4 Polynomials

4A Factorization, H.C.F. and L.C.M. of polynomials

4A.1 HKCEE MA 1980(1/1*/3) I 2

Factorize

(a) $a(3b-c)+c-3b$,

(b) x^4-1.

4A.2 HKCEE MA 1981(2/3) I 5

Factorize $(1+x)^4-(1-x^2)^2$.

4A.3 HKCEE MA 1983(A/B) – I – 1

Factorise $(x^2+4x+4)-(y-1)^2$.

4A.4 HKCEE MA 1984(A/B) I – 4

Factorize

(a) $x^2y+2xy+y$,

(b) $x^2y+2xy+y-y^3$.

4A.5 HKCEE MA 1985(A/B) I – 1

(a) Factorize a^4-16 and a^3-8.

(b) Find the L.C.M. of a^4-16 and a^3-8.

4A.6 HKCEE MA 1986(A/B) I 1

Factorize

(a) x^2-2x-3,

(b) $(a^2+2a)^2-2(a^2+2a)-3$.

4A.7 HKCEE MA 1987(A/B) I 1

Factorize

(a) x^2-2x+1,

(b) $x^2-2x+1-4y^2$.

4A.8 HKCEE MA 1993 – I 2(e)

Find the H.C.F. and L.C.M. of $6x^2y^3$ and $4xy^2z$.

4A.9 HKCEE MA 1995 I 1(b)

Find the H.C.F. of $(x-1)^3(x+5)$ and $(x-1)^2(x+5)^3$.

4A.10 HKCEE MA 1997 – I 1

Factorize

(a) x^2-9,

(b) $ac+bc-ad-bd$.

4A.11 HKCEE MA 2003 – I 3

Factorize

(a) $x^2-(y-x)^2$,

(b) $ab-ad-bc+cd$.

4A.12 HKCEE MA 2004 – I 6

Factorize

(a) $a^2-ab+2a-2b$,

(b) $169y^2-25$.

4A.13 HKCEE MA 2005 – I 3

Factorize

(a) $4x^2-4xy+y^2$,

(b) $4x^2-4xy+y^2-2x+y$.

4A.14 HKCEE MA 2007 I – 3

Factorize

(a) $r^2+10r+25$,

(b) $r^2+10r+25-s^2$.

4A.15 HKCEE MA 2009 – I 3

Factorize

(a) a^2b+ab^2,

(b) $a^2b+ab^2+7a+7b$.

4A.16 HKCEE MA 2010 – I – 3

Factorize

(a) $m^2+12mn+36n^2$,

(b) $m^2+12mn+36n^2-25k^2$.

4A.17 HKCEE MA 2011 – I – 3

Factorize

(a) $81m^2-n^2$,

(b) $81m^2\quad n^2+18m-2n$.

4A.18 HKDSE MA SP – I – 3

Factorize

(a) $3m^2 - mn - 2n^2$,

(b) $3m^2 - mn - 2n^2 - m + n$.

4A.19 HKDSE MA PP – I – 3

Factorize

(a) $9x^2 - 42xy + 49y^2$,

(b) $9x^2 - 42xy + 49y^2 - 6x + 14y$.

4A.20 HKDSE MA 2012 – I – 3

Factorize

(a) $x^2 - 6xy + 9y^2$,

(b) $x^2 - 6xy + 9y^2 + 7x - 21y$.

4A.21 HKDSE MA 2013 – I – 3

Factorize

(a) $4m^2 - 25n^2$,

(b) $4m^2 - 25n^2 + 6m - 15n$.

4A.22 HKDSE MA 2014 – I – 2

Factorize

(a) $a^2 - 2a - 3$,

(b) $ab^2 + b^2 + a^2 - 2a - 3$.

4A.23 HKDSE MA 2015 – I – 4

Factorize

(a) $x^3 + x^2y - 7x^2$,

(b) $x^3 + x^2y - 7x^2 - x - y + 7$.

4A.24 HKDSE MA 2016 – I – 4

Factorize

(a) $5m - 10n$,

(b) $m^2 + mn - 6n^2$,

(c) $m^2 + mn - 6n^2 - 5m + 10n$.

4A.25 HKDSE MA 2017 – I – 3

Factorize

(a) $x^2 - 4xy + 3y^2$,

(b) $x^2 - 4xy + 3y^2 + 11x - 33y$.

