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MATHEMATICS (SYLL 1)

數學 (課程一)

MARKING SCHEME

評卷參考

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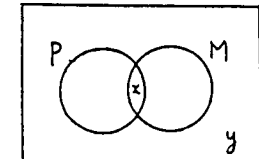
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SOLUTION STEPS	MARKS	NOTES
$1. r_1^3 : r_2^3 = 27 : 64$ $= 3^3 : 4^3$ $r_1 : r_2 = \sqrt[3]{27} : \sqrt[3]{64}$ $= 3 : 4$ $S_1 : S_2 = r_1^2 : r_2^2$ $= 9 : 16$ <p>Amount of paint required = $72 \times \frac{16}{9}$ kg or $x : 72 = 16 : 9$</p> $= 128 \text{ kg}$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>$V = \frac{4\pi r^3}{3}$ No mark</p> <p>for $72 \times \frac{r_2^2}{r_1^2}$</p> <p>5 分</p> <p>7.725</p>
$2. \text{Sum of roots} = 4$ $\text{Product of roots} = (2 + 3i)(2 - 3i)$ $= 13$ <p>The required equation is</p> $x^2 - 4x + 13 = 0$ <p>ALTERNATIVELY,</p> $[x - (2 + 3i)][x - (2 - 3i)] = 0$ $x^2 - 4x + 13 = 0$	<p>1A</p> <p>2A</p> <p>2M</p> <p>2M</p> <p>1A+2A</p>	<p>$k = (2+3i) + (2-3i)$ } 1分</p> <p>$= 4$</p> <p>$k = 4$ } 5分</p> <p>1A for -4</p> <p>2A for 13</p>
$3. \text{Probability} = \frac{2}{40} \times \frac{1}{39} \text{ or } 2 \times \frac{1}{40} \times \frac{1}{39}$ $= \frac{1}{780} \text{ or } 0.00128$ <p>ALTERNATIVELY,</p> <p>(This method is out of syllabus)</p> $\text{Probability} = \frac{1}{40^2}$ $= \frac{1}{1600}$ $= \frac{1}{780} \text{ or } 0.00128$	<p>2M+2A</p> <p>1A</p> <p>2M</p> <p>2A</p> <p>1A</p>	<p>2M for right approach</p> <p>5 分</p> <p>Prob = $\frac{1}{40} + \frac{1}{39}$ No mark</p> <p>Prob = $\frac{1}{40} + \frac{1}{39}$ 2分</p> <p>$\frac{1}{40} + \frac{1}{39} + \frac{1}{40}$ 2分</p> <p>Prob = $1 - P(\text{A chosen, B not chosen})$</p> <p>$- P(\text{B chosen, A not chosen})$</p> <p>$- P(\text{both A and B not chosen})$ 2分</p>

SOLUTION STEPS	MARKS	NOTES
4. $\cos(200^\circ + \theta) = \sin 120^\circ$ $= \frac{\sqrt{3}}{2}$ or 0.8660 _____ $= \cos 30^\circ$ _____ or $\cos 330^\circ$ _____	1A 1A 2A	May be omitted
$200 + \theta = 330^\circ$ _____ $\theta = 130^\circ$ _____	1A 2A	Accept $\theta = 130$
For each wrong answer, deduct 1 mark from * . (毋扣总分)		
5. $4^x = 10 - 4^{x+1}$ $4^x = 10 - 4^x(4)$ _____ $5(4^x) = 10$ _____ $4^x = 2$ _____ $x = \frac{1}{2}$ _____	2A 1A 1A 2A	For $4^{x+1} = 4^x(4)$ 又 $x = \frac{1}{2}$ 1A 在 checking 1A
6. (a) $43 \approx 44$ _____ (b) 20 or 21 students failed, _____ . 80 or 79 students passed. _____ (c) $Q_1 = 43$ $Q_3 = 70$ to 71 Interquartile range = $Q_3 - Q_1$ _____ $= 27$ to 28 _____	2A 1A 1A 1M 1A	May be omitted Awarded only if Q_1 and Q_3 are correct.

