8 Rate, Ratio and Variation

8A Rate and Ratio

8A.1 HKCEE MA 1980(1) I 8

A factory employs 10 skilled, 20 semi skilled, and 30 unskilled workers. The daily wages per worker of the three kinds are in the ratio 4 : 3 : 2. If a skilled worker is paid \$120 a day, find the mean daily wage for the 60 workers.

8A.2 HKCEE MA 1981(1/2/3) I 9

Normally, a factory produces 400 radios in x days. If the factory were to produce 20 more radios each day, then it would take 10 days less to produce 400 radios. Calculate x.

8A.3 HKCEE MA 1983(A/B) – I 4

If $a:b=3:4$ and $a:c=2:5$, find

(a) $a:b:c$,

(b) the value of $\dfrac{ac}{a^2+b^2}$.

8A.4 HKCEE MA 1989 – I – 1

The monthly income of a man is increased from \$8000 to \$9000.

(a) Find the percentage increase.

(b) After the increase, the ratio of his savings to his expenditure is 3 : 7 for each month. How much does he save each month?

8A.5 HKCEE MA 1989 – I 5

(a) Solve the simultaneous equations $\begin{cases} x+2y=5 \\ 5x-4y=4 \end{cases}$.

(b) Given that $\begin{cases} \dfrac{a}{c}+\dfrac{2b}{c}=5 \\ \dfrac{5a}{c}-\dfrac{4b}{c}=4 \end{cases}$, where a, b and c are non zero numbers, using the result of (a), find $a:b:c$.

8A.6 HKCEE MA 1991 I – 3

(Also as 2C.2.)

A man buys some British pounds (£) with 150 000 Hong Kong dollars (HK\$) at the rate £1 = HK\$15.00 and puts it on fixed deposit for 30 days. The rate of interest is 14.60% per annum.

(a) How much does he buy in British pounds?

(b) Find the amount in British pounds at the end of 30 days.
(Suppose 1 year = 365 days and the interest is calculated at simple interest.)

(c) If he sells the amount in (b) at the rate of £1 = HK\$14.50, how much does he get in Hong Kong dollars?

8A.7 HKCEE MA 1991 I – 4

Let $2a=3b=5c$.

(a) Find the ratio $a:b:c$.

(b) If $a \quad b+c=55$, find c.

8A.8 HKCEE MA 1995 – I 5

It is given that $x:(y+1)=4:5$.

(a) Express x in terms of y.

(b) If $2x+9y=97$, find the values of x and y.

8A.9 HKCEE MA 2005 – I 5

The ratio of the number of marbles owned by Susan to the number of marbles owned by Teresa is 5 : 2. Susan has n marbles. If Susan gives 18 of her own marbles to Teresa, both of them will have the same number of marbles. Find n.

8A.10 HKCEE MA 2011 – I 6

In a summer camp, the ratio of the number of boys to the number of girls is 7 : 6. If 17 boys and 4 girls leave the summer camp, then the number of boys and the number of girls are the same. Find the original number of girls in the summer camp.

8A.11 HKDSE MA PP I – 5

The ratio of the capacity of a bottle to that of a cup is 4 : 3. The total capacity of 7 bottles and 9 cups is 11 litres. Find the capacity of a bottle.

8A.12 HKDSE MA 2018 I 9

A car travels from city P to city Q at an average speed of 72 km/h and then the car travels from city Q to city R at an average speed of 90 km/h. It is given that the car travels 210 km in 161 minutes for the whole journey. How long does the car take to travel from city P to city Q?

8A.13 HKDSE MA 2019 – I 7

In a playground, the ratio of the number of adults to the number of children is 13 : 6. If 9 adults and 24 children enter the playground, then the ratio of the number of adults to the number of children is 8 : 7. Find the original number of adults in the playground.

8A.14 HKDSE MA 2020 I 4

Let a, b and c be non-zero numbers such that $\dfrac{a}{b}=\dfrac{6}{7}$ and $3a=4c$. Find $\dfrac{b+2c}{a+2b}$.

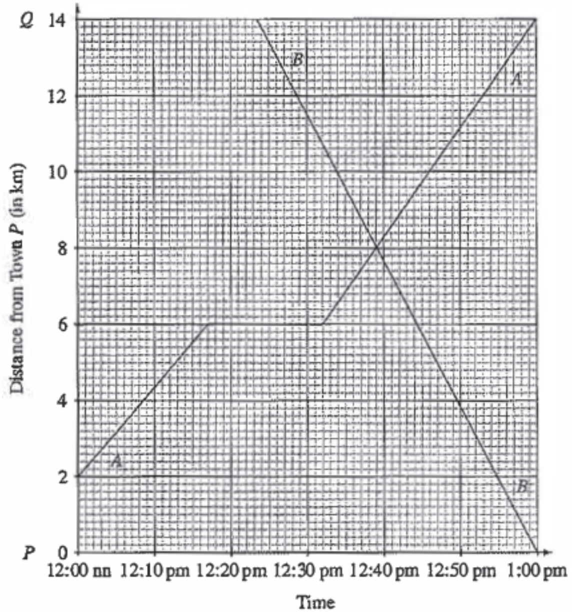
8B Travel graphs

8B.1 HKCEE MA 1984(B) – I – 3

The figure shows the travel graphs of two cyclists A and B travelling on the same road between towns P and Q, 14 km apart.

(a) For how many minutes does A rest during the journey?

(b) How many km away from P do A and B meet?



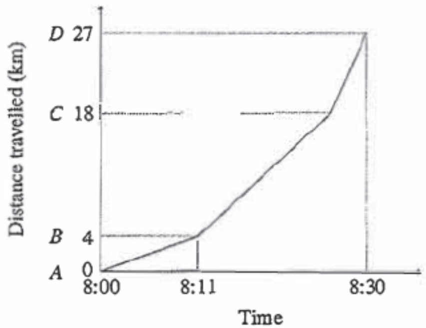
8B.2 HKDSE MA SP – I – 12

The figure shows the graph for John driving from town A to town D (via town B and town C) in a morning. The journey is divided into three parts: Part I (from A to B), Part II (from B to C) and Part III (from C to D).