4A.26 HKDSE MA 2018 – I – 5

Factorize

(a) $9r^3 - 18r^2s$,

(b) $9r^3 - 18r^2s - rs^2 + 2s^3$.

4A.27 HKDSE MA 2019 – I – 4

Factorize

(a) $4m^2 - 9$,

(b) $2m^2n + 7mn - 15n$,

(c) $4m^2 - 9 - 2m^2n - 7mn + 15n$.

4A.28 HKDSE MA 2020 – I – 2

Factorize

(a) $\alpha^2 + \alpha - 6$,

(b) $\alpha^4 + \alpha^3 - 6\alpha^2$.

4B Division algorithm, remainder theorem and factor theorem

4B.1 HKCEE MA 1980(1*/3) – I – 13(a)

It is given that $f(x)=2x^2+ax+b$.

(i) If $f(x)$ is divided by $(x-1)$, the remainder is -5. If $f(x)$ is divided by $(x+2)$, the remainder is 4. Find the values of a and b.

(ii) If $f(x)=0$, find the value of x.

4B.2 HKCEE MA 1981(2) – I – 3 and HKCEE MA 1981(3) – I – 2

Let $f(x)=(x+2)(x-3)+3$. When $f(x)$ is divided by $(x-k)$, the remainder is k. Find k.

4B.3 HKCEE MA 1984(A/B) – I – 1

If $3x^2-kx-2$ is divisible by $x-k$, where k is a constant, find the two values of k.

4B.4 HKCEE MA 1985(A/B) – I – 4

Given $f(x)=ax^2+bx-1$, where a and b are constants. $f(x)$ is divisible by $x-1$. When divided by $x+1$, $f(x)$ leaves a remainder of 4. Find the values of a and b.

4B.5 HKCEE MA 1987(A/B) – I – 2

Find the values of a and b if $2x^3+ax^2+bx-2$ is divisible by $x-2$ and $x+1$.

4B.6 HKCEE MA 1989 – I – 3

Given that $(x+1)$ is a factor of x^4+x^3-8x+k, where k is a constant,

(a) find the value of k,

(b) factorize x^4+x^3-8x+k.

4B.7 HKCEE MA 1990 – I – 7

(a) Find the remainder when $x^{1000}+6$ is divided by $x+1$.

(b) (i) Using (a), or otherwise, find the remainder when $8^{1000}+6$ is divided by 9.

(ii) What is the remainder when 8^{1000} is divided by 9?

4B.8 HKCEE MA 1990 – I – 11 (Continued from **15B.6**.)

A solid right circular cylinder has radius r and height h. The volume of the cylinder is V and the total surface area is S.

(a) (i) Express S in terms of r and h.

(ii) Show that $S=2\pi r^2+\frac{2V}{r}$.

(b) Given that $V=2\pi$ and $S=6\pi$, show that $r^3-3r+2=0$. Hence find the radius r by factorization.

(c) *[Out of syllabus]*

4B.9 HKCEE MA 1992 – I – 2(b)

Find the remainder when x^3-2x^2+3x-4 is divided by $x-1$.

4B.10 HKCEE MA 1993 – I – 2(d)

Find the remainder when x^3+x^2 is divided by $x-1$.

4B.11 HKCEE MA 1994 – I – 3

When $(x+3)(x-2)+2$ is divided by $x-k$, the remainder is k^2. Find the value(s) of k.

4B.12 HKCEE MA 1995 – I – 2

(a) Simplify $(a+b)^2-(a-b)^2$.

(b) Find the remainder when x^3+1 is divided by $x+2$.

4B.13 HKCEE MA 1996 – I – 4

Show that $x+1$ is a factor of x^3-x^2-3x-1.
Hence solve $x^3-x^2-3x-1=0$. (Leave your answers in surd form.)

4B.14 HKCEE MA 1998 – I – 9

Let $f(x)=x^3+2x^2-5x-6$.

(a) Show that $x-2$ is a factor of $f(x)$.

(b) Factorize $f(x)$.

4B.15 HKCEE MA 2000 – I – 6

Let $f(x)=2x^3+6x^2-2x-7$. Find the remainder when $f(x)$ is divided by $x+3$.

4B.16 HKCEE MA 2001 – I – 2

Let $f(x)=x^3-x^2+x-1$. Find the remainder when $f(x)$ is divided by $x-2$.

4B.17 HKCEE MA 2002 – I – 4

Let $f(x)=x^3-2x^2-9x+18$.