SOLUTION STEPS	MARKS	NOTES
7. Let U be the set of 42 students in the class, P be the set of students who have been to the Ocean Park, M be the set of students who have been to the Space Museum.		
 <p>Suppose $n(P \cap M) = x$ $n[U - (P \cup M)] = y$</p> <p>$n(P) = 28$ $n(M) = 34$</p>		
(a) $(28 - x) + 34 + y = 42$ _____ $x = 62 - 42 + y$ $= 20 + y$ For minimum x , $y = 0$. _____ $x_{\min} = 20$ _____	1A 1A 1A	If a cand. writes $n(P \cap M)$ $= n(P) + n(M) - n(P \cup M)$ $= 28 + 34 - 42$ $= 20$ $42 - 20 - 10 = 12$ $\therefore x_{\min} = 20$ 3分
(b) $y = 7$, $n(P \cup M) = 42 - 7$ $= 35$ $n(P \cap M) = n(P) + n(M) - n(P \cup M)$ _____ $= 28 + 34 - 35$ _____ $= 27$ _____	1A 1M 1A	For substitution
		$28 - x = 34 - x$ $x = 27$ 3分 又有 Ken Nguyen. 5分

SOLUTION STEPS	MARKS	NOTES
8. Constraints: $x, y \in \mathbb{N}$ $x + 2y \geq 48$ _____ $10x + 15y \leq 450$ _____ $x \geq y$ _____	1A 1A 1A	Deduct 1 mark for omitting 1 or more equality signs only if a cand. scored all these 3 marks. 8. 3 marks
Graphs of the lines: $x + 2y = 48$ _____ $10x + 15y = 450$ _____ $x = y$ _____	1A 1A 1A	Labelling of graphs may be omitted
Indicating the correct region. _____	1A	
Testing optimization: Profit Line: $300x + 400y = c$ _____ Graph of the line. _____	1A 2M	very simple. 2 marks
ALTERNATIVELY, Possible solutions: (16, 16), (18, 18), (36, 6) $f(x, y) = 300x + 400y$ _____ $f(16, 16) = 11200$ _____ $f(18, 18) = 13200$ _____ $f(36, 6) = 13200$ _____	1A 2M	
Number of single rooms = 36 _____ Number of double rooms = 6 _____	1A 1A	一定可以... 2 marks

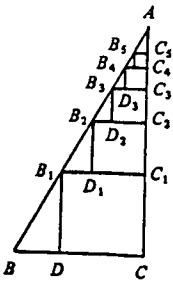
SOLUTION STEPS	MARKS	NOTES
9. Normal rate of production = $\frac{400}{x}$ (per day) _____ Faster rate of production = $\frac{400}{x} + 20$ (per day) _____ At the faster rate, no. of days taken = $\frac{400}{\frac{400}{x} + 20}$ $x - \frac{400}{\frac{400}{x} + 20} = 10$ _____ $x - \frac{20x}{20 + x} = 10$ _____	1A 1A 4A	Incorrect equation with no explanation, no mark.
ALTERNATIVELY, Normal rate of production = $\frac{400}{x}$ (per day) _____ In 10 days, $10\left(\frac{400}{x}\right)$ radios would be produced _____ This is balanced by producing 20 more radios each day for $(x - 10)$ days. $10\left(\frac{400}{x}\right) = 20(x - 10)$ _____ $2x^2 - 20x - 400 = 0$ _____	1A 1A 4A	
$x^2 - 10x - 200 = 0$ or $kx^2 - 10kx - 200k = 0$ _____ $(x - 20)(x + 10) = 0$ _____ $x = 20$ or -10 _____ Rejecting $x = -10$, $\therefore x = 20$ _____	3A 1A 1A 1M	If a candidate writes $(x - 20)(x + 10) = 0$ 1A $x = 20$ 2 marks
ALTERNATIVELY, Let y be the normal rate of production $xy = 400$ _____ $(x - 10)(y + 20) = 400$ _____	2A 2A	
Successfully eliminating one unknown, $(x - 10)\left(\frac{400}{x} + 20\right) = 400$ or $\left(\frac{400}{y} - 10\right)(y + 20) = 400$ _____ $x^2 - 10x - 200 = 0$ or $y^2 + 20y - 800 = 0$ _____ or $(y - 20)(y + 40) = 0$ _____ $y = 20$ or -40 (rejected) _____ (same as above) $y = 20$ _____ $x = 20$ _____	2A 3A 1A 1M 1A	For rejecting the -ve root.

