17 Counting Principles and Probability

17A Counting principles

17A.1 HKALE MS 1995 – 3

A teacher wants to divide a class of 18 students into 3 groups, each of 6 students, to do 3 different statistical projects.

(a) In how many ways can the students be grouped?

(b) If there are 3 girls in the class, find the probability that there is one girl in each group.

17A.2 HKALE MS 1999 – 6

At a school sports day, the timekeeping group for running events consists of 1 chief judge, 1 referee and 10 timekeepers. The chief judge and the referee are chosen from 5 teachers while the 10 timekeepers are selected from 16 students.

(a) How many different timekeeping groups can be formed?

(b) If it is possible to have a timekeeping group with all the timekeepers being boys, what are the possible numbers of boys among the 16 students?

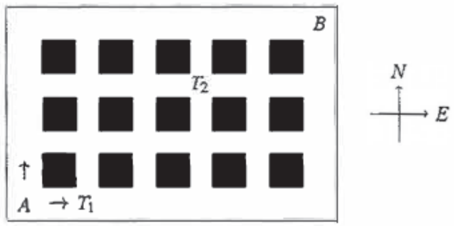
(c) *[Out of syllabus]*

17A.3 HKALE MS 2011 – 5 (Continued from 17B.31.)

The figure shows a board with routes blocked by shaded squares for an electronic toy car that goes from A to B. At each junction, the toy car will go either East or North as shown by the arrows at A. The toy car will choose randomly a route from A to B. There may be traps being set at some junctions. If the car reaches a trapped junction, it will stop and cannot reach B.

(a) If a trap is set at T_1, how many different routes are there for the toy car to go from A to B?

(b) If a trap is set at T_2, how many different routes are there for the toy car to go from A to B?



17A.4 HKDSE MA 2018 – I – 15

An eight-digit phone number is formed by a permutation of 2, 3, 4, 5, 6, 7, 8 and 9.

(a) How many different eight-digit phone numbers can be formed?

(b) If the first digit and the last digit of an eight-digit phone number are odd numbers, how many different eight digit phone numbers can be formed?

17A.5 HKDSE MA 2019 – I – 15

There are 21 boys and 11 girls in a class. If 5 students are selected from the class to form a committee consisting of at least 1 boy, how many different committees can be formed?

17B Probability (short questions)

17B.1 HKCEE MA 1981(1/3) – I – 3

There are 40 students in a class, including students A and B. If two students are to be chosen at random as class representatives, find the probability that both A and B are chosen.

17B.2 HKCEE MA 1982(1/3) – I – 6

If two dice are thrown once, find the probability that the sum of the numbers on the dice is

(a) equal to 4,

(b) less than 4,

(c) greater than 4.

17B.3 HKCEE MA 1996 – I – 7

The figure shows a circular dartboard. Its surface consists of two concentric circles of radii 12 cm and 2 cm respectively.

(a) Find the area of the shaded region on the dartboard.

(b) Two darts are thrown and hit the dartboard. Find the probability that

(i) both darts hit the shaded region;

(ii) only one dart hits the shaded region.

17B.4 HKCEE MA 1998 – I – 11

There are 8 white socks, 4 yellow socks and 2 red socks in a drawer. A boy randomly takes out 2 socks from the drawer.

(a) Find the probability that the socks taken out are both white.

(b) Find the probability that the socks taken out are of the same colour.

17B.5 HKCEE MA 1999 – I – 12

Mr. Sun is waiting for a bus at a bus stop. It is known that 75% of the buses are air-conditioned, of which 20% have Octopus machines installed. No Octopus machines have been installed on buses without air-conditioning.

(a) Find the probability that the next bus has an Octopus machine installed.

(b) The bus fare is \$3.00. Mr. Sun does not have an Octopus card but has two 1-dollar coins and three 2-dollar coins in his pocket. If he randomly takes out two coins, what is the probability that the total value of these coins is exactly \$3.00?

17B.6 HKCEE MA 2000 – I – 12

A box contains nine hundred cards, each marked with a different 3-digit number from 100 to 999. A card is drawn randomly from the box.

(a) Find the probability that two of the digits of the number drawn are zero.

(b) Find the probability that none of the digits of the number drawn is zero.

(c) Find the probability that exactly one of the digits of the number drawn is zero.

17B.7 HKCEE MA 2004 – I – 8

A box contains nine cards numbered 1, 2, 3, 4, 5, 6, 7, 8 and 9 respectively.

