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香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 六 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

數學（課程甲／乙）試卷一

MATHEMATICS (SYL A/B) I

評 卷 參 考

MARKING SCHEME

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

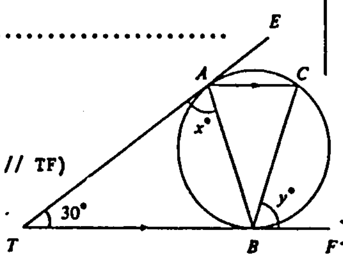
遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Note for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

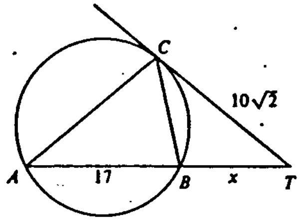
Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute misconduct on the part of the marker and is, moreover, in breach of the 1977 Hong Kong Examinations Authority Ordinace.

SOLUTIONS	MARKS	REMARKS
1. (a) $x^2 - 2x - 3 = (x + 1)(x - 3)$	2A	
(b) $(a^2 + 2a)^2 - 2(a^2 + 2a) - 3$		
$= [(a^2 + 2a) + 1][(a^2 + 2a) - 3]$	1M	
$= (a^2 + 2a + 1)(a^2 + 2a - 3)$		
$= (a + 1)^2\ (a - 1)(a + 3)$	1A+1A / 5	Only awarded if above correct.
Alternatively (for (b))		
Let $f(a) = (a^2 + 2a)^2 - 2(a^2 + 2a) - 3$		
$= (a^4 + 4a^3 + 4a^2) - 2(a^2 + 2a) - 3$		
$= a^4 + 4a^3 + 2a^2 - 4a - 3$		
$f(1) = 0 \Rightarrow (a - 1)$ is a factor of $f(a)$.	1M	For factor theorem
Similarly $(a + 1)$ is a factor	1A	Any two correct factors
$\therefore f(a) = (a - 1)(a + 1)(a^2 + 4a + 3)$		Only awarded if expansion correct.
$= (a - 1)(a + 1)^2(a + 3)$	1A	
2. Since TA and TB are tangents, $\angle TAB = \angle TBA$	1	optional
$x = \frac{1}{2}(180 - 30)$		
$= 75$	1A	
$\angle ACB = \angle ABT\ (= 75°)$	1	1 for reason
$\therefore \angle CBF = \angle ACB$ (as AC // TF)		
$y = 75$	2A / 5	
3. Total number of students $= 40 + x + 35$	1A	
$= 75 + x$		
Sum of all marks $= (40)(61) + 70x + (35)(50)$	1A	
$= 4190 + 70x$		
Overall mean mark $= \dfrac{4190 + 70x}{75 + x} = 60$	2M	有 sub
$4190 + 70x = (75 + x)(60)$		
$10x = 310$		
$x = 31$	1A / 5	



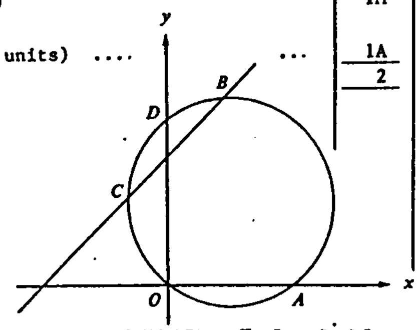
SOLUTIONS STEPS	MARKS	REMARKS
$\sin^2\theta + 7\sin\theta = 5\cos^2\theta$		or $(1-\cos^2\theta)+7\sqrt{1-\cos^2\theta}$ $= 5\cos^2\theta$
$= 5(1 - \sin^2\theta)$	1	
$6\sin^2\theta + 7\sin\theta - 5 = 0$		
$(2\sin\theta - 1)(3\sin\theta + 5) = 0$	1A	
$\sin\theta = \frac{1}{2}$ or $-\frac{5}{3}$ (rejected)	1A+1A	Accept $\sin\theta = \frac{1}{2}$
$\theta = 30°$ or $150°$ [or $\frac{\pi}{6}$, $\frac{5\pi}{6}$]	1A+1A 6	Deduct 1 mark for each extraneous solution.
[or 0.52, 2.62 (corr. to 2 d.p.)		
(Syll A)		OR
(a) $\log_2 8 + \log_2 \frac{1}{16} = 3 + (-4)$	1A+1A	$= \log_2 \frac{8}{16}$ 1A
$= -1$	1A	$= -1$ 2A
(b) $2\log_{10}x - \log_{10}y = 0$		
$\log_{10}x^2 - \log_{10}y = 0$	1A	
$\log_{10}x^2 = \log_{10}y$		OR $\log_{10}\frac{x^2}{y} = 0$
$x^2 = y$ .... (Show working)..	2A 6	$\frac{x^2}{y} = 1$ $y = x^2$ 2A
(Syll B)		
$z = \frac{kx^2}{y}$	2A	For "$z \propto x^2$ and $z \propto \frac{1}{y}$ $\Rightarrow z = kx^2$ and $z = \frac{k}{y}$ $z = \frac{kx^2}{y}$",
Substituting the values of x, y, z,		
$3 = \frac{k(1)^2}{2}$	1A	
$k = 6$	1A	Award 1A and follow through.
$\therefore z = \frac{6x^2}{y}$		OR
Putting $x = 2$, $y = 3$, $z = \frac{6(2)^2}{3}$	1A	$\frac{zy}{x^2} = k$
$= 8$.	1A 6	$\frac{z_1 y_1}{x_1^2} = \frac{z_2 y_2}{x_2^2}$ 2A $\frac{(3)(2)}{12} = \frac{z_2(3)}{2^2}$ 1A $z_2 = 8$ 1A