(a) For which part of the journey is the average speed the lowest? Explain your answer.

(b) If the average speed for Part II of the journey is 56 km/h, when is John at C?

(c) Find the average speed for John driving from A to D in m/s.



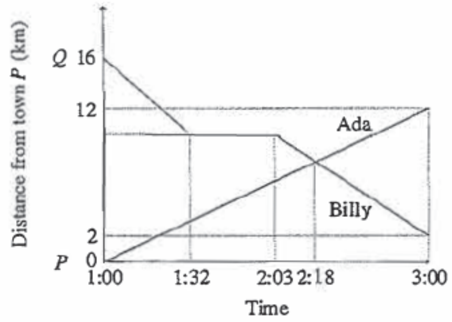
8B.3 HKDSE MA PP – I – 12

The figure shows the graphs for Ada and Billy running on the same straight road between town P and town Q during the period 1:00 to 3:00 in an afternoon. Ada runs at a constant speed. It is given that town P and town Q are 16 km apart.

(a) How long does Billy rest during the period?

(b) How far from town P do Ada and Billy meet during the period?

(c) Use average speed during the period to determine who runs faster. Explain your answer.



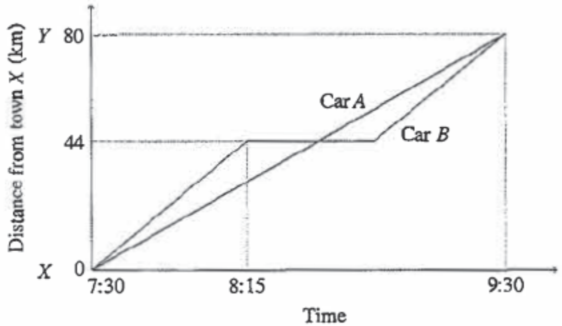
8B.4 HKDSE MA 2014 – I – 10

Town X and Town Y are 80 km apart. The figure shows the graphs for car A and car B travelling on the same straight road between town X and town Y during the period 7:30 to 9:30 in a morning. Car A travels at a constant speed during the period Car B comes to rest at 8:15 in the morning.

(a) Find the distance of car A from town X at 8:15 in the morning.

(b) At what time after 7:30 in the morning do car A and car B first meet?

(c) The driver of car B claims that the average speed of car B is higher than that of car A during the period 8:15 to 9:30 in the morning. Do you agree? Explain your answer.



8C Variation

8C.1 HKCEE MA 1982(1/2) – I 12 (To continue as 7C.2.)

The price of a certain monthly magazine is x dollars per copy. The total profit on the sale of the magazine is P dollars. It is given that $P = Y + Z$, where Y varies directly as x and Z varies directly as the square of x. When x is 20, P is 80 000; when x is 35, P is 87 500.

(a) Find P when $x = 15$.

8C.2 HKCEE MA 1984(B) I 14

A school and a youth centre agree to share the total expenditure for a camp in the ratio 3 : 1. The total expenditure $\$E$ for the camp is the sum of two parts: one part is a constant $\$C$, and the other part varies directly as the number of participants N. If there are 300 participants, the school has to pay \$7500. If there are 500 participants, the school has to pay \$12000.

(a) Find the total expenditure for the camp, when the school has to pay \$7500.

(b) Find the value of C.

(c) Express E in terms of N.

(d) If the youth centre has to pay \$4750, find the number of participants.

8C.3 HKCEE MA 1986(B) I 5

It is given that z varies directly as x^2 and inversely as y. If $x = 1$ and $y = 2$, then $z = 3$. Find z when $x = 2$ and $y = 3$.

8C.4 HKCEE MA 1987(B) I – 14 (To continue as 10C.3.)

Given $p = y + z$, where y varies directly as x, z varies inversely as x and x is positive. When $x = 2$, $p = 7$; when $x = 3$, $p = 8$.

(a) Find p when $x = 4$.

8C.5 HKCEE MA 1988 – I – 10 (To continue as 7C.3.)

A variable quantity y is the sum of two parts. The first part varies directly as another variable x, while the second part varies directly as x^2. When $x = 1$, $y = -5$; when $x = 2$, $y = -8$.

(a) Express y in terms of x. Hence find the value of y when $x = 6$.

8C.6 HKCEE MA 1991 – I 2

In a joint variation, x varies directly as y^2 and inversely as z. Given that $x = 18$ when $y = 3$, $z = 2$,

(a) express x in terms of y and z,

(b) find x when $y = 1$, $z = 4$.

8C.7 HKCEE MA 1994 I 4

Suppose x varies directly as y^2 and inversely as z. When $y = 3$ and $z = 10$, $x = 54$.

(a) Express x in terms of y and z.

(b) Find x when $y = 5$ and $z = 12$.

8C.8 HKCEE MA 1997 – I – 7 (Continued from 15C.5.)

The ratio of the volumes of two similar solid circular cones is 8 : 27.

(a) Find the ratio of the height of the smaller cone to the height of the larger cone.

(b) If the cost of painting a cone varies as its total surface area and the cost of painting the smaller cone is \$32, find the cost of painting the larger cone.

8C.9 HKCEE MA 1998 – I 12

The monthly service charge $\$S$ of mobile phone network A is partly constant and partly varies directly as the connection time t minutes. The monthly service charges are \$230 and \$284 when the connection times are 100 minutes and 130 minutes respectively.

(a) Express S in terms of t.

(b) The service charge of mobile phone network B only varies directly as the connection time. The charge is \$2.20 per minute. A man uses about 110 minutes connection time every month. Should he join network A or B in order to save money? Explain your answer.

