香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

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HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1982

MATHEMATICS (SYLL 1)

MARKING SCHEME

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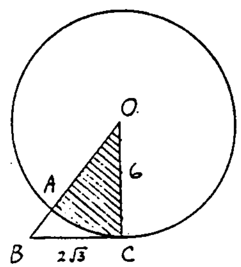
SOLUTION STEPS	MARKS	NOTES
(5 marks)		
$\dfrac{2+i}{1-3i}$		ALTERNATIVELY,
		Let $a + bi = \dfrac{2+i}{1-3i}$
$= \dfrac{(2+i)(1+3i)}{(1-3i)(1+3i)}$	2M	$(a+bi)(1-3i) = 2+i$
		$(a+3b)+(b-3a)i = 2+i$ 1A
		$a+3b = 2$ 1M
		$b-3a = 1$ 1M
$= \dfrac{-1+7i}{}$	1A	
10	1A	$a = -\dfrac{1}{10}$ 1A
$= -\dfrac{1}{10} + \dfrac{7}{10}i$	1A	$b = \dfrac{7}{10}$ 1A
(5 marks)		ALTERNATIVELY,
$4^{x-y} = 4$		$\dfrac{4^{x+y}}{4^{x-y}} = 4$ 1M
$4^{x+y} = 16$		
$x - y = 1$	1A	$4^{2y} = 4$ 1A
$x + y = 2$	1A	$2y = 1$ 1A
Solving,	1M	$y = \dfrac{1}{2}$ 1A
$x = 1\dfrac{1}{2}$	1A	$x = 1\dfrac{1}{2}$ 1A
$y = \dfrac{1}{2}$	1A	
(5 marks)		
$2x^2 - x < 36$		
$2x^2 - x - 36 < 0$	1A	
$(2x-9)(x+4) < 0$	2A	For factorization
$-4 < x < 4\dfrac{1}{2}$	2A	Accept $\begin{cases} -4 < x \\ x < 4\frac{1}{2} \end{cases}$
		"$-4 < x$ and $x < 4\frac{1}{2}$"
(6 marks)		



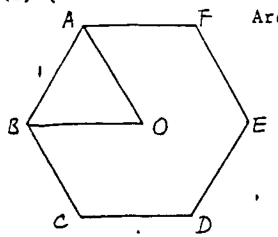
SOLUTION STEPS	MARKS	NOTES
$\angle C = 90°$		
$\tan \angle BOC = \dfrac{2\sqrt{3}}{6}$	2M	
$= \dfrac{1}{\sqrt{3}}$		
$\angle BOC = 30°$ or $\dfrac{\pi}{6}$	1A	
Area of sector $= \pi(6)^2 \times \dfrac{30}{360}$ or $\dfrac{1}{2}(6)^2 \dfrac{\pi}{6}$	1M+1A	
$= 3\pi$	1A	

SOLUTION STEPS	MARKS	NOTES
5. (6 marks)		
$2\sin^2\theta + 5\sin\theta - 3 = 0$		
$(2\sin\theta - 1)(\sin\theta + 3) = 0$	1M+1A	1M for attempting to factorize
$\sin\theta = \frac{1}{2}$ or $\sin\theta = -3$	1A	For $\sin\theta = \frac{1}{2}$
Rejecting $\sin\theta = -3$,	1A	If a cand. writes $\sin\theta = \frac{1}{2}$ only, award 2 marks.
$\theta = 30°$ or $150°$	1A+1A	Accept $\theta = 30°, 150°$. or $\theta = 30°$ and $150°$. General solution, no mark
If more than 2 answers given, deduct 1 mark for each wrong answer from the marks obtained in the answer only.		
6. (6 marks)		
(a) (1, 3), (3, 1), (2, 2)		
Probability $= \frac{3}{36}$	1A	For numerator
	1A	For denominator
$= \frac{1}{12}$ ≈ 0.083		
(b) (1, 1), (1, 2), (2, 1)		
Probability $= \frac{3}{36}$	1A	For numerator
	1A	For denominator
$= \frac{1}{12}$		
(c) Probability $= 1 - \frac{1}{12} - \frac{1}{12}$	1M	
$= \frac{5}{6}$	1A	
ALTERNATIVELY, (1, 4), (1, 5), (1, 6), ... , (6, 6)		
Probability $= \frac{30}{36}$	1A	For numerator
	1A	For denominator
$= \frac{5}{6}$		
If answer not simplified, deduct 1 mark for the whole question.		
7. (6 marks)		
(a) $x = 360 \times \frac{2}{12}$	1M	ALTERNATIVELY, put $x = 2k$, $y = 7k$, $z = 3k$, $2k + 7k + 3k = 360$
$= 60$	1A	
$y = 210$	1A	
$z = 90$	1A	
(b) Total number $= 240 \times \frac{12}{2}$	1M	ALTERNATIVELY, No. in Kowloon = 840
$= 1440$	1A	No. in N.T. = 360
		Total no. $= 840 + 360 + 240$ $= 1440$

