Form 5

HKCEE 1982
Mathematics II

82 1. $\dfrac{2a}{a^2-4b^2}+\dfrac{1}{2b-a}=$

A. $\dfrac{1}{a+2b}$
B. $\dfrac{2a-1}{(a+2b)(a-2b)}$
C. $\dfrac{2a+1}{(a+2b)(a-2b)}$
D. $\dfrac{3a+2b}{(a+2b)(a-2b)}$
E. $\dfrac{a+2b}{(a+2b)(a-2b)}$

82 2. $\dfrac{8^{2x}\cdot 4^{3x}}{2^x\cdot 16^{2x}}=$

A. 2^{3x}
B. 2^{2x}
C. 2^x
D. 8
E. 1

82 3. $(a^{-2}-3b^{-1})^{-1}=$

A. $\dfrac{3a^2+b}{a^2b}$
B. $\dfrac{3a^2-b}{a^2b}$
C. $\dfrac{3b-a^2}{a^2b}$
D. $\dfrac{3a^2b}{b-3a^2}$
E. $\dfrac{3a^2b}{3b-a^2}$

82 4. If $x=\dfrac{1}{\frac{1}{y}+\frac{2}{z}}$, then $y=$

A. $\dfrac{2x}{z}$
B. $\dfrac{z}{xz-z}$
C. $\dfrac{z-2x}{xz}$
D. $\dfrac{xz}{2x+z}$
E. $\dfrac{xz}{z-2x}$

82 5. If $10^{kx+a}=P$, then $x=$

A. $\dfrac{1}{k}(10^{P-a})$
B. $\log_{10}\dfrac{P-a}{k}$
C. $\dfrac{1}{k}\log_{10}P-a$
D. $\dfrac{1}{k}(\log_{10}P-a)$
E. $\dfrac{1}{k}(\log_{10}P+a)$

82 6. α and β are the roots of the equation $x^2-5x-7=0$. What is the equation whose roots are $\alpha+1$ and $\beta+1$?

A. $x^2-3x+3=0$
B. $x^2-3x-11=0$
C. $x^2-5x+1=0$
D. $x^2-7x-1=0$
E. $x^2-7x-7=0$

82 7. What are the roots of the equation $(x-3)^2(x+1)=-(x+1)^2(x-3)$?

A. 1 only
B. 1, −3 only
C. −1, 3 only

D. $1, -1, -3$
E. $1, -1, 3$

82 8. $5 - 9x - 2x^2 > 0$ is equivalent to

A. $x > \frac{1}{2}$
B. $x < -5$
C. $-5 < x < \frac{1}{2}$
D. $x < -5$ or $x > \frac{1}{2}$
E. $x > -5$ or $x < \frac{1}{2}$

82 9. What will \$P amount to in 3 years' time. If interest is compounded monthly at 12% per annum?

A. $\$P(1 + \frac{36}{100})$
B. $\$P(1 + \frac{1}{100})^{36}$
C. $\$P(1 + \frac{12}{100})^{36}$
D. $\$P(1 + \frac{12}{100})^{3}$
E. $\$P(1 + \frac{1}{100})^{3}$

82 10. A child spent $\frac{1}{10}$ of his saving on a shirt and $\frac{1}{5}$ of his savings on a pair of trousers. He then spent 30% of the rest of his savings on books. What percentage of his saving did he spend altogether?

A. 49.6%
B. 50.4%
C. 51%
D. 58%
E. 60%

82 11. The rent of a flat is raised by 30% every two years beginning from a fixed date. Counting from that date, after how many years will the rent just exceed twice the original rent?

A. 4 years
B. 5 years
C. 6 years
D. 7 years
E. Over 7 years

82 12. A man drives 20 km at 40km/h. At what speed must he drive on his return journey so that the average speed for the double journey is 60 km/h?

A. 50 km/h
B. 80 km/h
C. 100 km/h
D. 120 km/h
E. 160 km/h

82 13. The marked price of a book is \$240. If the book is sold at a discount of 20%, the profit will be 20% of the cost price. What is the cost price of the book?