8C.10 HKCEE MA 1999 I – 6

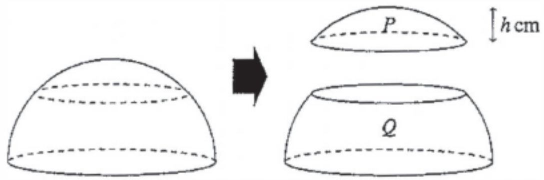
y varies partly as x and partly as x^2. When $x = 2$, $y = 20$ and when $x = 3$, $y = 39$. Express y in terms of x.

8C.11 HKCEE MA 2000 – I – 18 (To continue as 7D. 9.)

The figure shows a solid hemisphere of radius 10 cm. It is cut into two portions, P and Q, along a plane parallel to its base. The height and volume of P are h cm and V cm^3 respectively.

It is known that V is the sum of two parts. One part varies directly as h^2 and the other part varies directly as h^3. $V = \frac{29}{3}\pi$ when $h = 1$ and $V = 81\pi$ when $h = 3$.

(a) Find V in terms of h and π.



8C.12 HKCEE MA 2001 – I 13

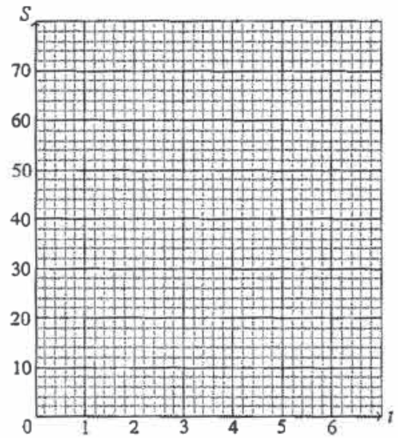
S is the sum of two parts. One part varies as t and the other part varies as the square of t. The table below shows certain pairs of the values of S and t.

S	0	33	56	69	72	65	48	21
t	0	1	2	3	4	5	6	7

(a) Express S in terms of t.

(b) Find the value(s) of t when $S = 40$.

(c) Using the data given in the table, plot the graph of S against t for $0 \le t \le 7$ in the following figure. Read from the graph the value of t when the value of S is greatest.



8C.13 HKCEE MA 2002 – I – 11 (To continue as **15C.8**.)

The area of a paper bookmark is A cm^2 and its perimeter is P cm. A is a function of P. It is known that A is the sum of two parts, one part varies as P and the other part varies as the square of P. When $P = 24$, $A = 36$ and when $P = 18$, $A = 9$.

(a) Express A in terms of P.

(b) (i) The best-selling paper bookmark has an area of 54 cm^2. Find the perimeter of this bookmark.

8C.14 HKCEE MA 2003 – I – 10 (To continue as **10C.5**.)

The speed of a solar-powered toy can is V cm/s and the length of its solar panel is L cm, where $5 \le L \le 25$. V is a function of L. It is known that V is the sum of two parts, one part varies as L and the other part varies as the square of L. When $L = 10$, $V = 30$ and when $L = 15$, $V = 75$.

(a) Express V in terms of L.

8C.15 HKCEE MA 2004 – I – 10 (To continue as **10C.6**.)

It is known that y is the sum of two parts, one part varies as x and the other part varies as the square of x. When $x = 3$, $y = 3$ and when $x = 4$, $y = 12$.

(a) Express y in terms of x.

8C.16 HKCEE MA 2005 – I – 10 (To continue as **4B.18**.)

It is known that $f(x)$ is the sum of two parts, one part varies as x^3 and the other part varies as x.
Suppose $f(2) = -6$ and $f(3) = 6$.

(a) Find $f(x)$.

8C.17 HKCEE MA 2006 – I – 15

The cost of a souvenir of surface area A cm^2 is \$$C$. It is given that C is the sum of two parts, one part varies directly as A while the other part varies directly as A^2 and inversely as n, where n is the number of souvenirs produced. When $A = 50$ and $n = 500$, $C = 350$; when $A = 20$ and $n = 400$, $C = 100$.

(a) Express C in terms of A and n.

(b) The selling price of a souvenir of surface area A cm^2 is \$$8A$ and the profit in selling the souvenir is \$$P$.

 (i) Express P in terms of A and n.

 (ii) Suppose $P : n = 5 : 32$. Find $A : n$.

 (iii) Suppose $n = 500$. Can a profit of \$100 be made in selling a souvenir? Explain your answer.

 (iv) Suppose $n = 400$. Using the method of completing the square, find the greatest profit in selling a souvenir.

8C.18 HKCEE MA 2007 – I – 14 (Continued from **4B.19**.)

(a) Let $f(x) = 4x^3 + kx^2 - 243$, where k is a constant. It is given that $x + 3$ is a factor of $f(x)$.

 (i) Find the value of k.

 (ii) Factorize $f(x)$.

(b) Let \$$C$ be the cost of making a cubical handicraft with a side of length x cm. It is given that C is the sum of two parts, one part varies as x^3 and the other part varies as x^2. When $x = 5.5$, $C = 7381$ and when $x = 6$, $C = 9072$.

 (i) Express C in terms of x.

 (ii) If the cost of making a cubical handicraft is \$972, find the length of a side of the handicraft.

8C.19 HKCEE MA 2010 – I – 10

The cost of a tablecloth of perimeter x metres is \$$C$. It is given that C is the sum of two parts, one part varies as x and the other part varies as x^2. When $x = 4$, $C = 96$ and when $x = 5$, $C = 145$.

(a) Express C in terms of x.

(b) If the cost of a tablecloth is \$288, find its perimeter.

8C.20 HKCEE MA 2011 – I – 11 (To continue as **7B.9**.)

It is given that $f(x)$ is the sum of two parts, one part varies as x^2 and the other part varies as x. Suppose that $f(-2) = 28$ and $f(6) = -36$.

(a) Find $f(x)$.

8C.21 HKDSE MA SP – I – 11

In a factory, the production cost of a carpet of perimeter s metres is \$$C$. It is given that C is a sum of two parts, one part varies as s and the other part varies as the square of s. When $s = 2$, $C = 356$; when $s = 5$, $C = 1250$.