(a) Find $f(2)$.

(b) Factorize $f(x)$.

4B.18 HKCEE MA 2005 – I – 10 (Continued from **8C.16**.)

It is known that $f(x)$ is the sum of two parts, one part varies as x^3 and the other part varies as x.
Suppose $f(2)=-6$ and $f(3)=6$.

(a) Find $f(x)$.

(b) Let $g(x)=f(x)-6$.

(i) Prove that $x-3$ is a factor of $g(x)$.

(ii) Factorize $g(x)$.

4B.19 HKCEE MA 2007 – I – 14 (To continue as **8C.18**.)

(a) Let $f(x)=4x^3+kx^2-243$, where k is a constant. It is given that $x+3$ is a factor of $f(x)$.

(i) Find the value of k.

(ii) Factorize $f(x)$.

4B.20 HKDSE MA SP – I – 10

(a) Find the quotient when $5x^3+12x^2-9x-7$ is divided by x^2+2x-3.

(b) Let $g(x)=(5x^3+12x^2-9x-7)-(ax+b)$, where a and b are constants. It is given that $g(x)$ is divisible by x^2+2x-3.

(i) Write down the values of a and b.

(ii) Solve the equation $g(x)=0$.

4B.21 HKDSE MA PP – I – 10

Let $f(x)$ be a polynomial. When $f(x)$ is divided by $x-1$, the quotient is $6x^2+17x-2$. It is given that $f(1)=4$.

(a) Find $f(-3)$.

(b) Factorize $f(x)$.

4B.22 HKDSE MA 2012 – I – 13 (To continue as **7B.17**.)

(a) Find the value of k such that $x-2$ is a factor of $kx^3-21x^2+24x-4$.

4B.23 HKDSE MA 2013 – I – 12

Let $f(x)=3x^3-7x^2+kx-8$, where k is a constant. It is given that $f(x)\equiv(x-2)(ax^2+bx+c)$, where a, b and c are constants.

(a) Find a, b and c.

(b) Someone claims that all the roots of the equation $f(x)=0$ are real numbers. Do you agree? Explain your answer.

4B.24 HKDSE MA 2014 – I – 7

Let $f(x)=4x^3\quad 5x^2-18x+c$, where c is a constant. When $f(x)$ is divided by $x-2$, the remainder is 33.

(a) Is $x+1$ a factor of $f(x)$? Explain your answer.

(b) Someone claims that all the roots of the equation $f(x)=0$ are rational numbers. Do you agree? Explain your answer.

4B.25 HKDSE MA 2015 – I 11

Let $f(x)=(x-2)^2(x+h)+k$, where h and k are constants. When $f(x)$ is divided by $x-2$, the remainder is -5. It is given that $f(x)$ is divisible by $x-3$.

(a) Find h and k.

(b) Someone claims that all the roots of the equation $f(x)=0$ are integers. Do you agree? Explain your answer.

4B.26 HKDSE MA 2016 – I – 14

Let $p(x)=6x^4+7x^3+ax^2+bx+c$, where a, b and c are constants. When $p(x)$ is divided by $x+2$ and when $p(x)$ is divided by $x-2$, the two remainders are equal. It is given that $p(x)\equiv(lx^2+5x+8)(2x^2+mx+n)$, where l, m and n are constants.

(a) Find l, m and n.

(b) How many real roots does the equation $p(x)=0$ have? Explain your answer.

4B.27 HKDSE MA 2017 I – 14

Let $f(x)=6x^3-13x^2-46x+34$. When $f(x)$ is divided by $2x^2+ax+4$, the quotient and the remainder are $3x+7$ and $bx+c$ respectively, where a, b and c are constants.

(a) Find a.

(b) Let $g(x)$ be a quadratic polynomial such that when $g(x)$ is divided by $2x^2+ax+4$, the remainder is $bx+c$.

(i) Prove that $f(x)-g(x)$ is divisible by $2x^2+ax+4$.

(ii) Someone claims that all the roots of the equation $f(x)-g(x)=0$ are integers. Do you agree? Explain your answer.

4B.28 HKDSE MA 2018 – I – 12

Let $f(x)=4x(x+1)^2+ax+b$, where a and b are constants. It is given that $x-3$ is a factor of $f(x)$. When $f(x)$ is divided by $x+2$, the remainder is $2b+165$.

(a) Find a and b.

(b) Someone claims that the equation $f(x)=0$ has at least one irrational root. Do you agree? Explain your answer.