A. \$153.6
B. \$160
C. \$192
D. \$200
E. \$240

82 14. A right circular cone of altitude $3r$ and base radius r has the same volume as a cube of side x. $x =$

A. πr^3
B. πr
C. $\frac{1}{3}\pi r$
D. $\sqrt[3]{3\pi}\, r$
E. $\sqrt[3]{\pi}\, r$

82 15. Some air escapes from a spherical balloon of volume a^3. The balloon keeps its spherical shape and is now of volume b^3. What is the percentage decrease in the radius?

A. $\frac{a-b}{a} \times 100\%$
B. $\frac{a-b}{b} \times 100\%$
C. $\sqrt[3]{\frac{a^3-b^3}{a^3}} \times 100\%$
D. $\sqrt[3]{\frac{a^3-b^3}{b^3}} \times 100\%$
E. $\frac{a^3-b^3}{a^3} \times 100\%$

82 16. Coffee A and coffee B are mixed in the ratio 1 : 2. A profit of 20% on the cost price is made by selling the mixture at \$36/kg. If the cost price of A is \$12/kg, what is the cost price of B?

A. \$18/kg
B. \$24/kg
C. \$39/kg
D. \$48/kg
E. \$66/kg

82 17. $(\sin\theta + \cos\theta)^2 - 1 =$

A. 0
B. 1
C. $2\cos^2\theta$
D. $2\sin\theta\cos\theta$
E. $-2\sin\theta\cos\theta$

82 18. If $\tan x = -\frac{3}{4}$ and x is an angle in the second quadrant, what is the value of $\sin x + \cos x$?

A. $-\frac{7}{5}$
B. $-\frac{1}{5}$
C. $\frac{1}{5}$
D. 1
E. $\frac{7}{5}$

82 19. If $A + B = 180^\circ$, which of the following is/are true?

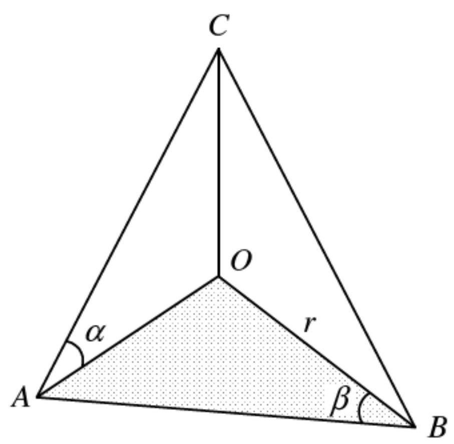
I. $\sin A = \sin B$
II. $\cos A = \cos B$
III. $\tan A = \tan B$

A. I only
B. II only
C. III only
D. I, II and III
E. None of them

82 20. From the top of a lighthouse, h metres high, the angle of depression of a boat is 20°. How far is the boat from the base of the lighthouse, which is at sea-level?

A. $h \sin 20^\circ$ m
B. $h \cos 20^\circ$ m
C. $h \tan 20^\circ$ m
D. $\frac{h}{\sin 20^\circ}$ m
E. $\frac{h}{\tan 20^\circ}$ n

82 21.



In the figure, OAB is a right-angled triangle in a horizontal plane with $\angle AOB = 90^{\circ}$. OC is a vertical line. If $OB = r$, $AC =$

A. $\dfrac{r\sin\beta}{\tan\alpha}$

B. $\dfrac{r\tan\alpha}{\cos\beta}$

C. $\dfrac{r\sin\beta}{\sin\alpha}$

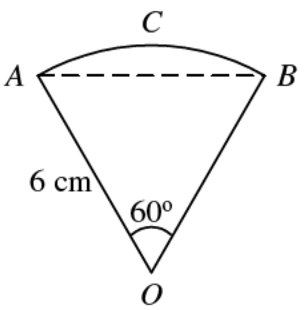
D. $\dfrac{r\cos\beta}{\tan\alpha}$

E. $\dfrac{r\tan\beta}{\cos\alpha}$

82
22. In a circle, the angle of a sector is 30° and the radius is 2 cm. The area of the sector is

A. 120 cm^2
B. 60 cm^2
C. $\dfrac{30}{\pi}$ cm^2
D. $\dfrac{2\pi}{3}$ cm^2
E. $\dfrac{\pi}{3}$ cm^2

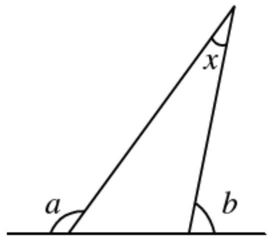
82
23.