(a) Find the production cost of a carpet of perimeter 6 metres.

(b) If the production cost of a carpet is \$539, find the perimeter of the carpet.

8C.22 HKDSE MA PP – I – 11

Let \$$C$ be the cost of manufacturing a cubical carton of side x cm. It is given that C is partly constant and partly varies as the square of x. When $x = 20$, $C = 42$; when $x = 120$, $C = 112$.

(a) Find the cost of manufacturing a cubical carton of side 50 cm.

(b) If the cost of manufacturing a cubical carton is \$58, find the length of a side of the carton.

8C.23 HKDSE MA 2012 – I – 11 (To continue as **15C.14**.)

Let \$$C$ be the cost of painting a can of surface area A m^2. It is given that C is the sum of two parts, one part is a constant and the other part varies as A. When $A = 2$, $C = 62$; when $A = 6$, $C = 74$.

(a) Find the cost of painting a can of surface area 13 m^2.

8C.24 HKDSE MA 2013 – I – 11

The weight of a tray of perimeter ℓ metres is W grams. It is given that W is the sum of two parts, one part varies directly as ℓ and the other part varies directly as ℓ^2. When $\ell = 1$, $W = 181$ and when $\ell = 2$, $W = 402$.

(a) Find the weight of a tray of perimeter 1.2 metres.

(b) If the weight of a tray is 594 grams, find the perimeter of the tray.

8C.25 HKDSE MA 2014 – I – 13

It is given that $f(x)$ is the sum of two parts, one part varies as x^2 and the other part is a constant. Suppose that $f(2) = 59$ and $f(7) = -121$.

(a) Find $f(6)$.

(b) $A(6, a)$ and $B(-6, b)$ are points lying on the graph of $y = f(x)$. Find the area of $\triangle ABC$, where C is a point lying on the x axis.

8C.26 HKDSE MA 2015 – I – 10

When Susan sells n handbags in a month, her income in that month is $\$S$. It is given that S is a sum of two parts: one part is a constant and the other part varies as n. When $n = 10$, $S = 10600$; when $n = 6$, $S = 9000$.

(a) When Susan sells 20 handbags in a month, find her income in that month.

(b) Is it possible that when Susan sells a certain number of handbags in a month, her income in that month is \$18000? Explain your answer.

8C.27 HKDSE MA 2016 – I – 8

It is given that $f(x)$ is the sum of two parts, one part varies as x and the other part varies as x^2. Suppose that $f(3) = 48$ and $f(9) = 198$.

(a) Find $f(x)$.

(b) Solve the equation $f(x) = 90$.

8C.28 HKDSE MA 2017 – I – 8

It is given that y varies inversely as $\sqrt{x}$. When $x = 144$, $y = 81$.

(a) Express y in terms of x.

(b) If the value of x is increased from 144 to 324, find the change in the value of y.

8C.29 HKDSE MA 2018 – I – 18 (To continue as **7B.21**.)

It is given that $f(x)$ partly varies as x^2 and partly varies as x. Suppose that $f(2) = 60$ and $f(3) = 99$.

(a) Find $f(x)$.

8C.30 HKDSE MA 2019 – I – 10

It is given that $h(x)$ is partly constant and partly varies as x. Suppose that $h(-2) = -96$ and $h(5) = 72$.

(a) Find $h(x)$.

(b) Solve the equation $h(x) = 3x^2$

8C.31 HKDSE MA 2020 – I – 10

The price of a brand X souvenir of height h cm is $\$P$. P is partly constant and partly varies as h^3. When $h = 3$, $P = 59$ and when $h = 7$, $P = 691$.

(a) Find the price of a brand X souvenir of height 4 cm. (4 marks)

(b) Someone claims that the price of a brand X souvenir of height 5 cm is higher than the total price of two brand X souvenirs of height 4 cm. Is the claim correct? Explain your answer. (2 marks)

8 Rate, Ratio and Variation

8A Rate and Ratio

8A.1 <u>HKCEE MA 1980(1) – I – 8</u>

Daily wage of a skilled worker = \$120

Daily wage of a semi-skilled worker $= \$120 \times \frac{3}{4} = \$90$

Daily wage of a unskilled worker $= \$120 \times \frac{2}{4} = \$60$

$\therefore$ Mean daily wage $= \dfrac{10 \times \$120 + 20 \times \$90 + 30 \times \$60}{10+20+30} = \$80$

8A.2 <u>HKCEE MA 1981(1/2/3) – I – 9</u>

Original rate $= \frac{400}{x}$ radios/day

New rate $= \left(\frac{400}{x} + 20\right)$ radios/day

$\therefore \left(\frac{400}{x}+20\right)(x \quad 10) = 400$

$(20+x)(x \quad 10) = 20x$

$x^2 - 10x - 200 = 0 \Rightarrow x = 50$ or -40 (rejected)

8A.3 <u>HKCEE MA 1983(A/B) – I – 4</u>

(a) $\begin{cases} a:b \;= 3:4 \;= 6:8 \\ a: \;\; c = 2: \;\; 5 = 6: \;\; 15 \end{cases} \Rightarrow a:b:c = 6:8:15$

(b) $\dfrac{ac}{a^2+b^2} = \dfrac{ac \times \frac{1}{a^2}}{(a^2+b^2)\times\frac{1}{a^2}} = \dfrac{\frac{c}{a}}{1+(\frac{b}{a})^2} = \dfrac{\frac{5}{2}}{1+(\frac{4}{3})^2} = \dfrac{9}{10}$

8A.4 <u>HKCEE MA 1989 – I – 1</u>

(a) % increase $= \dfrac{9000-8000}{8000} \times 100\% = 12.5\%$

(b) Amount saved $= \$9000 \times \dfrac{3}{3+7} = \$2700$

8A.5 <u>HKCEE MA 1989 – I – 5</u>

(a) $2(1)+(2) \Rightarrow 7x = 14 \Rightarrow x = 2 \Rightarrow y = \frac{3}{2}$

(b) From (a), $\frac{a}{c} = 2,\ \frac{b}{c} = \frac{3}{2}$.

i.e. $\begin{cases} a:c = 2:1 = 4:2 \\ b:c = 3:2 \end{cases} \Rightarrow a:b:c = 4:3:2$

8A.6 <u>HKCEE MA 1991 – I – 3</u>

(a) £150000 ÷ 15 = £10000

(b) Amount $= 10000 + 10000 \times 14.60\% \times \frac{30}{365} = (£)10120$

(c) $\$10120 \times 14.50 = \$146740$

8A.7 <u>HKCEE MA 1991 – I – 4</u>

(a) $2a = 3b \Rightarrow a:b \;= 3:2$

$3b = 5c \Rightarrow \;\; b:c = \;\; 5:3$

$\therefore a:b:c = 15:10:6$

(b) Let $a = 15k,\ b = 10k,\ c = 6k$.