(a) If one card is randomly drawn from the box, find the probability that the number drawn is odd.

(b) If two cards are randomly drawn from the box one by one with replacement, find the probability that the product of the numbers drawn is even.

17B.8 HKCEE MA 2006 – I – 8

(Continued from 18B.11.)

There are ten cards numbered 2, 3, 5, 8, 11, 11, 12, 15, 19 and k respectively, where k is a positive integer. It is given that the mean of the ten numbers is 11.

(a) Find the value of k.

(b) A card is randomly drawn from the ten cards. Find the probability that the number drawn is a multiple of 3.

17B.9 HKCEE MA 2008 – I – 5

A box contains three cards numbered 2, 3 and 4 respectively while a bag contains two balls numbered 6 and 7 respectively. If one card and one ball are randomly drawn from the box and the bag respectively, find the probability that the sum of the numbers drawn is 10.

17B.10 HKCEE MA 2009 – I – 5

The table below shows the distribution of the ages of all employees in a department of a company.

Employee \ Age (x)	$x < 30$	$30 \le x < 40$	$x \ge 40$
Administrative officer	7	21	30
Clerk	53	57	32

If an employee is randomly selected from the department, find the probability that the selected employee is an administrative officer under the age of 40.

17B.11 HKALE MS 1994 – 1

(a) Write down the sample space of the sex patterns of the children of a 2-child family in the order of their ages. (You may use B to denote a boy and G to denote a girl.)

(b) Assume that having a boy or having a girl is equally likely. It is known that a family has two children and they are not both girls.

(i) Write down the sample space of the sex patterns of the children in the order of their ages.

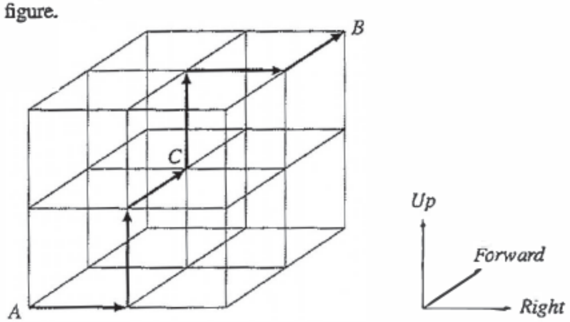
(ii) What is the probability that the family has two sons?

17B.12 HKALE MS 1994 – 3

Jack climbs along a cubical framework from a corner A to meet Jill at the opposite corner B. The framework, shown in the figure, is formed by joining bars of equal length. Jack chooses randomly a path of the shortest length to meet Jill. An example of such a path, which can be denoted by

$$Right - Up - Forward - Up - Right - Forward$$

is also shown in the figure.



(a) Find the number of shortest paths from A to B.

(b) If there is a trap at the centre C of the framework which catches anyone passing through it,

(i) find the number of shortest paths from A to C,

(ii) hence find the probability that Jack will be caught by the trap on his way to B.

17B.13 HKALE MS 1994 – 7

In asking some sensitive questions such as "Are you homosexual?", a *randomised response technique* can be applied: The interviewee will be asked to draw a card at random from a box with 1 red card and 2 black cards and then consider the statement 'I am homosexual' if the card is red and the statement 'I am not homosexual' otherwise. He will give the response either 'True' or 'False'. The colour of the card drawn is only known to the interviewee so that nobody knows which statement he has responded to. Suppose in a survey, 790 out of 1200 interviewees give the response 'True'.

(a) Estimate the percentage of persons who are homosexual.

(b) For an interviewee who answered 'True', what is the probability that he is really homosexual?

17B.14 HKALE MS 1995 – 5

An insurance company classifies the aeroplanes it insures into class L (low risk) and class H (high risk), and estimates the corresponding proportions of the aeroplanes as 70% and 30% respectively. The company has also found that 99% of class L and 88% of class H aeroplanes have no accident within a year. If an aeroplane insured by the company has no accident within a year, what is the probability that it belongs to

(a) class H?

(b) class L?

17B.15 HKALE MS 1996 – 6

A company buys equal quantities of fuses, in 100-unit lots, from two suppliers A and B. The company tests two fuses randomly drawn from each lot, and accepts the lot if both fuses are non-defective.
It is known that 4% of the fuses from supplier A and 1% of the fuses from supplier B are defective. Assume that the quality of the fuses are independent of each other.