SOLUTION STEPS

MARKS

NOTES

10. (a)



$$\triangle AB_1C_1 \sim \triangle ABC \text{ or } \frac{AC_1}{B_1C_1} = \frac{AC}{BC}$$

$$\therefore \frac{2a - b}{b} = \frac{2a}{a}$$

$$b = \frac{2}{3}a \quad 0.667$$

ALTERNATIVELY,

$$\triangle AB_1C_1 \sim \triangle B_1BD \text{ or } \frac{AC_1}{B_1C_1} = \frac{B_1D}{BD}$$

$$\therefore \frac{2a - b}{b} = \frac{b}{a - b}$$

$$2a^2 - 3ab = 0$$

$$b = \frac{2}{3}a \quad (a \neq 0)$$

ALTERNATIVELY,

$$\triangle B_1BD \sim \triangle ABC \text{ or } \frac{B_1D}{BD} = \frac{AC}{BC}$$

$$\therefore \frac{b}{a - b} = \frac{2a}{a}$$

$$b = \frac{2}{3}a$$

(b) (i) Similarly, $B_2C_2 = \frac{2}{3}b$

(ii) $B_2C_2 = \frac{2}{3}b$
 $= \frac{4}{9}a \quad 0.444a = 0.445a$

(c) (i) $B_5C_5 = a\left(\frac{2}{3}\right)^5 \text{ or } \frac{32}{243}a$
 $0.131a = 0.132a$

(ii) Mentioning that the areas of the squares form a G.P.

Area of first square = b^2 or $\frac{4}{9}a^2$

Common ratio = $\left(\frac{2}{3}\right)^2$

Sum of areas = $\frac{A}{1 - R}$

$$= \frac{b^2}{1 - \frac{4}{9}}$$

$$= \frac{4}{5}a^2 \quad 0.800a^2 = 0.801a^2$$

1M

1A

1A

1M

1A

1A

1M

1A

1A

1A

1A

2A

1

1A

1A

1M

1A

May be omitted

This line is suff. to indicate that the cand. knows the areas of the squares form a G.P.

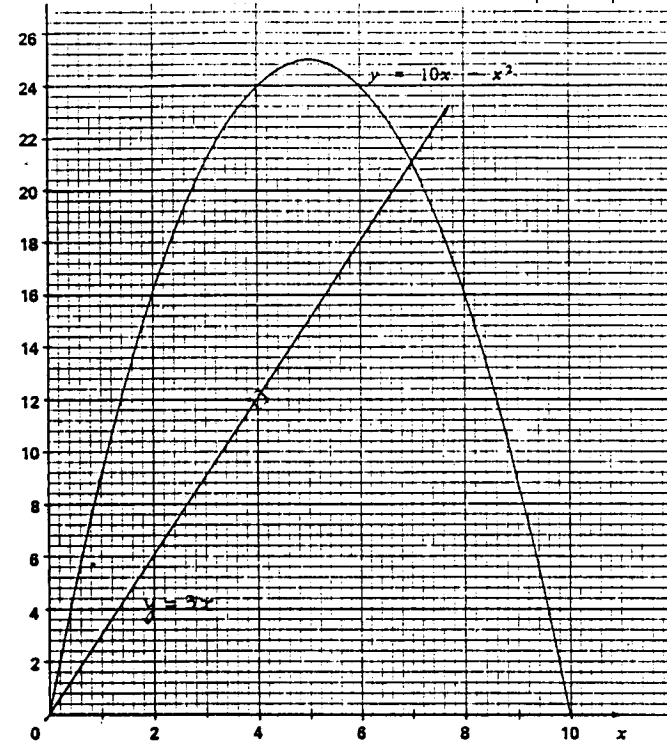
0.7779a^2 = 0.778a^2

SOLUTION STEPS

MARKS

NOTES

11.



