HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1985

數學(課程甲) 試卷一

MATHEMATICS (SYLLABUS A) PAPER I

8.30 am–10.30 am (2 hours)

This paper must be answered in English

Attempt ALL questions in Section A and any FIVE questions in Section B.

Full marks will not be given unless the method of solution is shown.

FORMULAS FOR REFERENCE

SPHERE	Surface area	=	$4\pi r^2$
	Volume	=	$\frac{4}{3}\pi r^3$
CYLINDER	Area of curved surface	=	$2\pi rh$
	Volume	=	$\pi r^2 h$
CONE	Area of curved surface	=	πrl
	Volume	=	$\frac{1}{3}\pi r^2 h$
PRISM	Volume	=	base area × height
PYRAMID	Volume	=	$\frac{1}{3}$ × base area × height

SECTION A Answer ALL questions in this section.
There is no need to start each question in this section on a fresh page.
Geometry theorems need not be quoted when used.

1. (a) Factorize $a^4 - 16$ and $a^3 - 8$.

   (b) Find the L.C.M. of $a^4 - 16$ and $a^3 - 8$.

(5 marks)

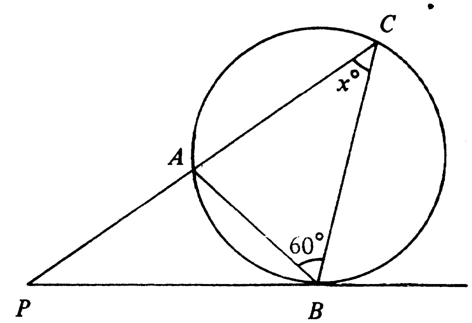
2. 

Figure 1

In Figure 1, PB touches the circle ABC at B. PAC is a straight line. $\angle ABC = 60°$. $AP = AB$. Find the value of x.

(5 marks)

3. Given a curve whose equation is $y = x^3 - hx + k$, where h and k are constants. The curve passes through $P(2, 3)$ and the slope of the curve at P is 10. Find the values of h and k.

(5 marks)

4. Given $f(x) = ax^2 + bx - 1$, where a and b are constants. $f(x)$ is divisible by $x - 1$. When divided by $x + 1$, $f(x)$ leaves a remainder of 4. Find the values of a and b.

(6 marks)

5. Let α and β be the roots of $x^2 + kx + 1 = 0$, where k is a constant.

(a) Find, in terms of k,

(i) $(\alpha + 2) + (\beta + 2)$,

(ii) $(\alpha + 2)(\beta + 2)$.

(b) Suppose $\alpha + 2$ and $\beta + 2$ are the roots of $x^2 + px + q = 0$, where p and q are constants. Find p and q in terms of k.

(6 marks)

6. Solve $2 \tan^2 \theta = 1 - \tan \theta$, where $0° \leqslant \theta < 360°$.
(Give your answers correct to the nearest degree.)

(6 marks)

7.

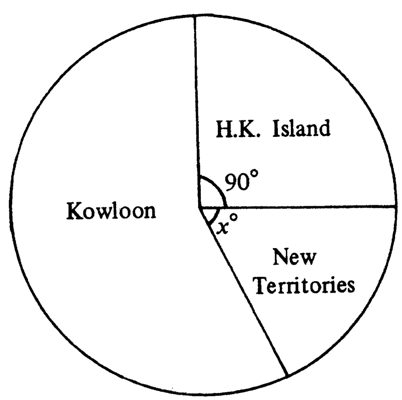


Figure 2

The pie-chart in Figure 2 shows the distribution of traffic accidents in Hong Kong in 1983. There were 4200 traffic accidents on H.K. Island, 9240 accidents in Kowloon and n accidents in the New Territories. Find n and x.

(6 marks)

**SECTION B Answer FIVE questions in this section.
Each question carries 12 marks.**

8.