(a) What is the probability that a lot will be accepted?

(b) What is the probability that an accepted lot came from supplier A?

17B.16 HKALE MS 1997 – 7

A brewery has a backup motor for its bottling machine. The backup motor will be automatically turned on if the original motor breaks down during operating hours. The probability that the original motor breaks down during operating hours is 0.15 and when the backup motor is turn on, it has a probability of 0.24 of breaking down. Only when both the original and backup motors break down is the machine not able to work.

(a) What is the probability that the machine is not working during operating hours?

(b) If the machine is working, what is the probability that it is operated by the original motor?

(c) The machine is working today. Find the probability that the first break down of the machine occurs on the 10th day after today.

17B.17 HKALE MS 1998 – 6

A factory produces 3 kinds of ice cream bars A, B and C in the ratio 1 : 2 : 5. It was reported that some ice cream bars produced on 1 May, 1998 were contaminated. All ice-cream bars produced on that day were withdrawn from sale and a test was carried out. The test results showed that 0.8% of kind A, 0.2% of kind B and 0% of kind C were contaminated.

(a) An ice-cream bar produced on that day is selected randomly. Find the probability that

(i) the bar is of kind A and is NOT contaminated,

(ii) the bar is NOT contaminated.

(b) If an ice cream bar produced on that day is contaminated, find the probability that it is of kind A.

17B.18 HKALE MS 1999 5

60% of passengers who travel by train use Octopus. A certain train has 12 compartments and there are 10 passengers in each compartment.

(a) What is the probability that exactly 5 of the passengers in a compartment use Octopus?
(b) What is the expected number of passengers using Octopus in a compartment?
(c) What is the probability that the third compartment is the first one to have exactly 5 passengers using Octopus?

17B.19 HKALE MS 2000 6

Mr. Chan has 6 cups of ice-cream in his refrigerator. There are 5 different flavours as listed: 1 cup of chocolate, 1 cup of mango, 1 cup of peach, 1 cup of strawberry and 2 cups of vanilla. Mr. Chan randomly chooses 3 cups of the ice-cream. Find the probability that

(a) there is no vanilla flavour ice-cream,
(b) there is exactly 1 cup of vanilla flavour ice-cream.

17B.20 HKALE MS 2000 – 8

A department store uses a machine to offer prizes for customers by playing games A or B. The probability of a customer winning a prize in game A is $\frac{5}{9}$ and that in game B is $\frac{5}{6}$. Suppose each time the machine randomly generates either game A or game B with probabilities 0.3 and 0.7 respectively.

(a) Find the probability of a customer winning a prize in 1 trial.
(b) The department store wants to adjust the probabilities of generating game A and game B so that the probability of a customer winning a prize in 1 trial is $\frac{2}{3}$. Find the probabilities of generating game A and game B respectively.

17B.21 HKALE MS 2001 – 6

3 students are randomly selected from 10 students of different weights. Find the probability that

(a) the heaviest student is in the selection,
(b) the heaviest one out of the 3 selected students is the 4th heaviest among the 10 students,
(c) the 2 heaviest students are not both selected.

17B.22 HKALE MS 2001 – 7

In the election of the Legislative Council, 48% of the voters support Party A, 39% Party B and 13% Party C. Suppose on the polling day, 65%, 58% and 50% of the supporting voters of Parties A, B and C respectively cast their votes.

(a) A voter votes on the polling day. Find the probability that the voter supports Party B.
(b) Find the probability that exactly 2 out of 5 voting voters support Party B.

17B.23 HKALE MS 2002 – 5

Twelve boys and ten girls in a class are divided into 3 groups as shown in the table below.

	Group A	Group B	Group C
Number of boys	6	4	2
Number of girls	2	3	5

To choose a student as the class representative, a group is selected at random, then a students is chosen at random from the selected group.

(a) Find the probability that a boy is chosen as the class representative.
(b) Suppose that a boy is chosen as the class representative. Find the probability that the boy is from Group A.

17B.24 HKALE MS 2002 – 8

A flower shop has 13 roses of which 2 are red, 5 are white and 6 are yellow. Mary selects 3 roses randomly and the colours are recorded.