4B.29 HKDSE MA 2019 – I – 11

Let $p(x)$ be a cubic polynomial. When $p(x)$ is divided by $x-1$, the remainder is 50. When $p(x)$ is divided by $x+2$, the remainder is 52. It is given that $p(x)$ is divisible by $2x^2+9x+14$.

(a) Find the quotient when $p(x)$ is divided by $2x^2+9x+14$.

(b) How many rational roots does the equation $p(x)=0$ have? Explain your answer.

4 Polynomials

4A Factorization, H.C.F. and L.C.M. of polynomials

4A.1 HKCEE MA 1980(1/1*/3) – I – 2

(a) $a(3b-c)+c-3b=(3b-c)(a-1)$

(b) $x^4-1=(x-1)(x+1)(x^2+1)$

4A.2 HKCEE MA 1981(2/3) – I – 5

$$(1+x)^4-(1-x^2)^2=[(1+x)^2]^2-(1-x^2)^2$$
$$=[(1+x)^2-(1-x^2)][(1+x)^2+(1-x^2)]$$
$$=(2x+2x^2)(2+2x)=4x(1+x)^2$$

4A.3 HKCEE MA 1983(A/B) – I – 1

$$(x^2+4x+4)-(y-1)^2=(x+2)^2-(y-1)^2$$
$$=[(x+2)-(y-1)][(x+2)+(y-1)]$$
$$=(x-y+3)(x+y+1)$$

4A.4 HKCEE MA 1984(A/B) – I – 4

(a) $x^2y+2xy+y=y(x^2+2x+1)=y(x+1)^2$

(b) $x^2y+2xy+y-y^3=y(x+1)^2-y^3$
$$=y[(x+1)^2-y^2]$$
$$=y(x+1-y)(x+1+y)$$

4A.5 HKCEE MA 1985(A/B) – I – 1

(a) $a^4-16=(a-2)(a+2)(a^2+4)$

$a^3-8=(a-2)(a^2+2a+4)$

(b) L.C.M. $=(a-2)(a+2)(a^2+4)(a^2+2a+4)$

4A.6 HKCEE MA 1986(A/B) – I – 1

(a) $x^2-2x-3=(x-3)(x+1)$

(b) $(a^2+2a)^2-2(a^2+2a)-3$
$$=[(a^2+2a)-3][(a^2+2a)+1]=(a+3)(a-1)(a+1)^2$$

4A.7 HKCEE MA 1987(A/B) – I – 1

(a) $x^2-2x+1=(x-1)^2$

(b) $x^2-2x+1-4y^2=(x-1)^2-(2y)^2$
$$=(x-1-2y)(x-1+2y)$$

4A.8 HKCEE MA 1993 – I – 2(c)

H.C.F. $=2xy^2$, L.C.M. $=12x^2y^3z$

4A.9 HKCEE MA 1995 – I – 1(b)