In the figure, $OACB$ is a sector of a circle of radius 6 cm. Arc ACB is longer than the chord AB by

A. $(\pi - 3)$ cm
B. $2(\pi - 3)$ cm
C. $3(\pi - 1)$ cm
D. $6(\pi - 1)$ cm
E. $3(2\pi - \sqrt{3})$ cm

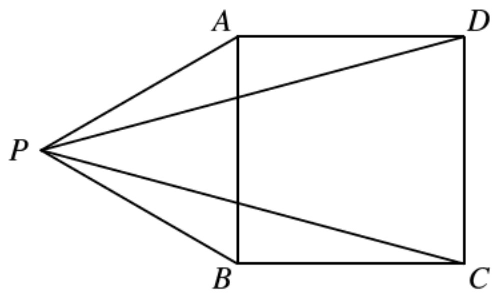
82
24.



In the figure, $x =$

A. $a - b$
B. $a + b - 180^{\circ}$
C. $a + b - 90^{\circ}$
D. $180^{\circ} - a + b$
E. $360^{\circ} - a - b$

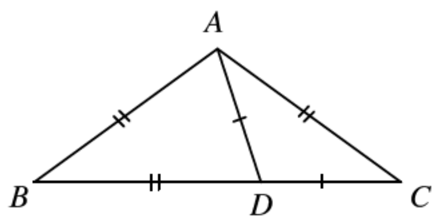
82
25.



In the figure, $ABCD$ is a square and PAB is an equilateral triangle. $\angle CPD$ =

A. 20°
B. 25°
C. 30°
D. 32°
E. 36°

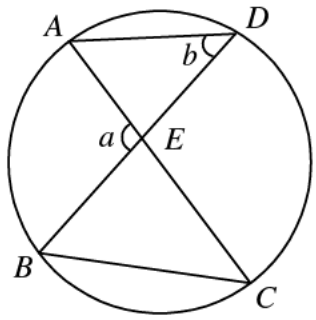
82
26.



In the figure, D is a point on BC such that $AD = CD$ and $AB = AC = BD$.
$\angle B =$

A. $22\frac{1}{2}^{\circ}$
B. 30°
C. 36°
D. 45°
E. 60°

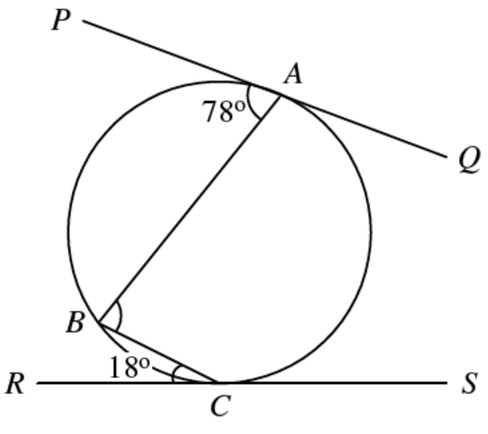
82 27.



In the figure, AKC and BKD are two chords of the circle. $\angle CBD =$

A. $a - b$
B. $a + b$
C. $a + b - 90^{\circ}$
D. $\frac{1}{2}a$
E. $\frac{1}{2}a + b$

82 28.



In the figure, PQ and RS touch the circle at A and C respectively. $\angle ABC =$

A. 48°
B. 60°
C. 84°
D. 90°
E. 96°

82 29. If $f(x) = 5x + 1$, then $f(x + 1) - f(x) =$

A. 1
B. 6
C. $4 \cdot 5^x$
D. $5 \cdot 5^x$
E. $4 \cdot 5^x + 1$

82 30. $\log_{10}(x^{\log_{10} x}) =$

A. $(\log_{10}x)^2$
B. $\log_{10}(x^2)$
C. $x\log_{10}x$
D. $\log_{10}(\log_{10}x)$
E. 10^{x^2}

82 31. The graphs of $y = \frac{x^2}{2}$ and $y = x + 2$ intersect at the points (x_1, y_1) and (x_2, y_2). Which of the following equations has roots x_1 and x_2?