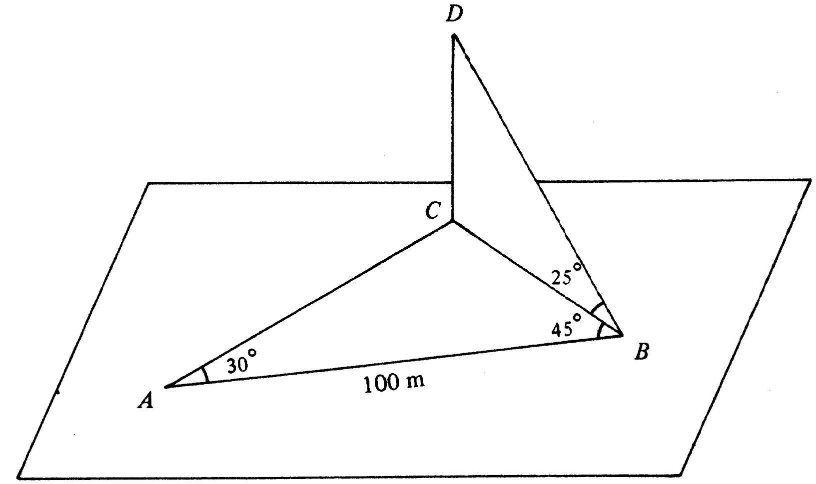


Figure 3

In Figure 3, A, B and C are three points in a horizontal plane. $AB = 100$ m. $\angle CAB = 30°$, $\angle ABC = 45°$.

(a) Find BC and AC, in metres, correct to 1 decimal place.

(5 marks)

(b) D is a point vertically above C. From B, the angle of elevation of D is $25°$.

(i) Find CD, in metres, correct to 1 decimal place.

(ii) X is a point on AB such that $CX \perp AB$.

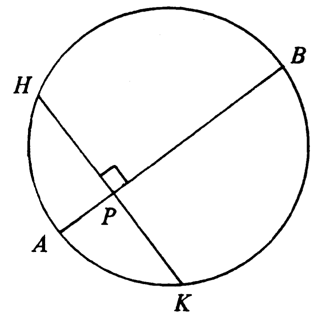
(1) Find CX, in metres, correct to 1 decimal place.

(2) Find the angle of elevation of D from X, correct to the nearest degree.

(7 marks)

9.

Figure 4



In Figure 4, $A(2\ ,\ 0)$ and $B(7\ ,\ 5)$ are the end-points of a diameter of the circle. P is a point on AB such that $\frac{AP}{PB} = \frac{1}{4}$.

(a) Find the equation of the circle. (3 marks)

(b) Find the coordinates of P. (2 marks)

(c) The chord HPK is perpendicular to AB.

(i) Find the equation of HPK.

(ii) Find the coordinates of H and K. (7 marks)

10. (a) If two dice are thrown,

(i) find the probability that the sum of the numbers on the two dice is greater than 9 ;

(ii) find the probability that the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal. (6 marks)

(b) In a game, two dice are thrown. In each throw, 2 points are gained if the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal; otherwise 1 point is lost. Using the result in (a)(ii), find the probability of

(i) losing a total of 2 points in two throws,

(ii) gaining a total of 1 point in two throws. (6 marks)

11.

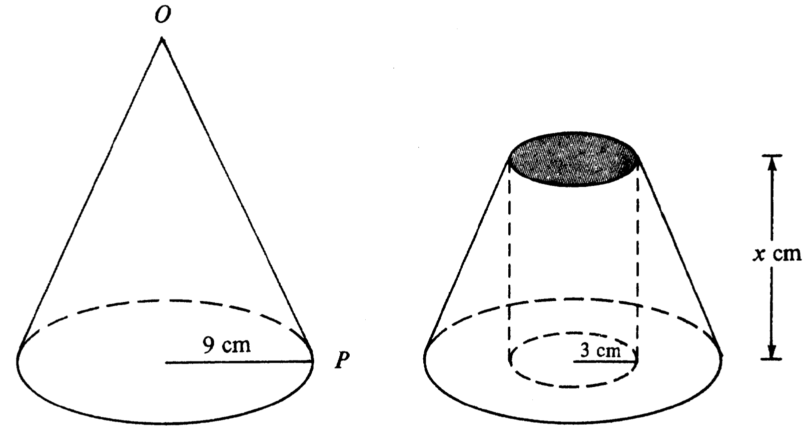


Figure 5(a) Figure 5(b)

Figure 5(a) shows a solid right circular cone. O is the vertex and P is a point on the circumference of the base. The area of the curved surface is $135\pi\ cm^2$ and the radius of the base is 9 cm.