SOLUTION STEPS	MARKS	NOTES
8. (a) (10 marks)		
$AC^2 = x^2 + x^2$ $= 2x^2$	1M	For Pythagoras' Theorem
$AB^2 = AC^2 + BC^2$ $= 2x^2 + y^2$	1M	For Pythagoras' Theorem
$2x^2 + y^2 = 9^2$	1A	
$8x + 4y + 9 = 69$	1A	
i.e. $2x + y = 15$		
Sub. $y = 15 - 2x$ in $2x^2 + y^2 = 9^2$,	1M	
$2x^2 + (15 - 2x)^2 = 81$	1A	
$6x^2 - 60x + 144 = 0$		
$x^2 - 10x + 24 = 0$	1A	
$(x - 4)(x - 6) = 0$	1A	
$x = 4$ or 6		
$x = 4$, $y = 7$.	1A	For both values
$x = 6$, $y = 3$.	1A	For both values
ALTERNATIVELY,		
Sub. $x = \frac{15 - y}{2}$ in $2x^2 + y^2 = 9^2$,	1M	
$2\left[\frac{15 - y}{2}\right]^2 + y^2 = 9^2$	1A	
$y^2 - 10y + 21 = 0$	1A	
$(y - 3)(y - 7) = 0$	1A	
$y = 3$ or 7		
$y = 7$, $x = 4$.	1A	For both values
$y = 3$, $x = 6$.	1A	For both values
(b) (2 marks)		
$\cos\theta = \frac{BC}{AB}$ or $\tan\theta = \frac{AC}{BC}$ or $\sin\theta = \frac{AC}{AB}$	1M	
$\cos\theta = \frac{7}{9}$ or $\tan\theta = \frac{\sqrt{32}}{7}$ or $\sin\theta = \frac{\sqrt{32}}{9}$		
$\theta \simeq 39°$	1A	

SOLUTION STEPS | MARKS | NOTES

9. (a) (6 marks)

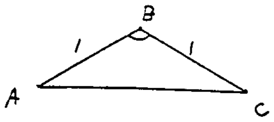


Area of $\triangle OAB = \frac{1}{2} \times 1 \times \sin 60°$ — 1A

$= \frac{\sqrt{3}}{4}$

Area of hexagon $= 6 \times \frac{\sqrt{3}}{4}$ — 1M

$= \frac{3\sqrt{3}}{2}$ — 1A



$\angle ABC = 120°$

$\frac{1}{2} AC = \cos 30°$ — 1M+1A

$= \frac{\sqrt{3}}{2}$

$AC = \sqrt{3}$ — 1A

NOTES: ALTERNATIVELY,

$AC^2 = 1^2 + 1^2 - 2 \times 1 \times 1 \times \cos 120°$ 1M+1A

or $\frac{AC}{\sin 120°} = \frac{1}{\sin 30°}$ 1M+1A

$AC = \sqrt{3}$ 1A

ALTERNATIVELY,

Area of $\triangle ABC = \frac{1}{2} \times 1 \times 1 \times \sin 120°$ — 1A

$= \frac{\sqrt{3}}{4}$

Area of hexagon $=$ 2 x Area of $\triangle ABC$ + Area of ACDF — 1M

$= 2 \times \frac{\sqrt{3}}{4} + 1 \times \sqrt{3}$

$= \frac{3\sqrt{3}}{2}$ — 1A

(b) (i) $\angle BAC = 30°$, $\angle APB = 120°$

$\angle BPQ = 60°$

Similarly, $\angle BQP = 60°$

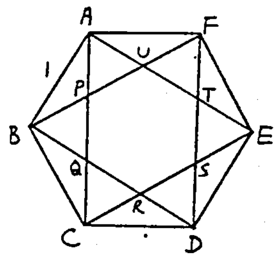
$\triangle BPQ$ is equilateral — 1

PQ = BP = BQ

But BP = AP, BQ = CQ

$PQ = \frac{1}{3} AC$ — 1M

$= \frac{\sqrt{3}}{3}$ — 1A



(ii) $\because$ ABCDEF and PQRSTU are similar

$\frac{\text{Area of PQRSTU}}{\text{Area of ABCDEF}} = \left(\frac{PQ}{AB}\right)^2$ — 2M

