RESTRICTED 內部文件

香 港 考 試 局
HONG KONG EXAMINATIONS AUTHORITY

一 九 八 八 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

數學
Mathematics

評卷參考
Marking Scheme

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Notes for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute misconduct on the part of the marker and is, moreover in breach of the 1977 Hong Kong Examinations Authority Ordinance.

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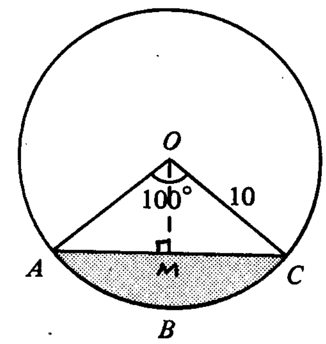
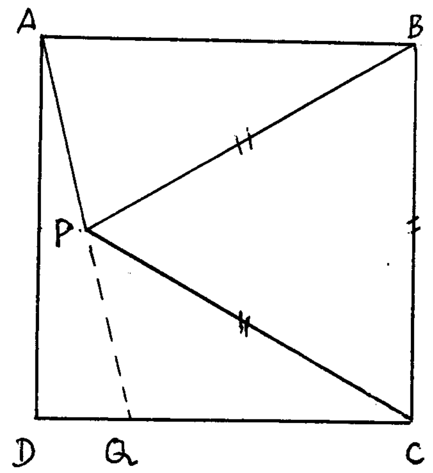
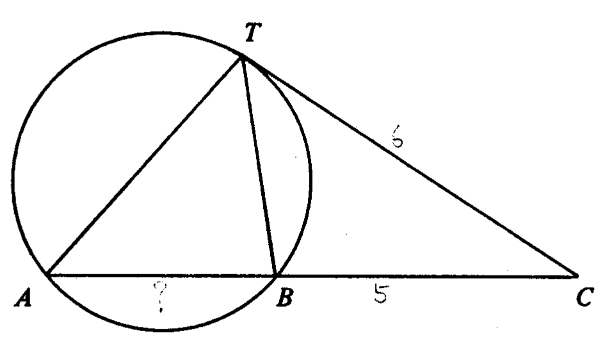
Solutions	Marks	Remarks
1. $a^2 - a - 6 = (a + 2)(a - 3)$)		
) .... any 1 all correct	2A+1A	2A for first correct part
$a^3 + 8 = (a + 2)(a^2 - 2a + 4)$)		
Their L.C.M. $= (a + 2)(a - 3)(a^2 - 2a + 4)$ the other correct	1M+1A	Both exp. must first be factorized.
$(= a^4 - 3a^2 + 8a - 24)$		
	5	PP-1 at most 3 per paper, at most 1 per question, at most 1 for the same type of pp.
2. (a) $\dfrac{\sin(180° - \theta)}{\sin(90° + \theta)} = \dfrac{\sin\theta}{\cos\theta}$ must be shown..........	1A 1A	EXCESS
$= \tan\theta$	1A	
(b) $\sin^2(\pi - \phi) + \sin^2(\frac{3\pi}{2} + \phi)$		
$= \sin^2\phi + \cos^2\phi$ $\quad \sin^2\phi - \cos^2\phi$ ... 0A	1A	For $\sin(\frac{3\pi}{2} + \phi) = -\cos\phi$
$= 1$	1A	
	5	
3. $2x^2 \geqslant 5x$		With-hold 1 mark if '=' omitted. If solved by equation, no marks awarded unless answer correct.
$2x^2 - 5x \geqslant 0$	1A	
$x(2x - 5) \geqslant 0$	1A	
Case (i) $\quad x \geqslant 0$ and $2x - 5 \geqslant 0$		Optional
i.e. $\quad x \geqslant \frac{5}{2}$		any 1 part without = , withhold 1 mark.
Case (ii) $\quad x \leqslant 0$ and $2x - 5 \leqslant 0$		
i.e. $\quad x \leqslant 0$		
Combining the two parts, we have $x \leqslant 0$ or $x \geqslant \frac{5}{2}$.	3A	For $x \leqslant 0$, $x \geqslant \frac{5}{2}$, 2
		$x \leqslant 0$ and $x \geqslant \frac{5}{2}$ 1
	5	
4. (a) If $9x^2 - (k + 1)x + 1 = 0$ has equal roots,		<u>Alt. Solution</u>:
$(k + 1)^2 - 36 = 0$	1A	$(k+1)^2 - 36 = 0$ 1A
$k^2 + 2k - 35 = 0$	1A	$k + 1 = \pm 6$ 1A+1A
$(k - 5)(k + 7) = 0$	1A	$k = 5$ or -7 1A
		$k+1 = 6$ 1A only
$k = 5$ or -7 both correct	1A	
(b) Putting $k = -7$ in (*)	1M	For Sub. negative value of k
$9x^2 + 6x + 1 = 0$		
$(3x + 1)^2 = 0$		L.S. $= (3x + 1)^2$
$x = -\frac{1}{3}$	1A	$x = -\frac{1}{3}$
subs. both for k=-7 and k=5 no mark	6	