SOLUTIONS	MARKS	REMARKS
6.		



SOLUTIONS	MARKS	REMARKS
(a) $\triangle$ CAT	2A	No marks if wrong reasons given
(b) $\frac{BT}{CT} = \frac{CT}{AT}$ (or $AT \cdot BT = CT^2$)	1	
$\frac{x}{10\sqrt{2}} = \frac{10\sqrt{2}}{17 + x}$		
$x^2 + 17x - 200 = 0$	1A	
$(x - 8)(x + 25) = 0$ (or $x = \frac{-17 \pm \sqrt{17^2 + 800}}{2}$)	1A	
$\therefore x = 8$ or -25 (rejected)	1A 6	Accept $x = 8$.
7. (a) $\frac{1}{m} + \frac{1}{n} = \frac{1}{a}$		
$\frac{n + m}{mn} = \frac{1}{a}$	1A	
$\frac{b}{mn} = \frac{1}{a}$	1M	For sub. $m+n = b$
$\therefore mn = ab$	1A	
(b) $m^2 + n^2 = (m + n)^2 - 2mn$	1A	
$= b^2 - 2ab$	1M+1A 6	1M for sub. $mn = ab$

SOLUTIONS	MARKS	REMARKS
(a) $y - x - 6 = 0$		
$x^2 + y^2 - 6x - 8y = 0$ ....(*)		
Putting $y = x + 6$ in (*), (or $x = y - 6$)		
$x^2 + (x + 6)^2 - 6x - 8(x + 6) = 0$	1M	
$2x^2 - 2x - 12 = 0$	1A	
$(x + 2)(x - 3) = 0$		
$x = -2$ or 3		
$y = 4$ or 9		
$\therefore$ B = (3, 9) , C = (-2, 4)	1A+1A	
	4	
(b) Putting $y = 0$ in (*),	1M	2 marks for either A or D
$x^2 - 6x = 0$		
$x = 0$ or 6		
A = (6, 0)	1A	
Putting $x = 0$ in (*),		
$y^2 - 8y = 0$		
$y = 0$ or 8		
$\therefore$ D = (0, 8)	1A	
	3	
(c) $\tan \angle ADO = \frac{OA}{OD}$	1M	for calculating $\angle$
$= \frac{3}{4}$		
$\angle ADO = 36.869°$		
$= 37°$ (correct to the nearest degree)	1A	
$\angle ABO = \angle ACO = \angle ADO = 37°$	1A+1A	both $\angle ABC$ and $\angle ACO$
	3	
(d) Area of Δ ACO $= \frac{1}{2}(6)(4)$	1M	$\frac{1}{2}AC \cdot OC \sin \angle ADO$ 1M
$= 12$ (sq. units)	1A	$= \frac{1}{2}\sqrt{80}\sqrt{20} \times 0.6$
	2	$=12$ ....1A (Ans. roundable to 12)