Area of PQRSTU $= \frac{3\sqrt{3}}{2}\left(\frac{\sqrt{3}}{3}\right)^2$

$= \frac{\sqrt{3}}{2}$ — 1A

ALTERNATIVELY,

Area of $\triangle OPQ = \frac{1}{2} \times \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{3} \sin 60°$ — 1A

$= \frac{\sqrt{3}}{12}$

Area of PQRSTU $= 6 \times \frac{\sqrt{3}}{12}$ — 1M

$= \frac{\sqrt{3}}{2}$ — 1A

NOTES: ALTERNATIVELY,

Area of $\triangle PQR$

$= \frac{1}{2} \times \frac{\sqrt{3}}{3} \times \frac{\sqrt{3}}{3} \times \sin 120°$ 1.

Area of PQRSTU

$= 2 \times \triangle PQR$ + area of PRSU 1

$= \frac{\sqrt{3}}{2}$ 1

SOLUTION STEPS | MARKS | NOTES

(a) (6 marks)

(i) Multiples of 3 :

Last term = 999

No. of terms $= \frac{1}{3} \times 999$

$= 333$ — 1A

Sum, $S_1 = \frac{n}{2}[a + \ell]$ or $\frac{n}{2}[2a + (n-1)d]$ — 1M

$= \frac{333}{2}[2(3) + (333 - 1)3]$ or — 1M

$\frac{333}{2}[3 + 999]$

$= 166\ 833$ — 1A

(ii) Multiples of 4 :

Last term = 1000

No. of terms $= \frac{1}{4} \times 1000$

$= 250$ — 1A

Sum, $S_2 = \frac{250}{2}[4 + 1000]$

$= 125\ 500$ — 1A

(b) (6 marks)

Multiples of 12 :

Last term = 83 x 12

No. of terms = 83 — 1A

Sum, $S_3 = \frac{83}{2}[12 + 83 \times 12]$

$= 41\ 832$ — 1A

$S_4 = 1 + 2 + 3 + \ldots + 1000$

$= \frac{1000}{2}[1 + 1000]$

$= 500\ 500$ — 1A

Required Sum $= S_4 - S_1 - S_2 + S_3$ — 2M

NOTES: If a cand. writes Required Sum = $S_4 - S_1 - S_2$, award 1M.

$= 500\ 500 - 166\ 833 - 125\ 500 + 41\ 832$

$= 249\ 999$ — 1A

SOLUTION STEPS	MARKS	NOTES
11. (a) (4 marks)		
Let α, β be the roots of $x^2 - 10x + k = 0$		
(i) $\alpha + \beta = 10$	1A	or sum of roots = 10
$OA + OB = 10$	1A	
(ii) $\alpha\beta = k$	1A	or product of roots = k
$OA \times OB = k$	1A	
(b) (4 marks)		
(i) $OM + ON = \frac{1}{2}(OA + OB)$	1M	
$= 5$	1A	
(ii) $OM \times ON = \frac{1}{2}OA \times \frac{1}{2}OB$	1M	
$= \frac{k}{4}$	1A	
(c) (4 marks)		
(i) From (b), $-p = OM + ON$		
$p = -5$	1M	
$r = OM \times ON$		
$= \frac{k}{4}$	1M	
(ii) $OM = 2$, $ON = 3$		
$\frac{k}{4} = (2)(3)$	1M	
$k = 24$	1A	
ALTERNATIVELY,		
$OM = 2$		
$M = (2, 0)$		
Sub. in $y = x^2 - 5x + \frac{k}{4}$	1M	
$0 = 4 - 10 + \frac{k}{4}$		
$k = 24$	1A	