Solutions	Marks	Remarks
5. (a) Area of OABC $= \pi 10^2 \times \frac{100°}{360°}$	1M	
$= 87.27$ (corr. to 2 d.p.) (or 87.28)	1A	
(b) Area of $\triangle$ OAC $= \frac{1}{2} \times 10 \times 10 \times \sin 100°$	1M	$\triangle = \frac{1}{2}AC \times OM$
$= 49.24$ (corr. to 2 d.p.)	1A	$= \frac{1}{2} \times 15.3209 \times 6.4279$ ... 1M $= 49.24$ ... 1A
(c) Area of minor segment ABC		
$= 87.27 - 49.24$	1M	
$= 38.03$ (corr. to 2 d.p.) (or 38.04)	1A	
	6	
6. $\log 2 = r$, $\log 3 = s$.		
(a) $\log 18 = \log 2\times 3^2$	1A	For $18 = 2 \times 3^2$
$= \log 2 + \log 3^2$) .... $= \log 2 + 2\log 3$)	1M	) $\log ab = \log a + \log b$ or) $\log a^2 = 2\log a$
$= r + 2s$	1A	
(b) $\log 15 = \log 3\times 5$		
$= \log 3 + \log 5$ 1A		
$= \log 3 + \log \frac{10}{2}$	1A	For $5 = \frac{10}{2}$ or $15 = \frac{30}{2}$
$= \log 3 + \log 10 - \log 2$	1A	
$= 1 - r + s$	1A	
	6	
7. (a) The coordinates of the centre are given by		
$x = -(-\frac{4}{2})$, $y = -\frac{10}{2}$	1M	$(x-2)^2 + (y+5)^2 = 25$; k+4
only answer correct 2A → i.e. $x = 2$, $y = -5$ 2,-5 without bracket PP-1	1A	
(b) As C touches the y-axis,		OR
its radius = 2	1M+1A	Subs. (0, -5) 1M $25 - 50 + k = 0$ $k = 25$ 1A
$4 + 25 - k = 2^2$	1M	$r = \sqrt{4 + 25 - 25}$ 1M $= 2$ 1A
$k = 25$	1A	OR
		Put $x = 0$, $y^2 + 10y + k = 0$ has equal roots. 1M $100 - 4k = 0$ $k = 25$ 1A r = etc.
	6	

Solutions	Marks	Remarks
8. (a) (i)		



Solutions	Marks	Remarks
		ABCD in must order
	1	For P
	1	For Q (between D, C)
(ii) Since Δ PBC is equilateral, $\angle$ PBC = 60°		Follow through even if diagram not accurate
angles written down on the diagram → no method mark; $\angle$ABP = 90° − 60° = 30°	1A	
As BA = BP , $\angle$ PAB = $\frac{1}{2}$ (180° − 30°)	1M	or equivalent
= 75°	1A	OR
Since AB // DC, $\angle$ PQC = 180° − 75°	1M	$\angle$PAD = 15°
= 105°	1A	$\angle$ PQC = 90° + 15° 1M; = 105° 1A
	7	
(b) (i) Δ TCB is similar to Δ ACT because	1	≈ ≅ } PP-1
both correct 1 mark: $\angle$ C is common.		
$\angle$ BTC = $\angle$ BAT (angle in alternate segment) — no need; $\angle$T no mark	1	Indication of 2 pairs of equal angles. Withheld if proving congruence.
△TCB ~ △ACT 1 mark (~~A.A.A.~~) no mark		
(ii) $\frac{AC}{CT} = \frac{CT}{BC}$	1A	Follow through even if (b)(i) wrong.
AC = $\frac{6^2}{5}$ = 7.2 correct substitution	1A	
$\therefore$ AB = 7.2 − 5		
= 2.2 (= $\frac{11}{5}$)	1A	
	5	