(a) Denote the red rose selected by R, the white rose by W and the yellow rose by Y.
List the sample space (i.e. the set of all possible combinations of the colours of roses selected, for example, $1R2W$ denotes that 1 red rose and 2 white roses are selected).
(b) Find the probability that Mary selects exactly one red rose.
(c) Given that Mary has selected exactly one red rose, find the probability that only one of the other two roses is white.

17B.25 HKALE MS 2003 – 12

A teacher randomly selected 7 students from a class of 13 boys and 17 girls to form a group to take part in a flag selling activity.

(a) Find the probability that the group consists of at least 1 boy and 1 girl.
(b) Given that the group consists of at least 1 boy and 1 girl, find the probability that there are more than 3 girls in the group.
(c) *[Out of syllabus]*

17B.26 HKALE MS 2004 – 6

David has forgotten his uncle's mobile phone number. He can only remember that the phone number is $98677XYZ$, where X, Y and Z are the forgotten digits. Find the probability that

(a) at least 2 of the forgotten digits are different;
(b) the forgotten digits are permutations of 2, 3 and 8;
(c) exactly 2 of the forgotten digits have already appeared among the first five digits of the phone number.

17B.27 HKALE MS 2004 – 10

A certain test gives a positive result in 94% of the people who have disease S. The test gives a positive result in 14% of the people who do not have disease S. In a city, 7.5% of the citizens have disease S.

(a) Find the probability that the test gives a positive result for a randomly selected citizen.
(b) Given that the test gives a positive result for a randomly selected citizen, find the probability that the citizen does not have disease S.
(c) *[Out of syllabus]*

17B.28 HKALE MS 2007 – 6

David has 10 shirts and 3 bags: 1 blue shirt, 4 yellow shirts, 5 white shirts, 1 yellow bag and 2 white bags. He randomly chooses 3 shirts from the 10 shirts and randomly puts the chosen shirts into 3 bags so that each bag contains 1 shirt.

(a) Find the probability that the yellow bag contains the blue shirt and each of the two white bags contains 1 yellow shirt.
(b) Find the probability that each of these three bags contains 1 shirt of a colour different from the bag.

17B.29 HKALE MS 2009 – 5

It is known that 36% of the customers of a certain supermarket will bring their own shopping bags. There are 3 cashiers and each cashier has 5 customers in queue.

(a) Find the probability that among all the customers in queue, at least 4 of them have brought their own shopping bags.
(b) If exactly 4 customers in queue have brought their own shopping bags, what is the probability that each cashier will have at least 1 customer who has brought his/her own shopping bag?

17B.30 HKALE MS 2011 4

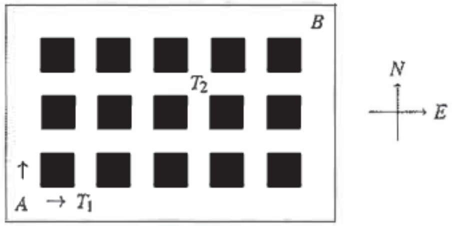
Peter and Susan play a shooting game. Each of them will shoot a target twice. Each shot will score 1 point if it hits the target. The one who has a higher score is the winner. It is known that the probabilities of hitting the target in one shot for Peter and Susan are 0.55 and 0.75 respectively.

(a) Find the probability that Susan will be the winner.

(b) Given that Peter scores at least 1 point, what is the probability that Susan is the winner?

17B.31 HKALE MS 2011 5 (Continued from **17A.3**.)

The figure shows a board with routes blocked by shaded squares for an electronic toy car that goes from A to B. At each junction, the toy car will go either East or North as shown by the arrows at A. The toy car will choose randomly a route from A to B. There may be traps being set at some junctions. If the car reaches a trapped junction, it will stop and cannot reach B.



(a) If a trap is set at T_1, how many different routes are there for the toy car to go from A to B?

(b) If a trap is set at T_2, how many different routes are there for the toy car to go from A to B?

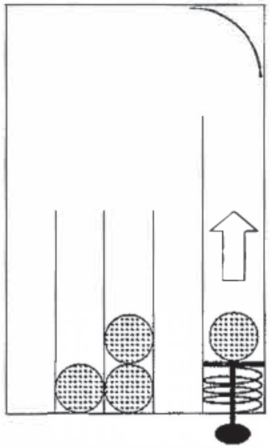
(c) If two traps are set at T_1 and T_2, find the probability that the toy car can reach B from A.