SOLUTIONS	MARKS	REMARKS
9. (a)(i) The common difference is $-1 - 2 = -3$	1A	
The nth term $= 2 + (n - 1)(-3)$		
$= 5 - 3n$	1A	
(ii) The sum of the first n terms S_n		
$= \frac{n}{2}[2 + (5 - 3n)]$ [or $\frac{n}{2}[2(2) + (n-1)(-3)]$]	1M	For formula
$= \frac{n}{2}(7 - 3n)$ (or $\frac{7n - 3n^2}{2}$)	1A	
(iii) $S_{30} = \frac{30}{2}(7 - 90)$ $(= -1245)$; $S_{20} = \frac{20}{2}(7 - 60)$ $(= -530)$	1M	for sub. formula; For either
		Alt. Solution:
$\therefore$ the sum from 21st term to 30th term		$T_{30} = -85$, $T_{21} = -58$ 1M
$= S_{30} - S_{20}$	1M	$\frac{10}{2}(-58+(-85))$ 1M
$= -1245 - (-530)$		
$= -715$	1A	$= -715$ 1A
	7	
(b) $\frac{n}{2}(7 - 3n) < -1000$	1M	
$3n^2 - 7n - 2000 > 0$ or $2000+7n-3n^2<0$	1A	
$(n - \frac{7 + \sqrt{24049}}{6})(n - \frac{7 - \sqrt{24049}}{6}) > 0$		
$(n - 27.01)(n + 24.68) > 0$		
$\therefore$ $n > 27.01$ (or $n \geq 27$) or $n < -24.68$ (or $n < -24$, or $n < -25$) may be omitted	1A+1A	1A for ends points, 1A for signs. Accept $n \geq 27.01$; $n > 27.01$, $n < -24.68$
The least value of n is 28.	1A	
	5	
Alt. Solution: (b)		
Testing	1M	
$S_{27} = -999$ (> -1000)	1A	
$S_{28} = -1078$ (< -1000)	1A	
$\therefore$ the least value of n is 28.	2A	
$\frac{n}{2}(7-3n) = -1000$		
$3n^2 - 7n - 2000 = 0$	1A	
$n = 27.01$ or -24.68	1A	
Least value of $n = 28$	1M+2A	

SOLUTIONS STEPS	MARKS	REMARKS
(a) $QSR = 180 - 50 - 35$		
$= 95°$	1A	
By the sine law,		
$\frac{500}{\sin 95°} = \frac{QS}{\sin 35°}$	1M	Correct formula
$QS = \frac{(500)(\sin 35°)}{\sin 95°}$	1A	
$(= 287.9\ m)$		Accept 287 to 288
$PS = QS \tan\angle POS$	1M	
$= 77.14$	1A	Any figure roundable to this answer
$= 77.1\ (m)$	1A	
P is 77.1 m from the plane.		
	6	
(b) By the sine law,		
$\frac{RS}{\sin 50°} = \frac{500}{\sin 95°}$	1M	
$RS = \frac{(500)(\sin 50°)}{\sin 95°}$	1A	
$(= 384.5)$		Accept 384 to 385
Let θ be the angle of elevation of P from R.		
$\tan\theta = \frac{PS}{RS}$	2M	for calculating θ.
$= \frac{77.1}{384.4}$ 384.5		
$= 0.2006$		
$\theta = 11.34°$	1A	Any figure roundable to this answer
$= 11°$ (correct to the nearest degree)	1A	Alt. Solution:
	6	

Alt. Solution:

Let $\angle RPS = \phi$

$\tan\phi = \frac{RS}{PS}$

$= 4.985$

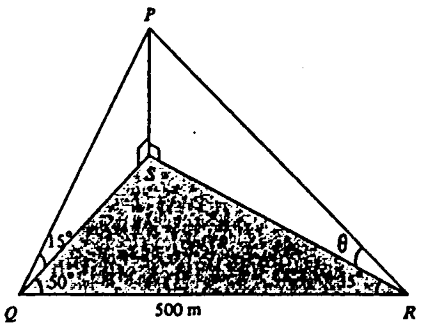
$\phi = 78.66°$

Angle of elevation

$= 90° - \phi$ 2M

$= 11.34°$ 1A

$= 11°$ (corr. to n.d.) 1A



SOLUTIONS STEPS	MARKS	REMARKS
11. (a)(i) Graphs of $x + y = 40$	1A	Correct to ± 'square'
$x + 3y = 60$	1A	Labelling not required
$7x + 2y = 140$	1A	
(ii) Region	3A	
	6	
(b) Let Workshops A and B operate for x and y days respectively. Then $x \geq 0$		
$y \geq 0$		
$x + y \geq 40$		
$x + 3y \geq 60$		
$7x + 2y \geq 140$		
Total expenditure $= 1000x + 2000y$ (dollars).......	2A	
Graph of $1000x + 2000y = 0$ (or equivalent)	1M + 1A	OR For testing any vertex 1M; For testing all other vertices (only if region correct) 1A
From the graph, the expenditure is a minimum when		
$(x, y) = (30, 10)$	2A	Only awarded if region correct
	6	

