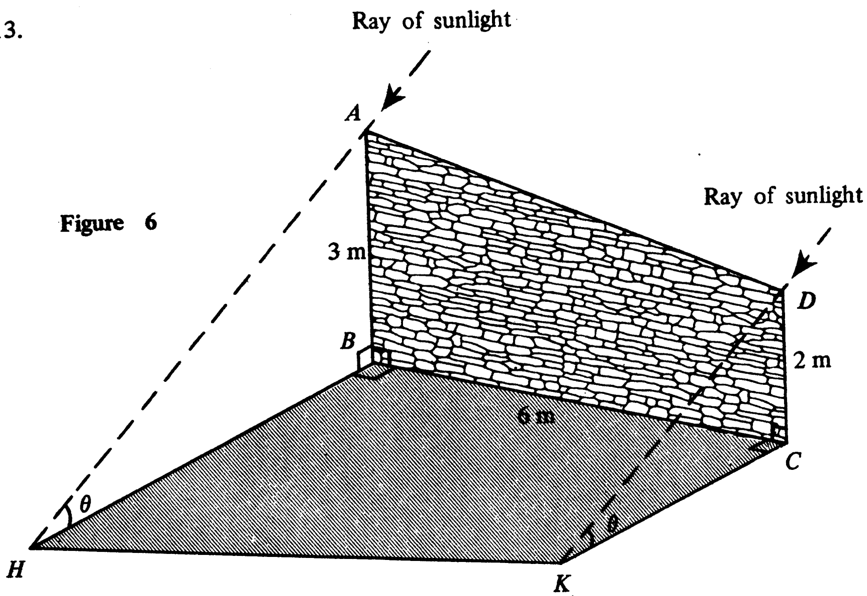

Figure 6

In Figure 6, $ABCD$ is a wall in the shape of a trapezium with AB and DC vertical. Rays of sunlight coming from the back of the wall cast a shadow $HBCK$ on the horizontal ground such that the edges HB and KC of the shadow are perpendicular to BC. Suppose the angle of elevation of the sun is θ, $AB = 3$ m, $CD = 2$ m and $BC = 6$ m.

(a) Express HB and KC in terms of θ.

(3 marks)

(b) (i) Find the area S_1 of the wall.

(ii) Find, in terms of θ, the area S_2 of the shadow.

Hence show that $\frac{S_1}{S_2} = \tan\theta$.

(3 marks)

(c) If $\theta = 30°$, find the length of the edge HK, leaving your answer in surd form.

(6 marks)

14. Figure 7 shows the graph of $y = x^3$ for $x \geqslant 0$.

(a) Let r be the real root of the equation $x^3 - \frac{4}{3}x - 6 = 0$.

(i) By adding a suitable straight line to the figure, find an interval of width 0.1 which contains r.

(ii) Use the method of bisection to find the value of r correct to two decimal places. Show your working in the form of a table.

(9 marks)

(b) Use (a) to find, correct to two decimal places, the real root of the equation $3(t + 1)^3 - 4(t + 1) - 18 = 0$.

(3 marks)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

14. (cont'd)

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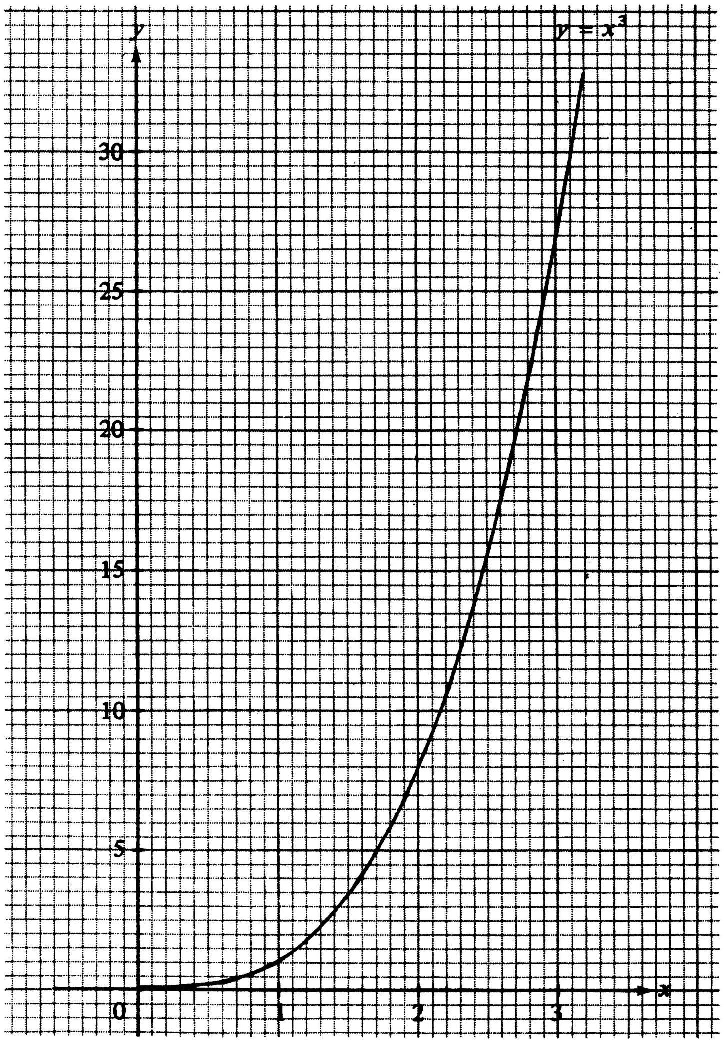

Figure 7

END OF PAPER