17B.32 HKALE MS 2013 4

In a game, a player will ping 4 balls one by one and each ball will randomly fall into 4 different slots as shown in the figure. A prize will be given if all the 4 balls are aligned in a horizontal or a vertical row.

(a) What is the probability that a player wins the prize?

(b) What is the probability that a player wins the prize given that first two balls are in two different slots?



17B.33 HKDSE MA SP – I – 16

A committee consists of 5 teachers from school A and 4 teachers from school B. Four teachers are randomly selected from the committee.

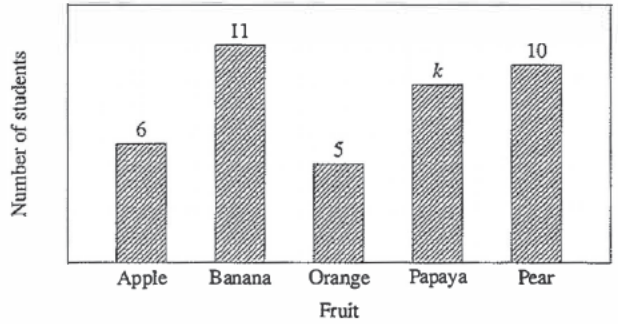
(a) Find the probability that only 2 of the selected teachers are from school A.

(b) Find the probability that the numbers of selected teachers from school A and school B are different.

17B.34 HKDSE MA PP – I – 13 (To continue as **18A.9**.)

The bar chart below shows the distribution of the most favourite fruits of the students in a group. It is given that each student has only one most favourite fruit.

Distribution of the most favourite fruits of the students in the group



If a student is randomly selected from the group, the probability that the most favourite fruit is apple is $\frac{3}{20}$.

(a) Find k.

(b) Suppose that the above distribution is represented by a pie chart.

17B.35 HKDSE MA PP – I – 16

There are 18 boys and 12 girls in a class. From the class, 4 students are randomly selected to form the class committee.

(a) Find the probability that the class committee consists of boys only.

(b) Find the probability that the class committee consists of at least 1 boy and 1 girl.

17B.36 HKDSE MA 2012 I – 16

There are 8 departments in a company. To form a task group of 16 members, 2 representatives are nominated by each department. From the task group, 4 members are randomly selected.

(a) Find the probability that the 4 selected members are nominated by 4 different departments.

(b) Find the probability that the 4 selected members are nominated by at most 3 different departments.

17B.37 HKDSE MA 2013 – I – 16

A box contains 5 white cups and 11 blue cups. If 6 cups are randomly drawn from the box at the same time,

(a) find the probability that at least 4 white cups are drawn;

(b) find the probability that at least 3 blue cups are drawn.

17B.38 HKDSE MA 2015 – I – 3

Bag A contains four cards numbered 1, 3, 5 and 7 respectively while bag B contains five cards numbered 2, 4, 6, 8 and 10 respectively. If one card is randomly drawn from each bag, find the probability that the sum of the two numbers drawn is less than 9.

17B.39 HKDSE MA 2015 – I – 16

A box contains 5 red bowls, 6 yellow bowls and 3 white bowls. If 4 bowls are randomly drawn from the box at the same time,

(a) find the probability that exactly 2 red bowls are drawn;

(b) find the probability that at least 2 red bowls are drawn.

17B.40 HKDSE MA 2016 – I – 9 (Continued from **18A.10**.)

The frequency distribution table and the cumulative frequency distribution table below show the distribution of the heights of the plants in a garden.

Height (m)	Frequency
0.1 – 0.3	a
0.4 – 0.6	4
0.7 – 0.9	b
1.0 – 1.2	c
1.3 – 1.5	15
1.6 – 1.8	3

Height less than (m)	Cumulative frequency
0.35	2
0.65	x
0.95	13
1.25	y
1.55	37
1.85	z

(a) Find x, y and z.

(b) If a plant is randomly selected from the garden, find the probability that the height of the selected plant is less than 1.25 m but not less than 0.65 m.

17B.41 HKDSE MA 2016 – I – 15

If 4 boys and 5 girls randomly form a queue, find the probability that no boys are next to each other in the queue.

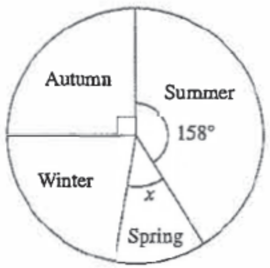
17B.42 HKDSE MA 2017 – I – 7

The pie chart shows the distribution of the seasons of birth of the students in a school.