(a) (i) Find the length of OP.

(ii) Find the height of the cone. (5 marks)

(b) The cone in Figure 5(a) is cut into two portions by a plane parallel to its base. The upper portion is a cone of base radius 3 cm. The lower portion is a frustum of height x cm.

(i) Find the value of x.

(ii) A right cylindrical hole of radius 3 cm is drilled through the frustum (see Figure 5(b)). Find the volume of the solid which remains in the frustum. (Give your answer in terms of π .)

(7 marks)

12. **If you attempt this question, you should refer to the separate supplementary leaflet provided.**

Figure 6 shows the graph of

$$y = x^3 + x \quad \text{for } -1 \leqslant x \leqslant 2 .$$

(a) (i) Draw a suitable straight line in Figure 6 and hence find, correct to 1 decimal place, the real root of the equation

$$x^3 + x - 1 = 0 .$$

(ii) By the method of magnification, find the real root of the equation in (i), correct to 2 decimal places.

(7 marks)

(b) (i) Expand and simplify the expression

$$(x + 1)^4 - (x - 1)^4 .$$

(ii) Using the result in (a)(ii), find, correct to 2 decimal places, the real root of the equation

$$(x + 1)^4 - (x - 1)^4 = 8 .$$

(5 marks)

13.

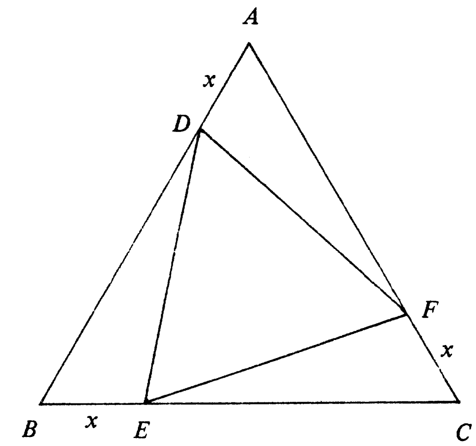


Figure 7

In Figure 7, ABC is an equilateral triangle. $AB = 2$. D, E, F are points on AB, BC, CA respectively such that $AD = BE = CF = x$.

(a) By using the cosine formula or otherwise, express DE^2 in terms of x.

(3 marks)

(b) Show that the area of $\Delta DEF = \frac{\sqrt{3}}{4}(3x^2 - 6x + 4)$.

Hence, by using the method of completing the square, find the value of x such that the area of ΔDEF is smallest.

(5 marks)

(c) If the area of $\Delta DEF \leqslant \frac{\sqrt{3}}{3}$, find the range of the values of x.

(4 marks)

14. $\$P$ is deposited in a bank at the interest rate of $r\%$ per annum compounded annually. At the end of each year, $\frac{1}{3}$ of the amount in the account (including principal and interest) is drawn out and the remainder is redeposited at the same rate.

Let $\$Q_1$, $\$Q_2$, $\$Q_3$, ... denote respectively the sums of money drawn out at the end of the first year, second year, third year, ...

(a) (i) Express Q_1 and Q_2 in terms of P and r.

(ii) Show that $Q_3 = \frac{4}{27}P(1 + r\%)^3$.

(5 marks)

(b) Q_1, Q_2, Q_3, ... form a geometric progression. Find the common ratio in terms of r.

(2 marks)

(c) Suppose $Q_3 = \frac{27}{128}P$.