A. $x^2 - x - 2 = 0$
B. $x^2 + x + 2 = 0$
C. $x^2 - 2x - 4 = 0$
D. $x^2 - 4x - 8 = 0$
E. $2x^2 - x - 2 = 0$

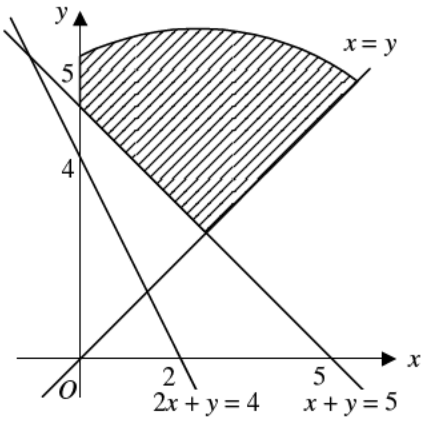
82 32. Let $a > 2$. The inequality $2x - 2a < ax + 5a$ is equivalent to

A. $x > \frac{7a}{2-a}$
B. $x < \frac{7a}{2-a}$
C. $x > \frac{-3a}{2-a}$
D. $x < \frac{-3a}{2-a}$
E. $x > \frac{-7a}{2-a}$

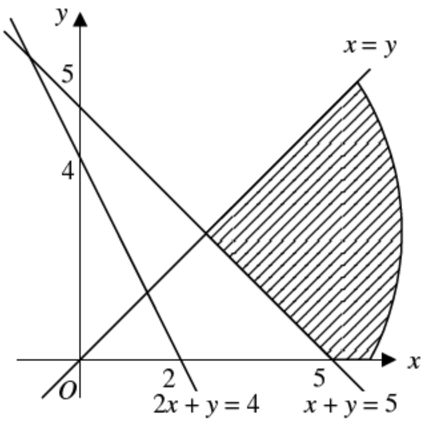
82
33. If $\begin{cases} x \geq 0, \\ y \geq 0, \\ x + y \leq 5, \\ 2x + y \geq 4, \\ x \geq y, \end{cases}$

in which of the following shaded regions do all the points satisfy the above inequalities?

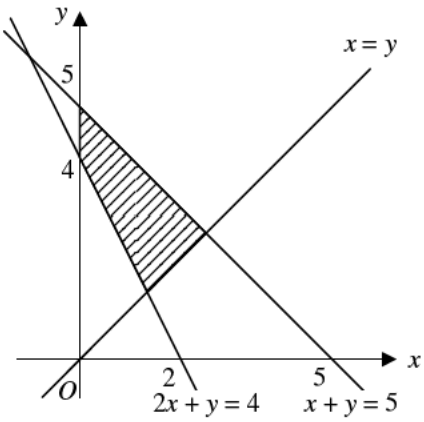
A.



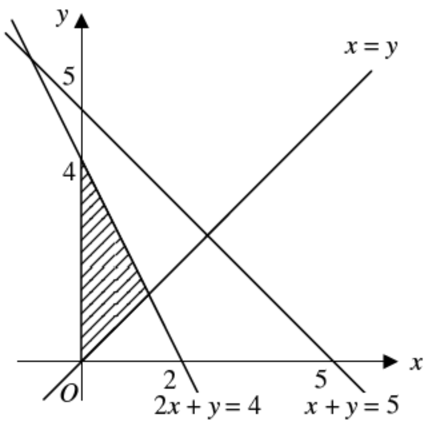
B.



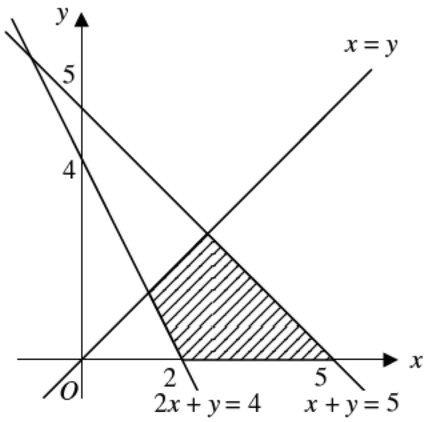
C.