Solutions	Marks	Remarks
9. (a) Between 100 and 999,		
the smallest multiple of 7 is 105,	1A	
the largest is 994.	1A	
	2	
(b) The number of multiples is $\frac{994-105}{7}+1$ must be correct	2M	OR 994= 105 + (n-1) X 7
= 128	1A	
The sum of these multiples		
= 105 + 112 + ... + 994		
= $\frac{128}{2}$ [105 + 994]	2M	
= 70336	1A	
	6	
(c) The sum of all positive 3-digit integers	1	
= 100 + 101 + ... + 999 or	~~1A~~	
= $\frac{900}{2}$ [100 + 999] all correct		
= 494,550	1A	
The required sum = 494,550 - 70,336	1M	
= 424,214	1A	
	4	

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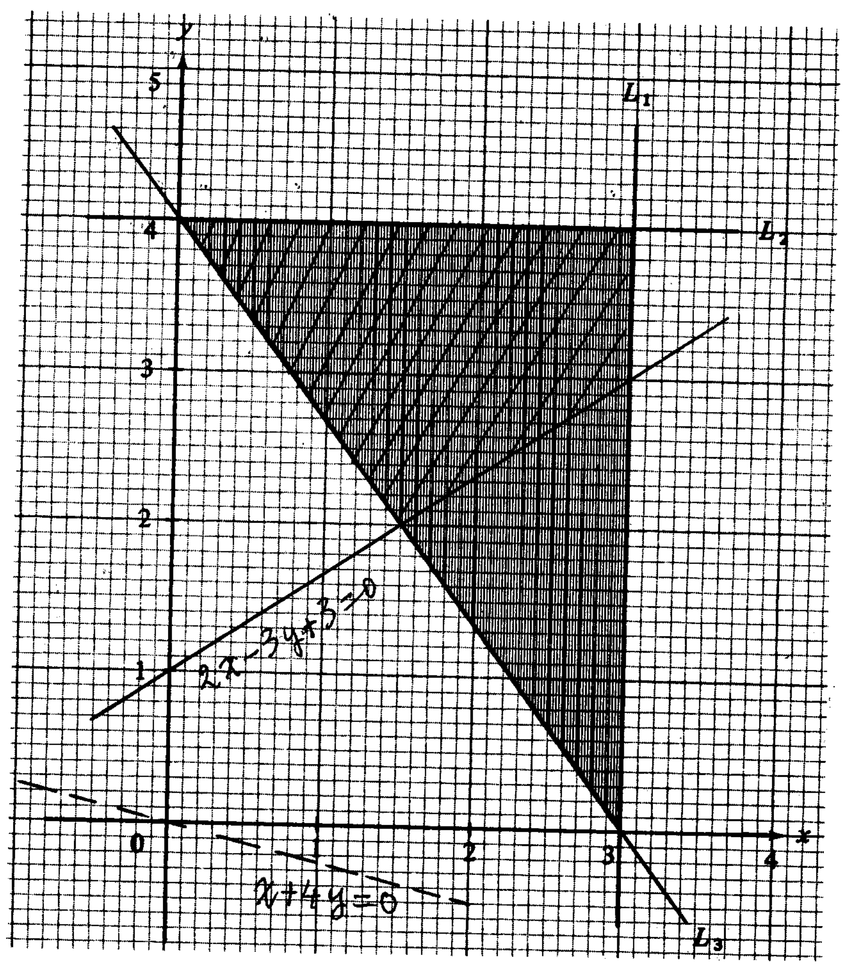
Solutions	Marks	Remarks
10. (a) Let $y = k_1x + k_2x^2$, where k_1 and k_2 are constants.		For $y=kx+kx^2$ or $y = kx+x^2$ or $y = x+kx^2$ 1
	2	
Putting $x = 1$, $y = -5$; $x = 2$, $y = -8$, we have	1M (for substitution)	$y = x + x^2$ no marks
$k_1 + k_2 = -5$	1A	no marks $\begin{cases} y = k_1x \\ y = k_2x^2 \end{cases}$
$2k_1 + 4k_2 = -8$	1A	
Solving, $k_1 = -6$, $k_2 = 1$	1A+1A	
$\therefore$ $y = -6x + x^2$		
Putting $x = 6$, we have $y = 0$.	1A	
	8	
(b) $y = -6x + x^2 = (x^2 - 6x + 9) - 9$	1M	Equality must hold.
$= (x - 3)^2 - 9$	1A	$y=(x+3)^2 - 9$ 0A
When $x = 3$, the value of y is least and the least value is -9.	1M+1A	least value of y is -9 1M 0A
	4	
11. (a) From the curve,		
(i) the median is 70 marks.	1A	
(ii) the 1st quartile is 50 marks.) the 3rd quartile is 86 marks.)	1A	for either
$\therefore$ the interquartile range $= 86 - 50$	1M	
$= 36$ marks	1A	
	4	
(b) (i) From the curve, the number of prize-winners = 60.	1A	
(ii) The probability that the student is a prize-winner $= \frac{60}{600}$ 1M $(= \frac{1}{10})$.	1M+1A	
(iii)(1) The probability that both are prize-winners is $\frac{60}{600}$ 1M $\times \frac{59}{599} = \frac{59}{5990}$ 1A $(=0.01)$	1M+1A	Accept $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$ 1M for product rule
(2) The probability that both are not prize-winners $= \frac{540}{600}$ 1A $\times \frac{539}{599}$ $(= \frac{4851}{5990})$ $(=0.81)$	1A	Accept $\frac{9}{10} \times \frac{9}{10}$
$\therefore$ the probability that at least one is a prize-winner $= 1 -$ 1M $\frac{4851}{5990}$	1M	**OR** $\frac{9}{10} \times \frac{60}{599} + \frac{1}{10} \times \frac{540}{599} + \frac{1}{10} \times \frac{59}{599}$ 1M+1A
1A $= \frac{1139}{5990}$ $(=0.19)$	1A	$= \frac{1139}{5990}$ 1A
	8	