(i) Find the value of r.

(ii) If $P = 10\,000$, find $Q_1 + Q_2 + Q_3 + \ldots + Q_{10}$. (Give your answer correct to the nearest integer.)

(5 marks)

END OF PAPER

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1985

數學(課程甲) 試卷一(附頁)

MATHEMATICS (SYLLABUS A) PAPER I
(SUPPLEMENTARY LEAFLET)

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

12. If you attempt this question, fill in the details in the first three boxes above and tie this sheet inside your answer book.

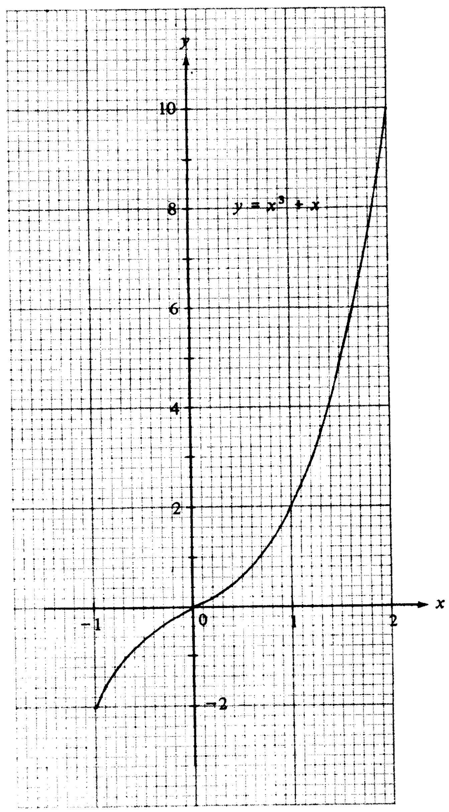


Figure 6

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1985

數學(課程乙) 試卷一
MATHEMATICS (SYLLABUS B) PAPER I

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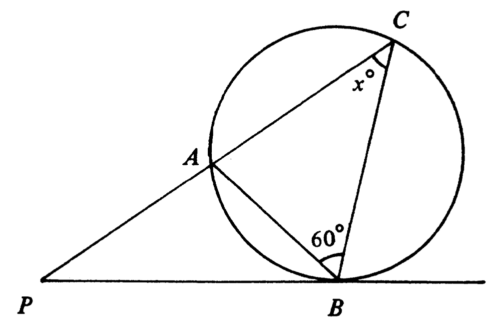

Figure 1

In Figure 1, PB touches the circle ABC at B. PAC is a straight line. $\angle ABC = 60°$. $AP = AB$. Find the value of x.

(5 marks)

3. Solve $2^{2x} - 3(2^x) - 4 = 0$.

(5 marks)

4. Given $f(x) = ax^2 + bx - 1$, where a and b are constants. $f(x)$ is divisible by $x - 1$. When divided by $x + 1$, $f(x)$ leaves a remainder of 4. Find the values of a and b.

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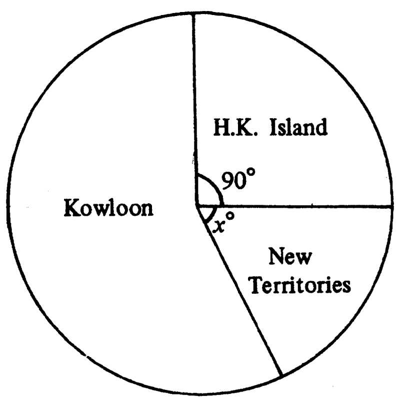