D.



E.



82
34. a, b and k are real numbers. If $k > 0$ and $a > b$, which of the following must be true?

I. $a^2 > b^2$
II. $-a < -b$
III. $ka > kb$

A. II only
B. III only
C. I and III only
D. II and III only
E. I, II and III

82
35. \$9000 is divided among A, B and C. A's share, B's share and C's share, in that order, form an arithmetic progression. If B's share is three times A's share, how much does C get?

A. \$1500
B. \$3000
C. \$4500
D. \$5000
E. \$6000

82 36. $1, -0.1, 0.01, -0.001, \ldots$ is a geometric progression. What is its sum to infinity?

A. 0
B. 1
C. 0.99
D. $\dfrac{10}{11}$
E. $\dfrac{10}{9}$

82 37. If $x \neq 0$, which of the following is/are geometric progression?

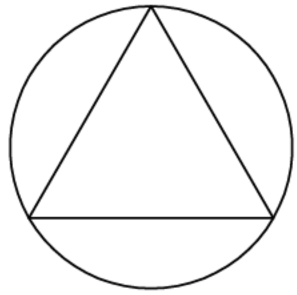
I. x, x^2, x^3, x^4
II. $x, 2x, 3x, 4x,$
III. $x, -x^2, x^3, -x^4$

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II and III

82 38. The average of x and y is a, the average of y and z is b, and the average of x and z is c. What is the average of x, y and z?

A. $\dfrac{1}{6}(a+b+c)$
B. $\dfrac{1}{3}(a+b+c)$
C. $\dfrac{1}{2}(a+b+c)$
D. $\dfrac{2}{3}(a+b+c)$
E. $\dfrac{3}{2}(a+b+c)$

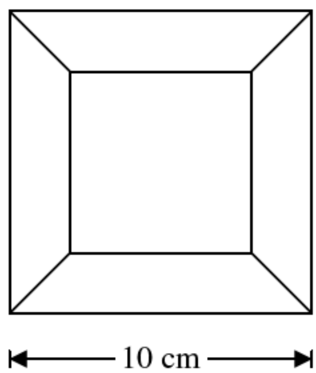
82 39.



In the figure an equilateral triangle is inscribed in a circle of radius a. What is the area of the triangle?

A. $\dfrac{3}{2}a^2$
B. $\dfrac{3\sqrt{3}}{4}a^2$
C. $\dfrac{3}{4}a^2$
D. a^2
E. $\dfrac{3\sqrt{3}}{2}a^2$

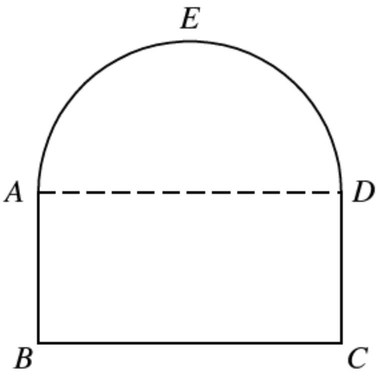
82 40.



Four identical trapeziums, each of area 16 cm^2, are drawn inside a square of side 10 cm as shown in the figure. What is the height of each trapezium?

A. $\dfrac{1}{2}$ cm
B. 1 cm
C. 2 cm
D. 3 cm
E. 4 cm

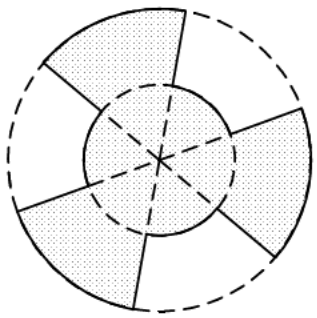
82
41.



The perimeter of the given figure $ABCDE$ is $2(\pi + 4)$ cm. The upper portion AED is a semi-circle and the lower portion $ABCD$ is a rectangle. $AB : BC = 1 : 2$. What is the area of the given figure?