Solutions	Marks	Remarks
12. (a) L_3 is given by $\frac{x}{3} + \frac{y}{4} = 1$	1M	or 2-pt form, etc.
$\frac{y-4}{x} = \frac{4}{3}$ wrong slope 1M		
i.e. $4x + 3y = 12$	1A	Must be in this form.
$\frac{x}{4} + \frac{y}{3} = 1$	2	
(b) The three constraints are $y \leqslant 4$	1A	Withhold 1 mark if '=' omitted.
$x \leqslant 3$	1A	
$4x + 3y \geqslant 12$	1A	or $4x + 3y - 12 \geqslant 0$.
	3	
(c) The line $x + 4y = c$ drawn in the diagram.	1M+1A	For 1A Drop of 2-3 verticle units for 10 horizontal units.
From the diagram, P is greatest when $x = 3$, $y = 4$ and least when $x = 3$, $y = 0$.		OR Testing any vertices 1M
only answer 2A The greatest value of $P = 19$,	1A	At (3, 0), P = 3. At (0, 4), P = 16.
the least value = 3.	1A	At (3, 4), P = 19. 1A
	4	test 2 points only 1M



Solutions	Marks	Remarks
(d) The line $2x - 3y + 3 = 0$ drawn in the diagram.	1A	±1 unit at (1.5, 2), (3, 3).
The shaded region.	1A	Should be reasonably shaded.
P is least when $x = \frac{3}{2}$, $y = 2$.		At (3, 3), P = 15.
The least value = $\frac{19}{2}$ (= 9.5)	1A	At (1.5, 2), P = 9.5.
	3	

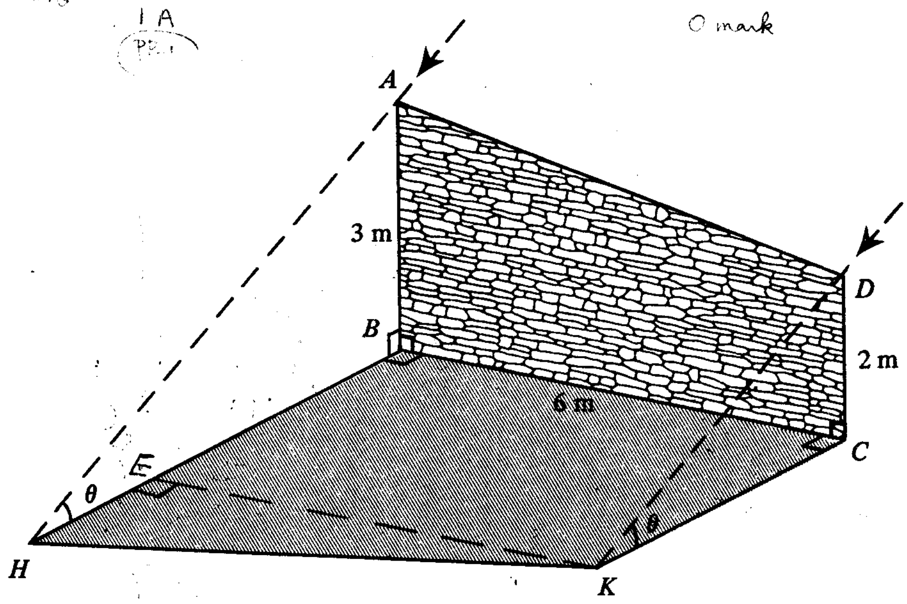
Solutions	Marks	Remarks
13. (a) $\frac{AB}{HB} = \tan\theta$	1M	any part in this question Wrong/no unit, pp-1. in the answer in each part
$HB = \frac{3}{\tan\theta}$ m .... (1M or)	1A	2 + 1
$\frac{DC}{KC} = \tan\theta$, $KC = \frac{2}{\tan\theta}$ m	1A	
	3	
(b) (i) $S_1 = \frac{6}{2}(3 + 2)$		
$= 15\ m^2$	1A	
(ii) $S_2 = \frac{6}{2}\left(\frac{3}{\tan\theta} + \frac{2}{\tan\theta}\right)$		
$= \frac{15}{\tan\theta}\ m^2$	1A	
$\therefore \frac{S_1}{S_2} = \frac{15}{\frac{15}{\tan\theta}} = \tan\theta$	1A	Must show working.
	3	