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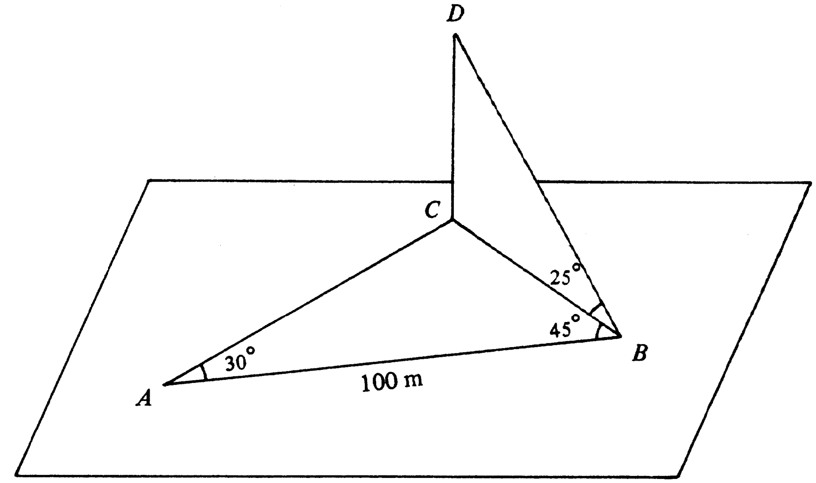

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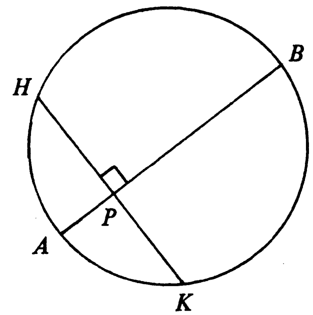

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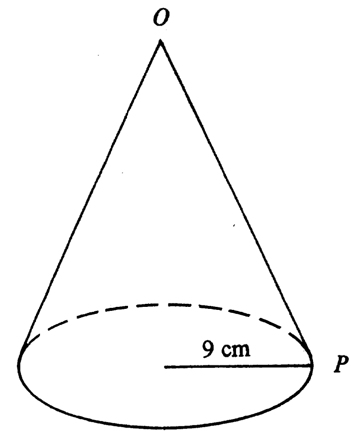

Figure 5(a)

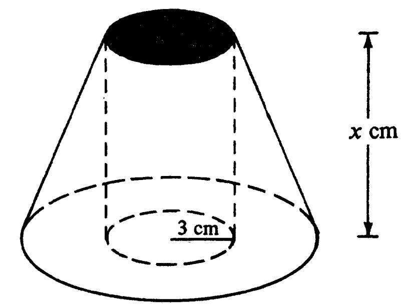

Figure 5(b)

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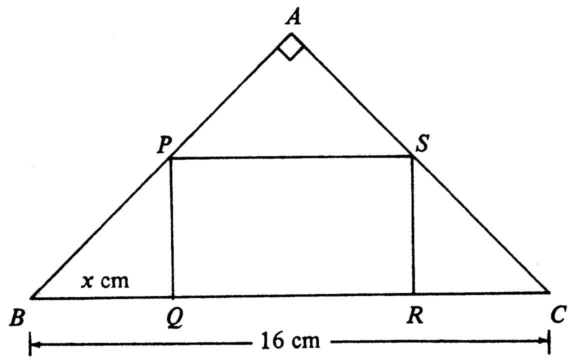

Figure 6(a)

In Figure 6(a), ABC is an isosceles triangle with $\angle A = 90^\circ$. $PQRS$ is a rectangle inscribed in $\triangle ABC$. $BC = 16$ cm, $BQ = x$ cm.

(a) Show that the area of $PQRS = 2(8x - x^2)\ \text{cm}^2$.

(2 marks)

(b) Figure 6(b) shows the graph of $y = 8x - x^2$ for $0 \leqslant x \leqslant 8$. Using the graph,

(i) find the value of x such that the area of $PQRS$ is greatest;

(ii) find the two values of x, correct to 1 decimal place, such that the area of $PQRS$ is $28\ \text{cm}^2$.

(5 marks)

(c) (i) If the area of $PQRS$ is greater than four times the area of $\triangle BPQ$ by $8\ \text{cm}^2$, show that $x^2 - 4x + 2 = 0$.

(ii) Draw a suitable straight line in Figure 6(b) and hence find, correct to 1 decimal place, the two roots of the equation in (c)(i).