H.C.F. $=(x-1)^2(x+5)$

4A.10 HKCEE MA 1997 – I – 1

(a) $x^2-9=(x-3)(x+3)$

(b) $ac+bc-ad-bd=c(a+b)-d(a+b)=(a+b)(c-d)$

4A.11 HKCEE MA 2003 – I – 3

(a) $x^2-(y-x)^2=[x-(y-x)][x+(y-x)]=y(2x-y)$

(b) $ab-ad-bc+cd=a(b-d)-c(b-d)=(b-d)(a-c)$

4A.12 HKCEE MA 2004 – I – 6

(a) $a^2-ab+2a-2b=a(a-b)+2(a-b)=(a-b)(a+2)$

(b) $169y^2-25=(13y)^2-5^2=(13y-5)(13y+5)$

4A.13 HKCEE MA 2005 – I – 3

(a) $4x^2-4xy+y^2=(2x-y)^2$

(b) $4x^2-4xy+y^2-2x+y=(2x-y)^2-(2x-y)$
$$=(2x-y)(2x-y-1)$$

4A.14 HKCEE MA 2007 – I – 3

(a) $r^2+10r+25=(r+5)^2$

(b) $r^2+10r+25-s^2=(r+5)^2-s^2=(r+5-s)(r+5+s)$

4A.15 HKCEE MA 2009 – I – 3

(a) $a^2b+ab^2=ab(a+b)$

(b) $a^2b+ab^2+7a+7b=ab(a+b)+7(a+b)$
$$=(a+b)(ab+7)$$

4A.16 HKCEE MA 2010 – I – 3

(a) $m^2+12mn+36n^2=(m+6n)^2$

(b) $m^2+12mn+36n^2-25k^2=(m+6n)^2-(5k)^2$
$$=(m+6n-5k)(m+6n+5k)$$

4A.17 HKCEE MA 2011 – I – 3

(a) $81m^2-n^2=(9m-n)(9m+n)$

(b) $81m^2-n^2+18m-2n=(9m-n)(9m+n)+2(9m-n)$
$$=(9m-n)(9m+n+2)$$

4A.18 HKDSE MA SP – I – 3

(a) $3m^2-mn-2n^2=(3m+2n)(m-n)$

(b) $3m^2-mn-2n^2-m+n=(3m+2n)(m-n)-(m-n)$
$$=(m-n)(3m+2n-1)$$

4A.19 HKDSE MA PP – I – 3

(a) $9x^2-42xy+49y^2=(3x-7y)^2$

(b) $9x^2-42xy+49y^2-6x+14y=(3x-7y)^2-2(3x-7y)$
$$=(3x-7y)(3x-7y-2)$$

4A.20 HKDSE MA 2012 – I – 3

(a) $x^2-6xy+9y^2=(x-3y)^2$

(b) $x^2-6xy+9y^2+7x-21y=(x-3y)^2+7(x-3y)$
$$=(x-3y)(x-3y+7)$$

4A.21 HKDSE MA 2013 – I – 3

(a) $4m^2-25n^2=(2m-5n)(2m+5n)$

(b) $4m^2-25n^2+6m-15n$
$$=(2m-5n)(2m+5n)+3(2m-5n)$$
$$=(2m-5n)(2m+5n+3)$$

4A.22 HKDSE MA 2014 – I – 2

(a) $a^2-2a-3=(a-3)(a+1)$

(b) $ab^2+b^2+a^2-2a-3=b^2(a+1)+(a-3)(a+1)$
$$=(a+1)(b^2+a-3)$$

4A.23 HKDSE MA 2015 – I – 4

(a) $x^3+x^2y-7x^2=x^2(x+y-7)$

(b) $x^3+x^2y-7x^2-x-y+7=x^2(x+y-7)-(x+y-7)$
$$=(x+y-7)(x^2-1)$$
$$=(x+y-7)(x-1)(x+1)$$

4A.24 HKDSE MA 2016 – I – 4

(a) $5m-10n=5(m-2n)$

(b) $m^2+mn-6n^2=(m+3n)(m-2n)$

(c) $m^2+mn-6n^2-5m+10n$
$$=(m+3n)(m-2n)-5(m-2n)=(m-2n)(m+3n-5)$$

4A.25 HKDSE MA 2017 – I – 3

(a) $x^2-4xy+3y^2=(x-3y)(x-y)$

(b) $x^2-4xy+3y^2+11x-33y=(x-3y)(x-y)+11(x-3y)$
$$=(x-3y)(x-y+11)$$

4A.26 HKDSE MA 2018 – I – 5

(a) $9r^3-18r^2s=9r^2(r-2s)$

(b) $9r^3-18r^2s-rs^2+2s^3=9r^2(r-2s)-s^2(r-2s)$
$$=(r-2s)(9r^2-s^2)$$
$$=(r-2s)(3r-s)(3r+s)$$

4A.27 HKDSE MA 2019 – I – 4

(a) $4m^2-9=(2m-3)(2m+3)$

(b) $2m^2n+7mn-15n=n(2m^2+7m-5)=n(2m-3)(m+5)$

(c) $4m^2-9-2m^2n-7mn+15n$
$$=(2m-3)(2m+3)-n(2m-3)(m+5)$$
$$=(2m-3)[(2m+3)-n(m+5)]$$
$$=(2m-3)(2m-mn-5n+3)$$

4A.28 HKDSE MA 2020 – I – 2

2a $\quad \alpha^2+\alpha-6=(\alpha+3)(\alpha-2)$

b $\quad \alpha^4+\alpha^3-6\alpha^2=\alpha^2(\alpha^2+\alpha-6)$
$$=\alpha^2(\alpha+3)(\alpha-2)$$

4B Division algorithm, remainder theorem and factor theorem

4B.1 HKCEE MA 1980(1*/3) – I – 13(a)