$\frac{15}{\frac{15}{\tan\theta}} = \tan\theta$ 1A (PP-1)

$\frac{15}{\left(\frac{15}{\tan\theta}\right)} = \tan\theta$; $\tan\theta = \tan\theta$ 0 mark

$\frac{15}{\frac{15}{\tan\theta}}$ $= \tan\theta$ 1A (PP-1)

$\frac{15}{\frac{15}{\tan\theta}}$ $= \tan\theta$ 1A (PP-1)



Solutions	Marks	Remarks
(c) Let $KE \perp BH$. K? = 6 —— 2 marks	1M	Construction of perpendicular line
$EK = BC = 6$ (m)	1A	
$HE = \frac{3}{\tan\theta} - \frac{2}{\tan\theta} = \left(\frac{3}{\tan 30°} - \frac{2}{\tan 30°}\right)$ m $(= \sqrt{3})$ (1M, 1M $\theta = 30°$)	1M+1M	HB = 5.1961... or 5.2
$\therefore HK = \sqrt{HE^2 + EK^2}$	1M	KC = 3.464... or 3.5
$= \sqrt{(\sqrt{3})^2 + 6^2}$		HE = 1.732
$= \sqrt{39}$ m	1A	HK = 6.24
	6	

Solutions	Marks	Remarks
14. (a) (i) $x^3 - \frac{4}{3}x - 6 = 0$ can be written as		
$x^3 = \frac{4}{3}x + 6$.	1M	
Consider the line $y = \frac{4}{3}x + 6$	1A+1A	1A for equation 1A for line drawn, ±1 vertical division about (0, 6), (3, 10)
It cuts the curve $y = x^3$ at $x = r$,		
where r lies between 2.0 and 2.1 .	1A	
(ii) Let $f(x) = x^3 - \frac{4}{3}x - 6$		
無 but $f(2) = -(= -0.67)$ $f(2.1) = +(=0.46)$ } both correct …………………	1M	Correct change of sign.

all correct give 5 marks

Interval	Mid-value x	f(x)
$2.00\bar{0} < r < 2.\overset{+}{1}00$	2.050 1M	1A $-(=-0.12)$
1M $2.05\bar{0} < r < 2.100$	2.075	$+(=0.17)$
$2.050 < r < 2.075$	2.063	$+(=0.02)$
$2.050 < r < 2.063$	2.057	$-(=-0.04)$
$2.057 < r < 2.063$		

Marks: 1M+1A; 1M

Remarks: 1M for choosing mid-value, 1A for correct sign. Next correct interval step.

$\therefore$ $r = 2.06$ (correct to 2 d.p.) — 1A

Total: 9

Alt. Solution:

$f(2) = -$
$f(2.5) = +$) …….. — 1M

2.25 0M+0A

Interval	Mid-value x	f(x)
$2.00\bar{0} < r < 2.\overset{+}{5}00$	2.2■50	+
$2.000 < r < 2.\overset{+}{2}25$	2.113	+
⋮	⋮	⋮

Marks: 1M+1A; 1M

$\therefore$ $r = 2.06$ (correct to 2 d.p.) — 1A

Solutions	Marks	Remarks
(b) Put $x = t + 1$	1A	
The given equation can be written		
as $3x^3 - 4x - 18 = 0$		
or $x^3 - \frac{4}{3}x - 6 = 0$		
By (a), the solution is		
$t = 2.06 - 1$ ………………………	1M	
$= 1.06$ (correct to 2 d.p.)	1A	
	3	

Solutions	Marks	Remarks

14.