SOLUTION STEPS	MARKS	NOTES
12. (a) (7 marks)		
$Y = k_1 x$ or $Z = k_2 x^2$	1M	
$P = Y + Z$		
$= k_1 x + k_2 x^2$	1M	
$80\ 000 = 20k_1 + 20^2 k_2$	1A	(for either equation)
$87\ 500 = 35k_1 + 35^2 k_2$		
Solving,	1M	
$k_1 = 6000$	1A	
$k_2 = -100$	1A	
$P = 6000x - 100x^2$		
When $x = 15$,		
$P = 6000(15) - 100(15)^2$		
$= 67\ 500$	1A	
(b) (3 marks)		
$P = 6000x - 100x^2$		
$= -100(x^2 - 60x)$		
$= -100[x^2 - 60x + 30^2 - 30^2]$	2M	For the method of completing square
$= -100[(x - 30)^2 - 900]$		
$= 90\ 000 - 100(x - 30)^2$		
$a = 90\ 000$, $b = 100$, $c = 30$	1A	All three answers must be correct.
(c) (2 marks)		
When $x = 30$, P is a maximum.	1M+1A	

$P = 300^2 - 300^2 + 2(300)(10x) - (10x)^2$

SOLUTION STEPS	MARKS	NOTES
13. (a) (3 marks)		
Centre = (0, 7)	1A	$r^2 = g^2 + f^2 - c$ 1M
Radius = 3	2A 1M+1A	
(b) (3 marks)		
Slope of L $= \frac{4}{3}$		
slope of L' $= -\frac{3}{4}$	1A	
Equation of L' : $\frac{y-7}{x-0} = -\frac{3}{4}$	1M	
$3x + 4y - 28 = 0$	1A	
(c) (3 marks)		
Solving L and L',	1M	
Point of intersection = (4, 4)	1A+1A	1A for each component
(d) (3 marks)		
Distance between (0, 7) and (4, 4)		
$= \sqrt{16 + 9}$		
$= 5$	1A	
Shortest distance between C and L		
$= 5 - 3$	1M	
$= 2$	1A	
ALTERNATIVELY,		
Distance from (0, 7) to L $= \left\lvert \frac{4(0) - 3(7) - 4}{\sqrt{3^2 + 4^2}} \right\rvert$		
$= 5$	1A	
Shortest distance between C and L		
$= 5 - 3$	1M	
$= 2$	1A	
ALTERNATIVELY,		
Solving L' and C,	1M	
Points of intersection are $\left(\frac{12}{5}, \frac{26}{5}\right)$ and $\left(-\frac{12}{5}, \frac{44}{5}\right)$	1A	
Distance between $\left(\frac{12}{5}, \frac{26}{5}\right)$ and (4, 4)		
$= 2$	1A	

SOLUTION STEPS	MARKS	NOTES
14. (a) (4 marks)		
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1M	Accept column or row vector notation
$= (3\vec{i} + 4\vec{j}) - (\vec{i} - \vec{j})$		
$= 2\vec{i} + 5\vec{j}$	1A	Missing of "$\longrightarrow$" for 3 times or more, deduct 1 mark for poor presentation
$\overrightarrow{CD} = \overrightarrow{OD} - \overrightarrow{OC}$	1M	
$= (9\vec{i} + 5\vec{j}) - (-3\vec{i} - 4\vec{j})$		
$= 12\vec{i} + 9\vec{j}$	1A	
(b) (4 marks)		
$\lvert\overrightarrow{CD}\rvert = \sqrt{12^2 + 9^2}$		
$= 15$	1A	
$\vec{u} = \frac{\overrightarrow{CD}}{\lvert\overrightarrow{CD}\rvert}$	2M	
$= \frac{1}{15}(12\vec{i} + 9\vec{j})$ or $\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$	1A	
(c) (4 marks)		
(i) $\overrightarrow{AB} \cdot \vec{u} = (2\vec{i} + 5\vec{j}) \cdot (\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j})$		
$= 2 \times \frac{4}{5} + 5 \times \frac{3}{5}$	1M	For evaluation of dot product
$= 4\frac{3}{5}$	1A	
(ii) $\lvert\overrightarrow{AE}\rvert = \lvert\overrightarrow{AB}\rvert \cos\angle BAE$	1M	May be omitted
$= \overrightarrow{AB} \cdot \vec{u}$		
$= 4\frac{3}{5}$	1A	
ALTERNATIVELY,		
Equation of CD : $3x - 4y - 7 = 0$		
Equation of BE : $4x + 3y - 24 = 0$	1A	
Solving,		
$E = \left(\frac{117}{25}, \frac{44}{25}\right)$		
$AE = 4\frac{3}{5}$	1A	