A. 8 cm^2
B. $2\pi \text{ cm}^2$
C. $4\pi \text{ cm}^2$
D. $4(\pi + 2) \text{ cm}^2$
E. $2(\pi + 4) \text{ cm}^2$

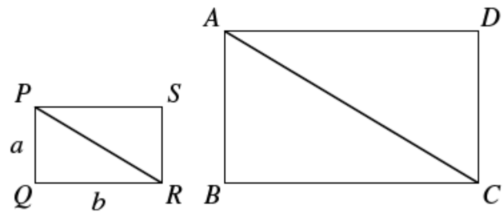
82
42.



In the figure, the two concentric circles are of radius 2 cm and 4 cm respectively. Each circle is divided into 6 equal parts by 6 radii. What is the area of the shaded region?

A. $12\pi \text{ cm}^2$
B. $10\pi \text{ cm}^2$
C. $9\pi \text{ cm}^2$
D. $6\pi \text{ cm}^2$
E. $2\pi \text{ cm}^2$

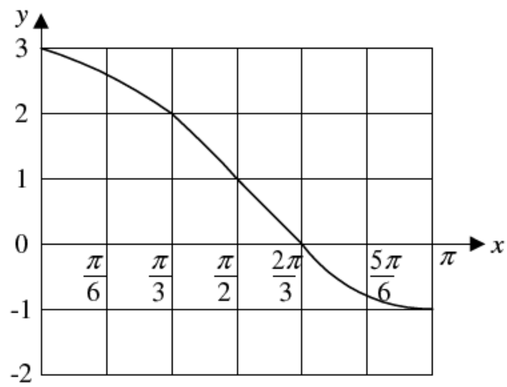
82
43.



In the figure, the rectangles are similar. $PQ = a$, $QR = b$. If $AC = 2PR$, what is the area of $ABCD$?

A. $2ab$
B. $4ab$
C. $2(a^2 + b^2)$
D. $2(a + b)\sqrt{a^2 + b^2}$
E. $2ab\sqrt{a^2 + b^2}$

82
44.



The above figure shows the graph of $y = a \cos x + 1$ for $0 \le x \le \pi$. $a =$

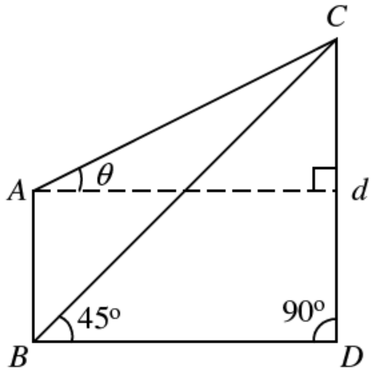
A. -1
B. 0
C. 1
D. 2
E. 3

82
45. $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} =$

A. 2
B. $4 \sin\theta \cos\theta$
C. $\dfrac{2\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$
D. $\dfrac{4\sin\theta\cos\theta}{\sin^2\theta - \cos^2\theta}$

E. $\dfrac{2}{\sin^2\theta - \cos^2\theta}$

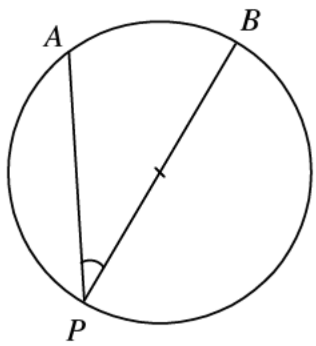
82
46.



AB and CD are two buildings of heights h and d respectively. The angles of elevation of C from A and B are respectively θ and 45°. $d =$

A. $h(1 - \tan\theta)$
B. $h(1 + \tan\theta)$
C. $h \tan\theta$
D. $\dfrac{h}{1+\tan\theta}$
E. $\dfrac{h}{1-\tan\theta}$

82
47.