Alternatively

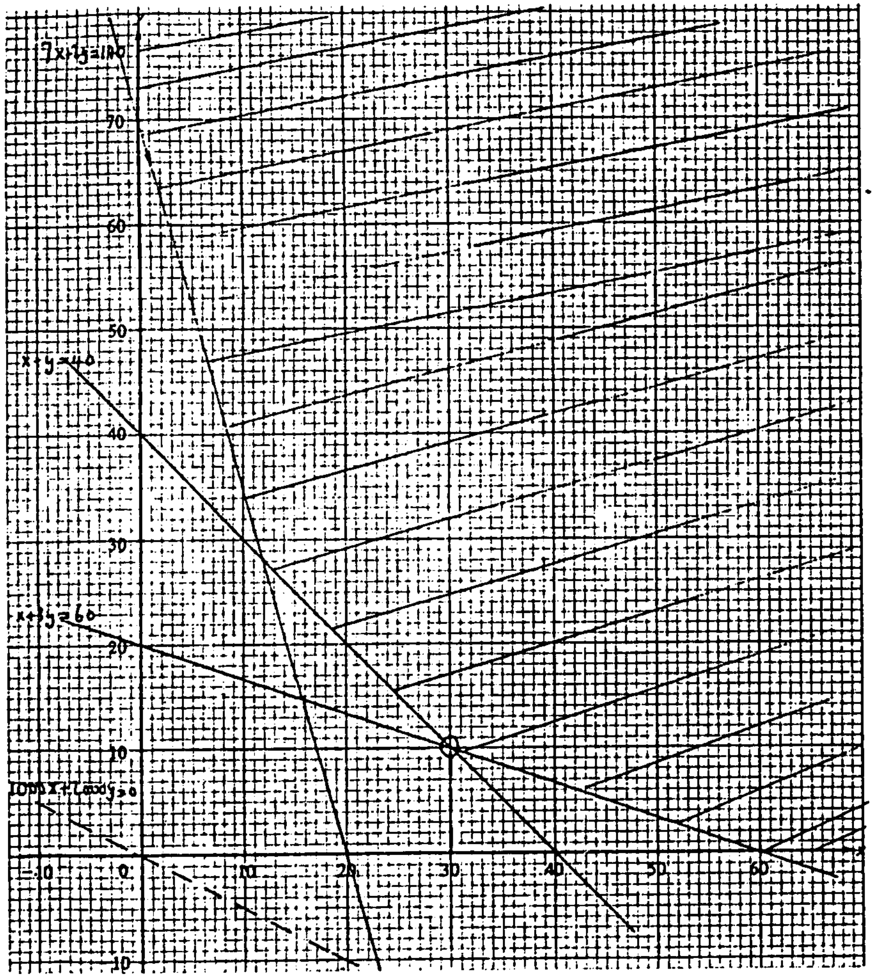
For testing vertices,

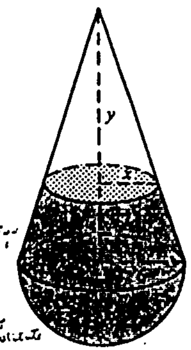
At (0, 70), exp. = 140 000

At (12, 28), exp. = 68 000

At (60, 0), exp. = 60 000

At (30, 10), exp. = 50 000



SOLUTIONS STEPS		MARKS	REMARKS
12. (a)(i)	Let h be the height of the cone.		
	Volume of cone $= \frac{1}{3}\pi 6^2 h$	1A	
	$(= 12\pi h)$		
	Volume of hemisphere $= (\frac{1}{2})(\frac{4}{3})\pi 6^3$	1A	
	$(= 144\pi)$		
	$12\pi h = (\frac{4}{3})(144\pi)$	1M	
	$h = (\frac{4}{3})(\frac{144}{12}) = 16$	1A	
(ii)	Volume of solid $= 12\pi h + 144\pi$	1M	OR $(144\pi)(\frac{7}{3})$ 1M
	$= 336\pi$	1A	$= 336\pi$ 1A
		6	
(b)(i)	By similar triangles,		
	$\frac{x}{y} = \frac{6}{h}$	1M	