(5 marks)

13.

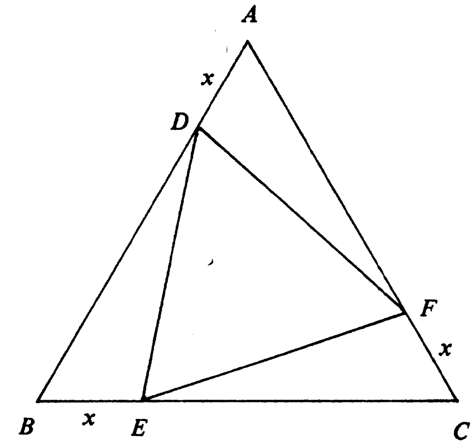

Figure 7

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(b) Q_1, Q_2, Q_3, ... form a geometric progression. Find the common ratio in terms of r.

(2 marks)

(c) Suppose $Q_3 = \frac{27}{128}P$.

(i) Find the value of r.

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(5 marks)

END OF PAPER

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數學(課程乙)　試卷一(附頁)

MATHEMATICS (SYLLABUS B) PAPER I (SUPPLEMENTARY LEAFLET)

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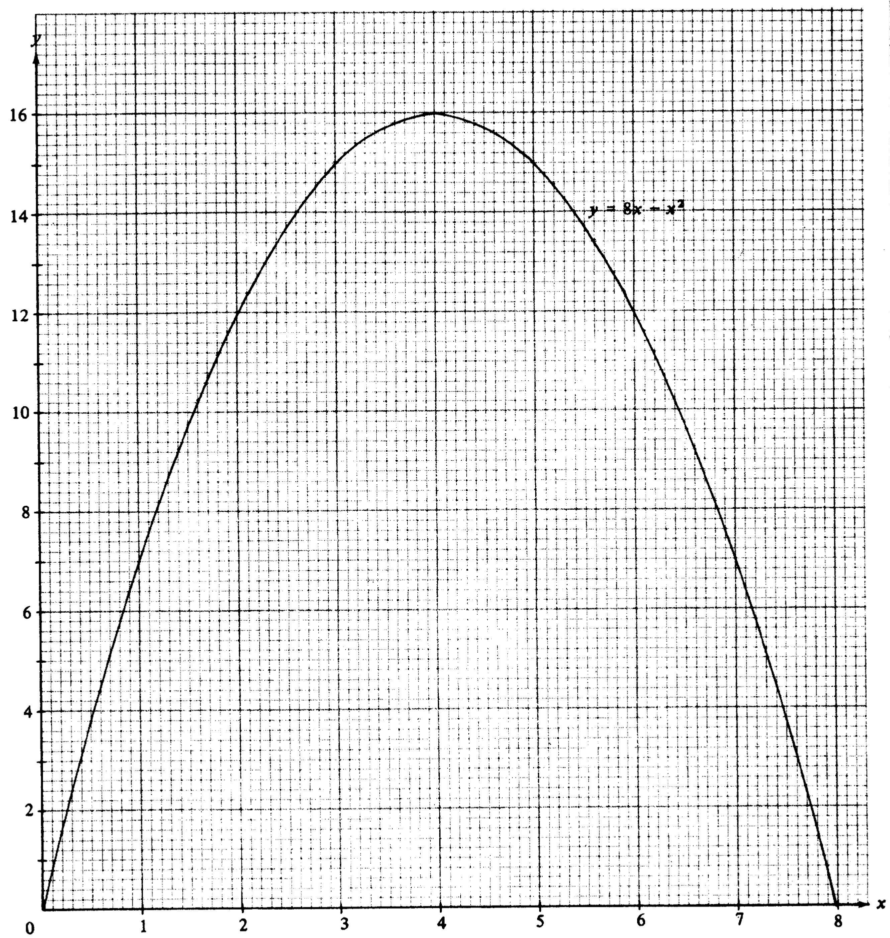

Figure 6(b)