In the figure, BP is a diameter of the circle. The minor arc AB and the radius are of equal length. $\angle APB =$

A. $\dfrac{1}{2}$ rad
B. 1 rad
C. $\dfrac{\pi}{6}$ rad
D. $\dfrac{\pi}{4}$ rad
E. $\dfrac{\pi}{3}$ rad

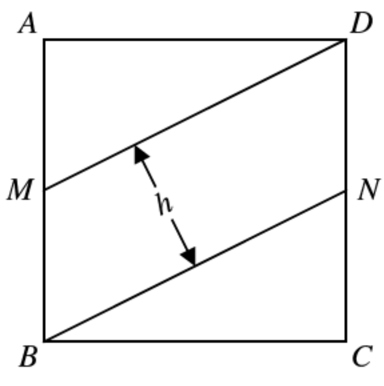
82
48. How many roots has the equation
$\sin\theta + \sin^2\theta = \cos^2\theta$
where $0^\circ \le \theta \le 360^\circ$?

A. 0
B. 1
C. 2
D. 3
E. 4

82
49. If $0 \le x \le \pi$ and $\sin x \le \cos x$, what is the range of x?

A. $0 \le x \le \dfrac{\pi}{4}$
B. $0 \le x \le \dfrac{\pi}{2}$
C. $\dfrac{\pi}{4} \le x \le \dfrac{\pi}{2}$
D. $\dfrac{\pi}{4} \le x \le \pi$
E. $\dfrac{\pi}{2} \le x \le \pi$

82
50.



In the figure, $ABCD$ is a square of side $2a$. M and N are the mid-points of AB and CD respectively. h is the height of the parallelogram $MBND$. $h =$

A. $\dfrac{1}{2}a$

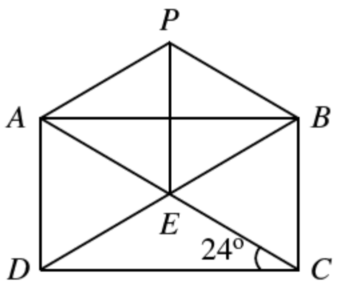
B. $\dfrac{2}{\sqrt{5}}a$

C. $\dfrac{\sqrt{5}}{2}a$

D. $\dfrac{2}{\sqrt{3}}a$

E. $\dfrac{\sqrt{2}}{4}a$

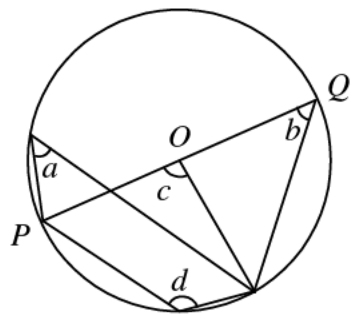
82
51.



In the figure, $ABCD$ is a rectangle. AC and BC intersect at K. PAK is an equilateral triangle. $\angle PBK =$

A. 48°
B. 50°
C. 52°
D. 54°
E. 60°

82
52.

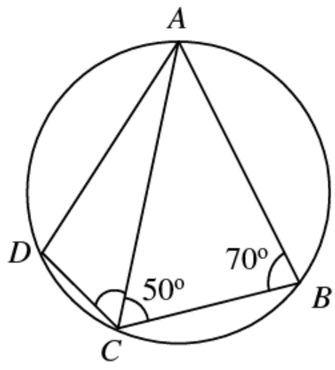


In the figure, O is the centre of the circle. PQ is a diameter. Which of the following is/are true?

I. $a = b$
II. $c = 2a$
III. $c + d = 180°$

A. I only
B. I and II only
C. I and III only
D. II and III only
E. I, II and III

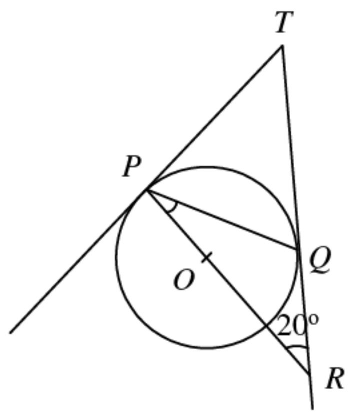
82
53.



in the figure, the length of the minor arc CD is half the length of the minor arc BC. $\angle ACD =$

A. 30°
B. 35°
C. 40°
D. 45°
E. 50°

82
54.



In the figure, TP and TQ touch the circle at P and Q respectively. R is the point on TQ produced such that PR passes through the centre O of the circle. $\angle QPR =$

A. 55°
B. 40°
C. 35°
D. 30°
E. 20°