(a) Perimeter = 20 cm.

Half of the perimeter = 10 cm

Length of the other side = $(10 - x)$ cm

Area, $y = x(10 - x)$

$$= 10x - x^2$$

1A

If omitted, deduct 1 mark for pp.

1A

(b) (i) $y = 22.4$ or 22.5

(ii) $x = 1.4$ or 8.6

(iii) 25 cm^2

(No unit, do not deduct mark. Wrong unit, -1 pp.)

2A

Accept $y = 22.5$

2A+1A

2A for 1st correct answer.

2A

If a cand. writes $x = 5$ only, award 1 mark.

(iv) $y = 3x$

Graph of $y = 3x$

$x = 7$

1A

1A

1A

Labelling not necessary

Awarded only if the line $y = 3x$ is shown.

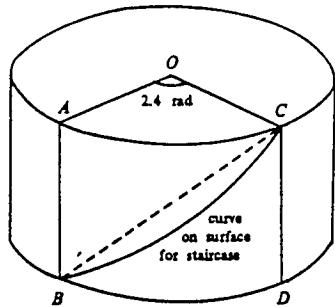
Accept ' $x = 7$ or $x = 0$ '

SOLUTION STEPS

MARKS

NOTES

2.



(a) $AC = 2 \times 10 \times \sin 1.2$ _____ 2M

ALTERNATIVELY,

$AC^2 = 10^2 + 10^2 - 2(10)(10) \cos 2.4$ _____ 2M

For cosine law

$BC^2 = AC^2 + AB^2$ _____ 1M

For Pythagoras' Theorem

$= 18.64^2 + 10^2$

$BC = 21.15$ _____ 1A

or any number which rounds off to 21.2

$= 21.2 \text{ (m)}$ _____ 1A

(b) $\widehat{AC} = r\theta$ _____ 1M

For correct formula

$= 10 \times 2.4$
 $= 24$

Area $= 24 \times 10$ _____ 1M

$= 240 \text{ (m}^2\text{)}$ _____ 1A

ALTERNATIVELY,

Area of curved surface
 $= 2\pi rh \times \frac{\theta}{2\pi}$ or $2\pi rh \times \frac{\theta}{360}$ _____ 1M

For correct formula

$= 2\pi \times 10 \times 10 \times \frac{2.4}{2\pi}$ or $2\pi \times 10 \times 10 \times \frac{137.51}{360}$ _____ 1A

For substitution

$= 240 \text{ (m}^2\text{)}$ _____ 1A

(c) Length of curve _____ 2M

$= \sqrt{(\text{arc } AC)^2 + AB^2}$ _____

Pythagoras Th.

$= \sqrt{24^2 + 10^2}$ _____ 1A

$= 26 \text{ (m)}$ _____ 1A

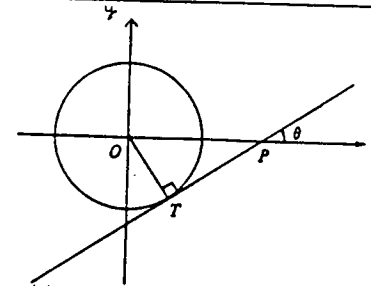
26 单位, 全数加分.

SOLUTION STEPS

MARKS

NOTES

13.



(a) $x^2 + y^2 = 15^2$ or $x^2 + y^2 = 225$ _____ 1A

(b) $\frac{OT}{OP} = \sin \theta$ _____ 1M

$OP = \frac{OT}{\sin \theta}$

$= \frac{15}{\frac{3}{5}}$

$= 25$ _____ 2A

(c) $P = (25, 0)$ _____

TP: $\frac{y-0}{x-25} = \frac{3}{4}$ _____ 1M

For point-slope form

$3x - 4y - 75 = 0$ _____ 1A

ALTERNATIVELY,

(b) and (c) _____

OT: $\frac{y}{x} = -\frac{4}{3}$ _____ 1A

$4x + 3y = 0$

Solving with $x^2 + y^2 = 15^2$, _____ 1M

$x^2 + \frac{16}{9}x^2 = 225$

$x = 9$ or -9 (rejected)