$a - b + c = 55$

$15k - 10k + 6k = 55 \Rightarrow k = 5$

$\therefore c = 6k = 30$

8A.8 <u>HKCEE MA 1995 – I – 5</u>

(a) $\dfrac{x}{y+1} = \dfrac{4}{5} \Rightarrow 5x = 4(y+1) \Rightarrow x = \dfrac{4}{5}(y+1)$

(b) $2x + 9y = 97$

$2 \cdot \frac{4}{5}(y+1) + 9y = 97 \Rightarrow \frac{53}{5}y = \frac{477}{5} \Rightarrow y = 9$

$\therefore x = \frac{4}{5}(9+1) = 8$

8A.9 <u>HKCEE MA 2005 – I – 5</u>

Teresa has $\frac{2}{5}n$ marbles.

$n - 18 = \frac{2}{5}n + 18 \Rightarrow \frac{3}{5}n = 36 \Rightarrow n = 60$

8A.10 <u>HKCEE MA 2011 – I – 6</u>

Let there be x girls and $\frac{7}{6}x$ boys originally.

$\frac{7}{6}x - 17 = x - 4 \Rightarrow x = 78$

$\therefore$ There were 78 girls originally.

8A.11 <u>HKDSE MA PP – I – 5</u>

Let the capacity of a bottle and a cup be x litres and $\frac{3}{4}x$ litres respectively.

$7x + 9\left(\frac{3}{4}x\right) = 11 \Rightarrow \frac{55}{4}x = 11 \Rightarrow x = 0.8$

$\therefore$ The capacity of a bottle is 0.8 litres.

8A.12 <u>HKDSE MA 2018 – I – 9</u>

Let x mins be the time taken from P to Q. Then the car took $(161-x)$ mins from Q to R.

$72 \times \left(\frac{x}{60}\right) + 90 \times \left(\frac{161-x}{60}\right) = 210$

$\frac{483}{2} \quad \frac{3}{10}x = 210 \Rightarrow x = 105$

$\therefore$ The car takes 105 ~~mins from P to Q~~.

8A.13 <u>HKDSE MA 2019 – I – 7</u>

Let the original numbers of adults and children be $13k$ and $6k$ respectively.

$\dfrac{13k+9}{6k+24} = \dfrac{8}{7} \Rightarrow 91k - 48k = 192 - 63 \Rightarrow k = 3$

$\therefore$ Original number of adults was $13(3) = 39$.

8A.14 <u>HKDSE MA 2020 – I – 4</u>

$\frac{a}{b} = \frac{6}{7}$

$b = \frac{7}{6}a$

$3a = 4c$

$c = \frac{3}{4}a \quad \rightarrow \quad \dfrac{b+2c}{a+2b} = \dfrac{\frac{7}{6}a + 2\left(\frac{3}{4}a\right)}{a + 2\left(\frac{7}{6}a\right)}$

$= \frac{4}{5}$

8B Travel graphs

8B.1 <u>HKCEE MA 1984(B) – I – 3</u>

(a) Rested from 12:17 p.m. to 12:32 p.m. $\Rightarrow$ 15 min

(b) 8 km

8B.2 <u>HKDSE MA SP – I – 12</u>

(a) Part I since the slope of the graph is the smallest.

(b) Time for Part II $= (18-4) \div 56 = \frac{1}{4}$ (hours)

$\therefore$ The time at C is 8:26.

(c) Average speed $= \dfrac{27 \times 1000\text{ m}}{30 \times 60\text{ s}} = 15$ m/s

8B.3 <u>HKDSE MA PP – I – 12</u>

(a) Billy rested from 1:32 to 2:03 $\Rightarrow$ 31 min

(b) They meet at 2:18.

$\because$ Speed of Ada $= \frac{12}{2} = 6$ (km/h)

$\therefore$ Dist. from P when they meet $= 6 \times \frac{60+18}{60} = 7.8$ (km)

(c) Average speed of Billy $= (16-2) \div 2 = 7$ (km/h) > 6 km/h

$\therefore$ Billy runs faster.

8B.4 <u>HKDSE MA 2014 – I 10</u>

(a) Speed of $A = \frac{80}{2} = 40$ (km/h)

$\therefore$ Dist. from X at 8:15 $= 40 \times \frac{45}{60} = 30$ (km)

(b) They meet when A is 44 km from X.

Time taken by $A = \frac{44}{40} = 1.1$ (hour) = 1 hr 6 mins

$\therefore$ The time is 8:36.

(c) Dist. travelled by $B = 80 \quad 44 = 36$ (km)

Dist. travelled by $A = 80 - 30 = 50$ (km)

$\therefore$ A has a higher speed as the time taken is the same.

$\therefore$ NO

8C Variation

8C.1 <u>HKCEE MA 1982(1/2) – I – 12</u>

(a) Let $P = ax + bx^2$.

$\begin{cases} 80000 = 20a + 400b \Rightarrow a + 20b = 4000 \\ 87500 = 35a + 1225b \Rightarrow a + 35b = 2500 \end{cases}$

$\Rightarrow \begin{cases} a = 6000 \\ b = -100 \end{cases} \Rightarrow P = 6000x - 100x^2$

Hence, when $x = 15$, $P = 5000(15) - 100(15)^2 = 67500$.