	$= \frac{3}{8}$ $(= \frac{6}{16} = 0.375)$	1A	
(ii)	Since the two parts are equal in volume,		
	$\frac{1}{3}\pi x^2 y = (\frac{1}{2})(336\pi)$	1M	
	But $x = \frac{3}{8}y$,		
	$\frac{1}{3}\pi(\frac{3}{8}y)^2 y = (\frac{1}{2})(336\pi)$	1M	
	$y^3 = \frac{(64)(336)}{(3)(2)}$ $(=3584)$		
	$y = 8\sqrt[3]{7}$ $(=15.304)$	1A	Any number roundable to 15.3
	$y = 15.3$ (correct to 1 decimal place)	1A	
		6	

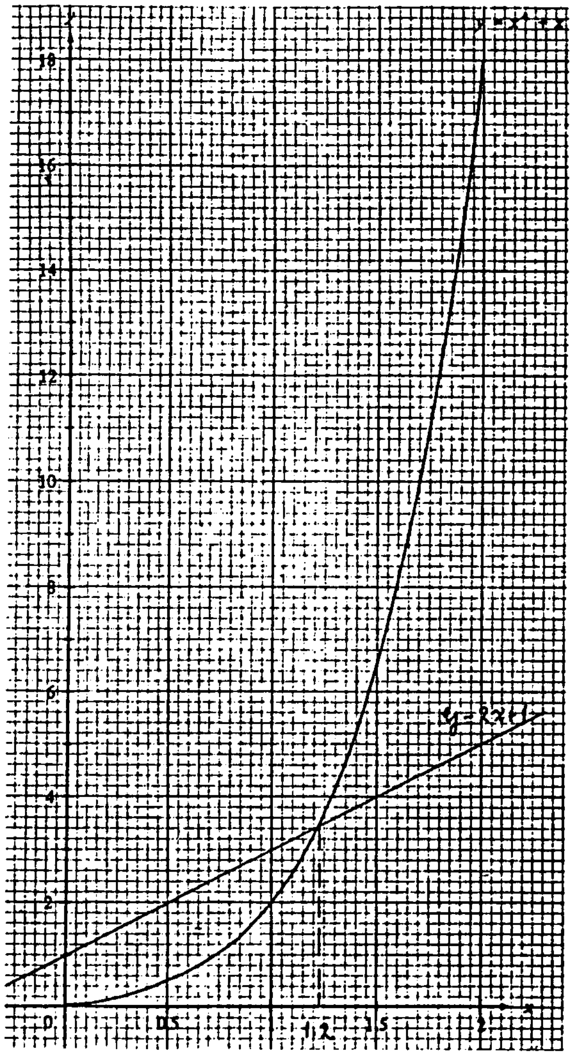
Alt. Solution:

(b)(ii)	$\frac{1}{3}\pi x^2 y = \frac{1}{3}\pi(6^2)(16) + \frac{2}{3}\pi(6^3) - \frac{1}{3}\pi x^2 y$	1M	
	$2\pi x^2 y = \pi(6^2)(16) + 2\pi(6^3)$		
	But $x = \frac{3}{8}y$		
	$2\pi(\frac{3}{8}y)^2 y = 1008\pi$	1M	
	$y^3 = 3584$		
	$y = 8\sqrt[3]{7}$ $(= 15.304)$	1A	
	$= 15.3$ (corr. to 1 d.p.)	1A	