(a) (i) $\begin{cases}5=f(1)=24a+b \Rightarrow a+b=7\\4=f(-2)=8-2a+b \Rightarrow 2a-b=4\end{cases}$
$$\Rightarrow \begin{cases}a=-1\\b=-6\end{cases}$$

(ii) $f(x)=0$
$$2x^2-x-6=0$$
$$(2x+3)(x-2)=0 \Rightarrow x=-\frac{3}{2}\text{ or }2$$

4B.2 HKCEE MA 1981(2) – I – 3 and 1981(3) – I – 2

$$k=f(k)=(k+2)(k-3)+3$$
$$k=k^2-k-3$$
$$k^2-2k-3=0$$
$$(k-3)(k+1)=0 \Rightarrow k=3\text{ or }-1$$

4B.3 HKCEE MA 1984(A/B) – I – 1

$\because x-k$ is a factor

$\therefore 3(k)^2-k(k)-2=0 \Rightarrow k^2=1 \Rightarrow k=\pm 1$

4B.4 HKCEE MA 1985(A/B) – I – 4

$$\begin{cases}0=f(1)=a+b-1 \Rightarrow a+b=1\\4=f(-1)=a-b-1 \Rightarrow a-b=5\end{cases} \Rightarrow \begin{cases}a=3\\b=-2\end{cases}$$

4B.5 HKCEE MA 1987(A/B) – I – 2

$$\begin{cases}2(2)^3+a(2)^2+b(2)-2=0\\2(-1)^3+a(-1)^2+b(-1)-2=0\end{cases}$$
$$\Rightarrow \begin{cases}4a+2b=-14\\a-b=4\end{cases} \Rightarrow \begin{cases}a=-1\\b=-5\end{cases}$$

4B.6 HKCEE MA 1989 – I – 3

(a) $(-1)^4+(-1)^3-8(-1)+k=0 \Rightarrow k=-8$

(b) $x^4+x^3-8x+k=x^4+x^3-8x-8$
$$=x^3(x+1)-8(x+1)$$
$$=(x+1)(x^3-8)$$
$$=(x+1)(x-2)(x^2+2x+4)$$

4B.7 HKCEE MA 1990 – I – 7

(a) Remainder $=(-1)^{1000}+6=7$

(b) (i) By (a), the remainder when $(8)^{1000}+6$ is divided by $(8)+1=9$ is 7.

(ii) Remainder $=7-6=1$

4B.8 HKCEE MA 1990 – I – 11

(a) (i) $S=2\pi r^2+2\pi rh$

(ii) $V=\pi r^2h \Rightarrow h=\frac{V}{\pi r^2}$
$$\therefore S=2\pi r^2+2\pi r\left(\frac{V}{\pi r^2}\right)=2\pi r^2+\frac{2V}{r}$$

(b) $6\pi=2\pi r^2+\frac{2(2\pi)}{r}$
$$3r=r^3+2 \Rightarrow r^3-3r+2=0$$
Since $(1)^3-3(1)+2=0$, $r-1$ is a factor.
$$\therefore r^3-3r+2=(r-1)(r^2+r-2)=0$$
$$(r-1)(r+2)(r-1)=0$$
$$r=-2\text{ (rej.) or }1$$

4B.9 HKCEE MA 1992 – I – 2(b)

Remainder $=(1)^3-2(1)^2+3(1)-4=-2$

4B.10 HKCEE MA 1993 – I – 2(d)

Remainder $=(1)^3+(1)^2=2$

4B.11 HKCEE MA 1994 – I – 3

Remainder $=k^2=(k+3)(k-2)+2$

$k^2+k-4=k^2 \Rightarrow k=4$

4B.12 HKCEE MA 1995 – I – 2

(a) $(a+b)^2-(a-b)^2=[(a+b)-(a-b)][(a+b)+(a-b)]$

$=(2b)(2a)=4ab$

(b) Remainder $=(-2)^3+1=-7$

4B.13 HKCEE MA 1996 – I – 4

$\because (-1)^3-(-1)^2-3(-1)-1=0$

$\therefore x+1$ is a factor.

$x^3-x^2-3x-1=0$

$(x+1)(x^2-2x-1)=0$

$x=-1$ or $\dfrac{2\pm\sqrt{4+4}}{2}=-1$ or $1\pm\sqrt{2}$

4B.14 HKCEE MA 1998 – I – 9

(a) $\because f(2)=(2)^3+2(2)^2-5(2)-6=0$

$\therefore x-2$ is a factor.