8C.2 <u>HKCEE MA 1984(B) – I – 14</u>

(a) Total expenditure $= \$7500 \div \frac{3}{4} = \$10000$

(b) Let $E = C + kN$.

$\begin{cases} 7500 \div \frac{3}{4} = C + k(300) \Rightarrow C + 300k = 10000 \\ 12000 \div \frac{3}{4} = C + k(500) \Rightarrow C + 500k = 16000 \end{cases}$

$\Rightarrow \begin{cases} c = 1000 \\ k = 30 \end{cases} \Rightarrow E = 1000 + 30N$

i.e. $C = 1000$

(c) $E = 1000 + 30N$

(d) $4750 \div \frac{1}{4} = 1000 + 30N \Rightarrow N = 60$

$\therefore$ The number of participants is 60.

8C.3 <u>HKCEE MA 1986(B) – I – 5</u>

Let $z = \frac{kx^2}{y}$. Then (3) $\frac{k(1)^2}{(2)} \Rightarrow k = 6$

$\therefore z = \frac{6x^2}{y}$

Hence, when $x = 2$ and $y = 3$, $z = \frac{6(2)^2}{(3)} = 8$.

8C.4 <u>HKCEE MA 1987(B) – I – 14</u>

(a) Let $p = ax + \frac{b}{x}$.

$\begin{cases} 7 = 2a + \frac{b}{2} \Rightarrow 4a + b = 14 \\ 8 = 3a + \frac{b}{3} \Rightarrow 9a + b = 24 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = 6 \end{cases}$

$\therefore p = 2x + \frac{6}{x}$.

When $x = 4$, $p = 2(4) + \frac{6}{\cancel{(4)}} = \frac{19}{2}$.

8C.5 <u>HKCEE MA 1988 – I – 10</u>

(a) Let $y = ax + bx^2$

$\begin{cases} 5 = a + b \\ -8 = 2a + 4b \end{cases} \Rightarrow \begin{cases} a = -6 \\ b = 1 \end{cases} \Rightarrow y = x^2 \quad 6x$

Hence, when $x = 6$, $y = (6)^2 - 6(6) = 0$

8C.6 <u>HKCEE MA 1991 – I – 2</u>

(a) Let $x = \frac{ky^2}{z} \Rightarrow 18 = \frac{k(3)^2}{2} \Rightarrow k = 4 \Rightarrow x = \frac{4y^2}{z}$

(b) $x = \frac{4(1)^2}{(4)} = 1$

8C.7 <u>HKCEE MA 1994 – I – 4</u>

(a) Let $x = \frac{ky^2}{z} \Rightarrow (54) = \frac{k(3)^2}{(10)} \Rightarrow k = 60$

$\therefore x = \frac{60y^2}{z}$

(b) $x = \frac{60(5)^2}{(12)} = 125$

8C.8 HKCEE MA 1997 – I – 7

(a) Required ratio $= \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

(b) Cost of painting larger cone $= \$32 \times \left(\frac{3}{2}\right)^2 = \$72$

8C.9 HKCEE MA 1998 – I – 12

(a) Let $S = a + bt$.
$\begin{cases} 230 = a + 100b \\ 284 = a + 130b \end{cases} \Rightarrow \begin{cases} a = 50 \\ b = 1.8 \end{cases}$
$\therefore\ S = 50 + 1.8t$

(b) Charge under $A = 50 + 1.8(110) = (\$)248$
Charge under $B = 2.20 \times 110 = (\$)232 < 248$
$\therefore$ He should join B to save money.

8C.10 HKCEE MA 1999 – I – 6

Let $y = ax + bx^2$.
$\begin{cases} 20 = 2a + 4b \\ 39 = 3a + 9b \end{cases} \Rightarrow \begin{cases} a = 5 \\ b = 3 \end{cases} \Rightarrow y = 5x + 3x^2$

8C.11 HKCEE MA 2000 – I – 18

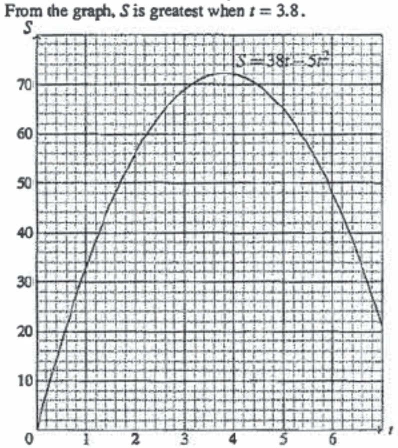
(a) Let $V = ah^2 + bh^3$.
$\begin{cases} \frac{29\pi}{3} = a + b \\ 81\pi = 9a + 27b \end{cases} \Rightarrow \begin{cases} a = 10\pi \\ b = -\frac{\pi}{3} \end{cases}$
$\therefore\ V = 10h^2 - \frac{\pi}{3}h^3$

8C.12 HKCEE MA 2001 – I – 13

(a) Let $S = ht + kt^2$.
$\begin{cases} 33 = h + k \\ 56 = 2h + 4k \end{cases} \Rightarrow \begin{cases} h = 38 \\ k = -5 \end{cases} \Rightarrow S = 38t - 5t^2$

(b) $40 = 38t - 5t^2$
$5t^2 - 38t + 40 = 0$
$t = \frac{38 \pm \sqrt{644}}{10} \left(= \frac{19 \pm \sqrt{161}}{5} \right)$

(c) From the graph, S is greatest when $t = 3.8$.



8C.13 HKCEE MA 2002 – I – 11

(a) Let $A = hP + kP^2$.
$\begin{cases} 36 = 24h + 576k \\ 9 = 18h + 324k \end{cases} \Rightarrow \begin{cases} h = -\frac{5}{2} \\ k = \frac{1}{6} \end{cases} \Rightarrow A = -\frac{5}{2}P + \frac{1}{6}P^2$

(b) (i) $54 = -\frac{5}{2}P + \frac{1}{6}P^2$
$P^2 - 15P - 324 = 0 \Rightarrow P = 27$ or -12 (rejected)
$\therefore$ The perimeter is 27 cm.