$y = -12$

$\therefore T = (9, -12)$

TP: $\frac{y+12}{x-9} = \frac{3}{4}$ _____ 1M

For point-slope form

$3x - 4y - 75 = 0$ _____ 1A

When $y = 0$, $x = 25$

$\therefore OP = 25$ _____ 1A

SOLUTION STEPS

MARKS

NOTES

(d) $y = \frac{3}{4}x$ or $3x - 4y = 0$

1A

(e) Let $C = (a, b)$,

$b = \frac{3}{4}a$ _____ (1)

1A

$(25 - a)^2 + (0 - b)^2 = 15^2$ _____ (2)

1A

ALTERNATIVELY,

Let $C = (a, b)$

$b = \frac{3}{4}a$ _____ (1)

1A

CP : $\frac{y - 0}{x - 25} = -\frac{4}{3}$

$4x + 3y - 100 = 0$

$4a + 3b - 100 = 0$ _____ (2)

1A

Solving (1) and (2),

1M

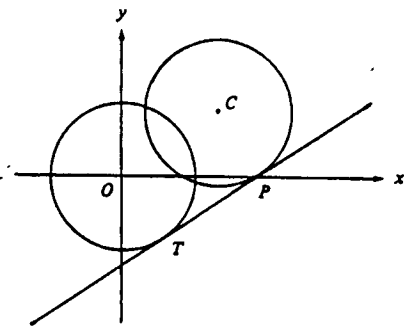
$a = 16, b = 12.$

1A

equation of the circle :

$(x - 16)^2 + (y - 12)^2 = 15^2$ _____

1A



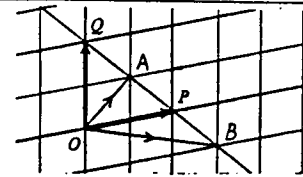
For both answers correct

SOLUTION STEPS

MARKS

NOTES

14. (a)



Vector \vec{OA} _____

1A

Vector \vec{OB} _____

1A

(b) $\vec{OT} = \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OB}$ _____
 $= \frac{1}{2}\vec{OA} + \frac{1}{2}\vec{OB}$ _____
 $= \frac{1}{2}\vec{OA} + \frac{1}{2}(\vec{OA} - \vec{OP})$ _____
 $= \frac{3}{2}\vec{OA} - \frac{1}{2}\vec{OP}$ _____
 $= \frac{3}{2}\vec{a} - \frac{1}{2}\vec{b}$ _____

1M

For expressing OT as a sum of 2 vectors.

2A

ALTERNATIVELY,

Using Figure 6,

Correct position of T (may be omitted)

$\vec{OT} = -\frac{1}{2}\vec{b} + \frac{3}{2}\vec{a}$ _____

1A

2A

(c) (i) $\vec{OR} = r\vec{p} + (1 - r)\vec{q}$ _____
 $= r(6\vec{i} + 2\vec{j}) + (1 - r)(5\vec{j})$ _____
 $= 6r\vec{i} + (5 - 3r)\vec{j}$ _____

1A

$\vec{PQ} = \vec{OQ} - \vec{OP}$ _____
 $= 5\vec{j} - (6\vec{i} + 2\vec{j})$ _____
 $= -6\vec{i} + 3\vec{j}$ _____

1M

1A

$\vec{PQ} \cdot \vec{OR} = (-6\vec{i} + 3\vec{j}) \cdot [6r\vec{i} + (5 - 3r)\vec{j}]$ _____
 $= -36r + 3(5 - 3r)$ _____
 $= 15 - 45r$ _____

1M

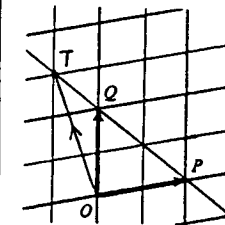
1A

For evaluation of dot product

(ii) If $\vec{PQ} \perp \vec{OR}$,
 then $\vec{PQ} \cdot \vec{OR} = 0$ _____
 $15 - 45r = 0$ _____
 $r = \frac{1}{3}$ _____

1M

1A



Column or row vector notation accepted.