(b) $f(x)=(x-2)(x^2+4x+3)=(x-2)(x+1)(x+3)$

4B.15 HKCEE MA 2000 – I – 6

Remainder $=f(-3)=2(-3)^3+6(-3)^2-2(-3)-7=-1$

4B.16 HKCEE MA 2001 – I – 2

Remainder $=f(2)=(2)^3-(2)^2+(2)-1=5$

4B.17 HKCEE MA 2002 – I – 4

(a) $f(2)=(2)^3-2(2)^2-9(2)+18=0$

(b) $\because f(2)=0$

$\therefore x-2$ is a factor of $f(x)$.

$f(x)=(x-2)(x^2-9)=(x-2)(x-3)(x+3)$

4B.18 HKCEE MA 2005 – I – 10

(a) Let $f(x)=hx^3+kx$.

$\begin{cases}-6=f(2)=8h+2k \Rightarrow 4h+k=-3\\ 6=f(3)=27h+3k \Rightarrow 9h+k=2\end{cases} \Rightarrow \begin{cases}h=1\\ k=-7\end{cases}$

$\therefore f(x)=x^3-7x$

(b) $g(x)=x^3-7x-6$

(i) $\because g(3)=(3)^3-7(3)-6=0$

$\therefore x-3$ is a factor of $g(x)$.

(ii) $g(x)=(x-3)(x^2+3x+2)=(x-3)(x+1)(x+2)$

4B.19 HKCEE MA 2007 – I – 14

(a) (i) $0=f(-3)=4(-3)^3+k(-3)^2-243 \Rightarrow k=39$

(ii) $f(x)=(x+3)(4x^2+27x-81)$

$=(x+3)(4x-9)(x+9)$

4B.20 HKDSE MA SP – I – 10

(a)

$$\begin{array}{r} 5x+2 \\ x^2+2x-3\,\big)\,5x^3+12x^2-9x^2-7 \\ 5x^3+10x^2-15x \\ \hline 2x^2+6x-7 \\ 2x^2+4x-6 \\ \hline 2x-1 \end{array}$$

$\therefore$ Quotient $=5x+2$

(b) (i) From (a),

$5x^3+12x^2-9x-7=(5x+2)(x^2+2x-3)+(2x-1)$

Hence, $(5x^3+12x^2-9x-7)-(2x-1)$ is a multiple of x^2+2x-3.

$\therefore a=2,\ b=-1$

(ii) $(5x+2)(x^2+2x-3)=0$

$x=-\dfrac{2}{5}$ or $(x+3)(x-1)=0 \Rightarrow x=-\dfrac{2}{5}$ or -3 or 1

4B.21 HKDSE MA PP – I – 10

(a) Since it is given that the remainder when $f(x)$ is divided by $x-1$ is 4,

$f(x)=(x-1)(6x^2+17x-2)+4$

$\therefore f(-3)=(-3-1)[6(-3)^2+17(-3)-2]+4=0$

(b) From (a), $x+3$ is a factor of $f(x)$.

$\therefore f(x)=6x^3+11x^2-19x+6$

$=(x+3)(6x^2-7x+2)=(x+3)(3x-1)(x-2)$

4B.22 HKDSE MA 2012 – I – 13

(a) $0=k(2)^3-21(2)^2+24(2)-4 \Rightarrow k=5$

4B.23 HKDSE MA 2013 – I – 12

(a) Given: $x-2$ is a factor.

$\therefore 0=3(2)^3-7(2)^2+k(2)-8 \Rightarrow k=6$

Hence, $f(x)=3x^3-7x^2+6x-8=(x-2)(3x^2-x+4)$

$\Rightarrow a=3,\ b=-1,\ c=4$

(b) Δ of $3x^2-x+4=-47<0$

$\therefore$ Roots for $3x^2-x+4=0$ are not real.

Hence, $f(x)=0$ only has 1 real root. Disagreed.

4B.24 HKDSE MA 2014 – I – 7

(a) $-33=f(2)=32-20-36+c \Rightarrow c=-9$

$\Rightarrow f(x)=4x^3-5x^2-18x-9$

$\because f(-1)=-4-5+18-9=0,$

$\therefore x+1$ is a factor of $f(x)$.

(b) $f(x)=(x+1)(4x^2-9x-9)=(x+1)(4x+3)(x-3)$

$\therefore$ The roots are -1, $\dfrac{-3}{4}$ and 3, which are all rational. Yes.