8C.14 HKCEE MA 2003 – I – 10

(a) Let $V = hL + kL^2$.
$\begin{cases} 30 = 10h + 100k \\ 75 = 15h + 225k \end{cases} \Rightarrow \begin{cases} h = -1 \\ k = 0.4 \end{cases} \Rightarrow V = 0.4L^2 - L$

8C.15 HKCEE MA 2004 – I – 10

(a) Let $y = hx + kx^2$.
$\begin{cases} 3 = 3h + 9k \\ 12 = 4h + 16k \end{cases} \Rightarrow \begin{cases} h = -5 \\ k = 2 \end{cases} \Rightarrow y = 2x^2 - 5x$

8C.16 HKCEE MA 2005 – I – 10

(a) Let $f(x) = hx^3 + kx$.
$\begin{cases} -6 = f(2) = 8h + 2k \Rightarrow 4h + k = -3 \\ 6 = f(3) = 27h + 3k \Rightarrow 9h + k = 2 \end{cases} \Rightarrow \begin{cases} h = 1 \\ k = -7 \end{cases}$
$\therefore\ f(x) = x^3 - 7x$

8C.17 HKCEE MA 2006 – I – 15

(a) Let $C = hA + \frac{kA^2}{n}$.
$\begin{cases} 350 = 50h + \frac{k(50)^2}{500} \Rightarrow 10h + k = 70 \\ 100 = 20h + \frac{k(20)^2}{400} \Rightarrow 20h + k = 100 \end{cases}$
$\Rightarrow \begin{cases} h = 3 \\ k = 40 \end{cases} \Rightarrow C = 3A + \frac{40A^2}{n}$

(b) (i) $P = 8A - C = 5A - \frac{40A^2}{n}$

(ii) $5A - \frac{40A^2}{n} = P$
$5\left(\frac{A}{n}\right) - 40\left(\frac{A}{n}\right)^2 = \frac{P}{n} = \frac{5}{32}$ (both sides $\div n$)
$256\left(\frac{A}{n}\right)^2 - 32\left(\frac{A}{n}\right) + 1 = 0$
$\left[16\left(\frac{A}{n}\right) - 1\right]^2 = 0 \Rightarrow \frac{A}{n} = \frac{1}{16}$

(iii) Put $n = 500$ and $P = 100$.
$100 = 5A - \frac{2}{25}A^2 \Rightarrow 2A^2 - 125A + 2500 = 0$
$\because\ \Delta = -4375 < 0$
$\therefore$ Not possible.

(iv) Put $n = 400$.
$P = 5A - \frac{1}{10}A^2 = \frac{-1}{10}(A^2 - 50A)$
$= \frac{-1}{10}(A^2 - 50A + 25^2 - 25^2)$
$= \frac{-1}{10}(A - 25)^2 + 62.5$
$\therefore$ Greatest profit is \$62.5.

8C.18 HKCEE MA 2007 – I – 14

(a) (i) $0 = f(-3) = 4(-3)^3 + k(-3)^2 - 243 \Rightarrow k = 39$

(ii) $f(x) = (x+3)(4x^2 + 27x - 81)$
$= (x+3)(4x - 9)(x + 9)$

(b) (i) Let $C = hx^3 + kx^2$.
$\begin{cases} 7381 = h(5.5)^3 + k(5.5)^2 \\ 9077 = h(6)^3 + k(6)^2 \end{cases} \Rightarrow \begin{cases} h = 16 \\ k = 156 \end{cases}$
$\therefore\ C = 16x^3 + 156x^2$

(ii) $972 = 16x^3 + 156x^2$
$4x^3 + 39x^2 - 243 = 0$
$x = -3$ (rej.) or -9 (rej.) or 2.25

8C.19 HKCEE MA 2010 – I – 10

(a) Let $C = hx + kx^2$.
$\begin{cases} 96 = 4h + 16k \\ 145 = 5h + 25k \end{cases} \Rightarrow \begin{cases} h = 4 \\ k = 5 \end{cases} \Rightarrow C = 4x + 5x^2$

(b) $4x + 5x^2 = 288$
$5x^2 + 4x - 288 = 0 \Rightarrow x = 7.2$ or -8 (rejected)

8C.20 HKCEE MA 2011 – I – 11

(a) Let $f(x) = hx^2 + kx$.
$\begin{cases} 28 = f(-2) = 4h - 2k \\ 36 = f(6) = 36h + 6k \end{cases} \Rightarrow \begin{cases} h = 1 \\ k = -12 \end{cases}$
$\therefore\ f(x) = x^2 - 12x$

(b) (i) $f(x) = x^2 - 12x = (x - 6)^2 - 36 \Rightarrow k = -36$

(ii) Put $x = 10$.
$y = 3(10 - 6)^2 - 36 = 2 \Rightarrow A = (10, 2)$
$y = (10)^2 - 12(10) = -20 \Rightarrow D = (10, -20)$
Since the graphs are symmetric about the common axis of symmetry $x = 6$,
$B = (6 - (10 - 6), 2) = (2, 2)$
$C = (10 - (10 - 6), -20) = (2, -20)$
$\therefore$ Area of $ABCD = (2 - (-20))(10 - 2) = 176$

8C.21 HKDSE MA SP – I – 11

(a) Let $C = hs + ks^2$.
$\begin{cases} 356 = 2h + 4k \\ 1250 = 5h + 25k \end{cases} \Rightarrow \begin{cases} h = 130 \\ k = 24 \end{cases} \Rightarrow C = 130s + 24s^2$
$\therefore$ When $s = 6$, cost $= 130(6) + 24(6)^2 = (\$)1644$

(b) $130s + 24s^2 = 539$
$24s^2 + 130s - 539 = 0 \Rightarrow s = \frac{11}{4}$ or $-\frac{49}{6}$ (rejected)
$\therefore$ The perimeter is $\frac{11}{4}$ m.