SOLUTIONS STEPS	MARKS	REMARKS
a) If a block is picked out at random, the probability that it is		Simplification of answers not necessary
(i) of red colour is $\frac{(5)(3)}{75} = \frac{1}{5}$	2A	Accept simply giving $\frac{1}{5}$
(ii) of blue colour and shape C is $\frac{3}{75} = \frac{1}{25}$ (or $(\frac{1}{5})(\frac{1}{5}) = \frac{1}{25}$)	2A	
(iii) of size S, shape A or E but not yellow is $\frac{(2)(4)}{75} = \frac{8}{75}$ (or $(\frac{1}{3})(\frac{2}{5})(\frac{4}{5}) = \frac{8}{75}$)	2A 6	
b)(i) The probability that the first is of size L and the second of size S $= (\frac{1}{3})(\frac{1}{3})$ $= \frac{1}{9}$	2A	
(ii) The probability that one is of size L and the other of size S $= (2)(\frac{1}{9})$ $= \frac{2}{9}$	2A	
(iii) The probability that they are both of size L $= (\frac{1}{3})(\frac{1}{3}) = \frac{1}{9}$ The probability that they are both of the same size $= (3)(\frac{1}{9}) = \frac{1}{3}$ The probability that they are of different sizes $= 1 - \frac{1}{3} = \frac{2}{3}$	2A 6	OR $(3)(\frac{2}{9})$ $= \frac{2}{3}$ 2A

SOLUTIONS STEPS	MARKS	REMARKS
14. (Syll A)		
(a) $x^4 - x - 1 = 0$ ....(1) $x^4 + x = 2x + 1$ ....(2)	1M + 1A	Writing L.S. as x^4+x (may show working on graph)
The line $y = 2x + 1$ drawn in Fig. 6	1A	±1 'square' at (0, 1), (1.5, 4)
The curve $y = x^4 + x$ meets the line $y = 2x + 1$ at $x = 1.2$ for $0 \leq x \leq 2$.		
The required root is 1.2 (corr. to 1 d.p.)	1A 4	(Explanation not necessary)
(b) Consider $y = x^4 - x - 1$ Testing for change of sign of y		
x = 1.22, y: −	1M	Change of sign (1 d.p.)
x = 1.23, y: +	1M	Change of sign (2 d.p.)
x = 1.225, y: +	1A	Checking sign at 1.221 to 1.225 ($y > 0$).
$\therefore$ $x = 1.22$ (correct to 2 decimal places)	1A 4	Award only if above correct.
Alt. Solution: Graphical method 1st mag. at 1st d.p. 2nd mag. at 2nd d.p. 1.220 to 1.225 x = 1.22	 1M 1M 1A 1A	
(c) Putting $x = y + 1$ $(y + 1 - 1)^4 = y + 1$ $y^4 = y + 1$ ....(*)	1A	
$y^4 - y - 1 = 0$ By (b), solution of (*) is $y = 1.22$ (correct to 2 decimal places)	1M	
$x = 1.22 + 1$	1M	
$= 2.22$ (correct to 2 decimal places)	1A 4	

(Syll A)



SOLUTIONS STEPS	MARKS	REMARKS
14. (Syll B)		
(a) $y = ax^2 + bx + c$		If 'c' not found first, award at most 3 marks for this part.
Since the curve passes through (0, 6),		
Substituting these values of x, y,		
$6 = a(0)^2 + b(0) + c$		
$c = 6$	1A	
Substituting the coordinates of (3, 0), (-2, 0),	1M	無利用 c=6. 扣1分
$9a + 3b + 6 = 0$ ....(i)		
$4a - 2b + 6 = 0$ ....(ii)		
2 X (i) + 3 X (ii) gives		
$18a + 12a + 12 + 18 = 0$		
$a = -1$	1A	
$\therefore 2b = 4a + 6 = 2$		
$b = 1$	1A	
The curve is given by $y = -x^2 + x + 6$.	4	
(b)(i) $(x + 2)(x - 3) = -1$		
$x^2 - x - 6 = -1$		
$-x^2 + x + 6 = 1$	1M	Writing L.S. same as result in (a)
Draw the line $y = 1$	1A	For line (可不用寫出)
one obtains $x = -1.8$ or 2.8	1A+1A	(有線或其它說明)
(ii) $x^2 - 2x - 1 = 0$		
$-x^2 + 2x + 1 = 0$		
$-x^2 + x + 6 = -x + 5$	1M	Writing L.S. same as result in (a)
Drawing the line $y = -x + 5$,	1A	For line through (3,2) and (0, 5), ±1 'square'
one obtains $x = -0.4$ or 2.4	1A+1A	
	8	用 ordered pair 作 ans 答案扣2分.

$x = -2$ or 3
$(x + 2)(x - 3) = 0$
$x^2 - x - 6 = 0$
$-x^2 + x + 6 = 0$
$\therefore a = -1, b = 1, c = 6$ 4分

$x = -2$ or 3
$x^2 - x - 6 = 0$
$a = 1, b = -1, c = -6$. 0分

Syu 8)