4B.25 HKDSE MA 2015 – I – 11

(a) $\begin{cases}-5=f(2)=k\\ 0=f(3)=(3-2)^2(3+h)+k\end{cases} \Rightarrow \begin{cases}h=2\\ k=-5\end{cases}$

(b) $f(x)=(x-2)^2(x+2)-5=x^3-2x^2-3x+3$

$=(x-3)(x^2+x-1)$

$\therefore$ The roots of $f(x)=0$ are 3 and $\dfrac{-1\pm\sqrt{1+4}}{2}=\dfrac{-1\pm\sqrt{5}}{2}$, which are not integers. Disagreed.

4B.26 HKDSE MA 2016 – I – 14

(a) $p(-2)=p(2)$

$96-56+4a-2b+c=96+56+4a+2b+c$

$b=-28$

Thus, we have

$6x^4+7x^3+ax^2-28x+c\equiv(lx^2+5x+8)(2x^2+mx+n)$

$\Rightarrow \begin{cases}6=2l \Rightarrow l=3\\ 7=(3)m+10 \Rightarrow m=-1\\ -28=8(-1)+5n \Rightarrow n=-4\end{cases}$

(b) $p(x)=(3x^2+5x+8)(2x^2-x-4)$

Δ of $3x^2+5x+8=-71<0 \Rightarrow$ No real root

Δ of $2x^2-x-4=33<0 \Rightarrow$ 2 distinctreal roots

$\therefore p(x)=0$ has 2 real roots.

4B.27 HKDSE MA 2017 – I – 14

(a) Usi ngthedi visionalalgorithm,

$f(x)\equiv(3x+7)(2x^2+ax+4)+(bx+c) \Rightarrow$

$6x^3-13x^2-46x+34\equiv(3x+7)(2x^2+ax+4)+(bx+c)$

Method 1

Expand and compare coefficients of like terms.

Method 2

$\begin{cases}f(0)=34=28+c \Rightarrow c=6\\ f(1)=-19=10(6+a)+(b+6) \Rightarrow 10a+b=-85\\ f(2)=-62=13(12+2a)+(2b+6)\Rightarrow 13a+b=-112\end{cases}$

$\Rightarrow b=5,\ a=-9$

(b) (i) $\begin{cases}f(x)=(3x+7)(2x^2-9x+4)+(bx+c)\\ g(x)=k(2x^2-9x+4)+(bx+c)\end{cases}$

$f(x)-g(x)=(3x+7)(2x^2-9x+4)-k(2x^2-9x+4)$

$=(2x^2-9x+4)(3x+7-k)$,

which has a factor of $2x^2-9x+4$ indeed.

(ii) Roots of $2x^2-9x+4=(2x-1)(x-4)$ are 4 and $\dfrac{1}{2}$, whichis not an integer. Disagreed.

4B.28 HKDSE MA 2018 – I – 12

(a) $\begin{cases}0=f(3)=192+3a+b \Rightarrow 3a+b=-192\\ 2b+165=f(-2)=-8-2a+b \Rightarrow 2a+b=-173\end{cases}$

$\Rightarrow \begin{cases}a=-19\\ b=-135\end{cases}$

(b) $f(x)=4x(x+1)^2-19x-135=4x^3+8x^2-15x-135$

$=(x-3)(4x^2+20x+45)$

Roots of $f(x)=0$ are 3 and $\dfrac{-20\pm\sqrt{400-720}}{8}$ which are unreal. Disagreed.

4B.29 HKDSE MA 2019 – I – 11

(a) Let $p(x)=(ax+b)(2x^2+9x+14)$.

$\begin{cases}50=p(1)=25(a+b) \Rightarrow a+b=2\\ -52=p(-2)=4(-2a+b) \Rightarrow 2a-b=-13\end{cases}$

$\Rightarrow \begin{cases}a=5\\ b=-3\end{cases} \Rightarrow$ Required quotient $=ax+b=5x-3$

(b) $p(x)=0 \Rightarrow 5x-3=0$ or $2x^2+9x+14=0$

$\because \Delta$ of $2x^2+9x+14=-31<0$

$\therefore 2x^2+9x+14=0$ has no real rt. and thus no rational rt.

$\therefore$ The only real root of $p(x)=0$ is $\dfrac{3}{5}$ whichi srati onal.

i.e. There is 1 rati onal root.