8C.22 HKDSE MA PP – I – 11

(a) Let $C = h + kx^2$.
$\begin{cases} 42 = h + 400k \\ 112 = h + 14400k \end{cases} \Rightarrow \begin{cases} h = 40 \\ k = 0.005 \end{cases} \Rightarrow C = 40 + 0.005x^2$
$\therefore$ When $x = 50$, cost $= 40 + 0.005(50)^2 = (\$)52.5$.

(b) $40 + 0.005x^2 = 58$
$0.005x^2 = 18 \Rightarrow x = 60$
$\therefore$ The length of a side is 60 cm.

8C.23 HKDSE MA 2012 – I – 11

(a) Let $C = h + kA$.
$\begin{cases} 62 = h + 2k \\ 74 = h + 6k \end{cases} \Rightarrow \begin{cases} h = 56 \\ k = 3 \end{cases} \Rightarrow C = 56 + 3A$
$\therefore$ When $A = 13$, cost $= 56 + 3(13) = (\$)95$

8C.24 HKDSE MA 2013 – I – 11

(a) Let $W = h\ell + k\ell^2$.
$\begin{cases} 181 = h + k \\ 402 = 2h + 4k \end{cases} \Rightarrow \begin{cases} h = 161 \\ k = 20 \end{cases} \Rightarrow W = 161\ell + 20\ell^2$
$\therefore$ When $\ell = 1.2$, weight $= 161(1.2) + 20(1.2)^2 = 222$ (g)

(b) $161\ell + 20\ell^2 = 594$
$20\ell^2 + 161\ell - 594 = 0 \Rightarrow \ell = \frac{11}{4}$ or $-\frac{54}{5}$ (rejected)
$\therefore$ The perimeter is $\frac{11}{4}$ m.

8C.25 HKDSE MA 2014 – I – 13

(a) Let $f(x) = hx^2 + k$.
$\begin{cases} 59 = f(2) = 4h + k \\ -121 = f(7) = 49h + k \end{cases} \Rightarrow \begin{cases} h = -4 \\ k = 75 \end{cases}$
$\therefore\ f(x) = -4x^2 + 75$
$\therefore\ f(6) = -4(6)^2 + 75 = -69$

(b) From (a), $a = b = -69$.
$\therefore$ Area of $\Delta ABC = \frac{(6 - (-6))(69)}{2} = 414$

8C.26 HKDSE MA 2015 – I – 10

(a) Let $S = h + kn$.
$\begin{cases} 16600 = h + 10k \\ 9000 = h + 6k \end{cases} \Rightarrow \begin{cases} h = -2400 \\ k = 1900 \end{cases}$
$\therefore\ S = -2400 + 1900n$
$\therefore$ When $n = 20$, income $= -2400 + 1900(20) = (\$)35600$

(b) $18000 = -2400 + 1900n \Rightarrow n = \frac{204}{19}$, not an integer
$\therefore$ NOT possible

8C.27 HKDSE MA 2016 – I – 8

(a) Let $f(x) = hx + kx^2$.
$\begin{cases} 48 = f(3) = 3h + 9k \\ 198 = f(9) = 9h + 81k \end{cases} \Rightarrow \begin{cases} h = 13 \\ k = 1 \end{cases}$
$\therefore\ f(x) = 13x + x^2$

(b) $13x + x^2 = 90$
$x^2 + 13x - 90 = 0 \Rightarrow x = 5$ or -18

8C.28 HKDSE MA 2017 – I – 8

(a) Let $y = \frac{k}{\sqrt{x}} \Rightarrow 81 = \frac{k}{\sqrt{144}} \Rightarrow k = 972$
$\therefore\ y = \frac{972}{\sqrt{x}}$

(b) Change of $y = \frac{972}{\sqrt{(324)}} - 81 = -27$

8C.29 HKDSE MA 2018 – I – 18

(a) Let $f(x) = hx^2 + kx$.
$\begin{cases} 60 = f(2) = 4h + 2k \\ 99 = f(3) = 9h + 3k \end{cases} \Rightarrow \begin{cases} h = 3 \\ k = 24 \end{cases}$
$\therefore\ f(x) = 3x^2 + 24x$

8C.30 <u>HKDSE MA 2019 – I – 10</u>

(a) Let $h(x) = a + bx$.

$$\begin{cases} -96 = h(-2) = a - 2b \\ 72 = h(5) = a + 5b \end{cases} \Rightarrow \begin{cases} a = \quad 48 \\ b = 24 \end{cases}$$

$\therefore\ h(x) = \quad 48 + 24x$

(b) $-48 + 24x = 3x^2 \Rightarrow x^2 - 8x + 16 = 0$

$\Rightarrow x = 4$ (repeated)

8C.31 <u>HKDSE MA 2020 – I – 10</u>

10a	Let $P=k_1+k_2h^3$, where k_1 and k_2 are non-zero constants.
	Sub. $h=3$ and $P=59$,
	$59=k_1+k_2(3)^3$
	$k_1+27k_2=59$ ------(1)
	Sub. $h=7$ and $P=691$,
	$691=k_1+k_2(7)^3$
	$k_1+343k_2=691$ ------(2)
	(2)–(1):
	$316k_2=632$
	$k_2=2$
	Sub. $k_2=2$ into (1).
	$k_1+27(2)\quad 59$
	$k_1=5$
	Therefore, $P=5+2h^3$.
	When $h=4$,
	$P=5+2(4)^3$
	$=133$
	Therefore, the price of a brand X souvenir is $133.
b	When $h\quad 5$,
	$P=5+2(5)^3$
	255
	<266
	$=2\times 133$
	Hence, the price of a brand X souvenir of height 5 cm is lower than the total price of two brand X souvenirs of height 4 cm
	Consequently, the claim is not correct.