If a student is randomly selected from the school, then the probability that the selected student was born in spring is $\frac{1}{9}$.

(a) Find x.

(b) In the school, there are 180 students born in winter. Find the number of students in the school.



Distribution of the seasons of birth of the students in a school

17B.43 HKDSE MA 2017 – I – 11 (Continued from **18C.48**.)

The stem-and leaf diagram shows the distribution of the hourly wages (in dollars) of the workers in a group.

It is given that the mean and the range of the distribution are \$70 and \$22 respectively.

Stem (tens)	Leaf (units)
6	1 1 1 3 4 6 8 9 9
7	a 7 7 8
8	1 b

(a) Find the median and the standard deviation of the above distribution.

(b) If a worker is randomly selected from the group, find the probability that the hourly wage of the selected worker exceeds \$70.

17B.44 HKDSE MA 2017 – I – 17

In a bag, there are 4 green pens, 7 blue pens and 8 black pens. If 5 pens are randomly drawn from the bag at the same time,

(a) find the probability that exactly 4 green pens are drawn;

(b) find the probability that exactly 3 green pens are drawn;

(c) find the probability that not more than 2 green pens are drawn.

17B.45 HKDSE MA 2018 – I – 4

A box contains n white balls, 5 black balls and 8 red balls. If a ball is randomly drawn from the box, then the probability of drawing a red ball is $\frac{2}{5}$. Find the value of n.

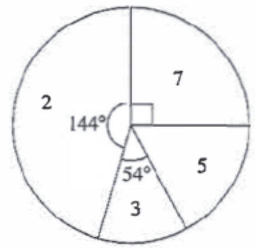
17B.46 HKDSE MA 2019 – I – 8 (Continued from **18B.19**.)

The pie chart below shows the distribution of the numbers of rings owned by the girls in a group.

(a) Write down the mode of the distribution.

(b) Find the mean of the distribution.

(c) If a girl is randomly selected from the group, find the probability that the selected girl owns more than 3 rings.



Distribution of the numbers of rings owned by the girls in the group

17B.47 HKDSE MA 2020 – I – 15

In a box, there are 3 blue plates, 7 green plates and 9 purple plates. If 4 plates are randomly selected from the box at the same time, find

(a) the probability that 4 plates of the same colour are selected; (3 marks)

(b) the probability that at least 2 plates of different colours are selected. (2 marks)

17C Probability (structural questions)

17C.1 HKCEE MA 1980(1/3) – I – 14

The examination for a professional qualification consists of a theory paper and a practical paper. To obtain the qualification, a candidate has to pass both papers. If a candidate fails in either paper, he needs only sit that paper again.

The probabilities of passing the theory paper for two candidates A and B are both $\frac{9}{10}$ and their probabilities of passing the practical paper are both $\frac{2}{3}$. Find the probabilities of the following events:

(a) Candidate A obtaining the qualification by sitting each paper only once.

(b) Candidate A failing in one of the two papers but obtaining the qualification with one re examination.

(c) At least one of the candidates A and B obtaining the qualification without any re examination.

17C.2 HKCEE MA 1983(A/B) I 11

In a short test, there are 3 questions. For each question, 1 mark will be awarded for a correct answer and no marks for a wrong answer. The probability that John correctly answers a question in the test is 0.6. Find the probability that

(a) John gets 3 marks in the test,

(b) John gets no marks in the test,

(c) John gets 1 mark in the test.

(d) John gets 2 marks in the test.

17C.3 HKCEE MA 1984(A/B) I – 11

(a) There are two bags. Each bag contains 1 red, 1 black and 1 white ball. One ball is drawn randomly from each bag. Find the probability that

  (i) the two balls drawn are both red;

  (ii) the two balls drawn are of the same colour;

  (iii) the two balls drawn are of different colours.

(b) A box contains 2 red, 2 black and 3 white balls. One ball is drawn randomly from the box. After putting the ball back into the box, one ball is again drawn randomly. Find the probability that

  (i) the two balls drawn are both red;

  (ii) the two balls drawn are of the same colour;

  (iii) the two balls drawn are of different colours.

17C.4 HKCEE MA 1985(A/B) – I – 10

(a) If two dice are thrown,

  (i) find the probability that the sum of the numbers on the two dice is greater than 9;

  (ii) find the probability that the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal.

(b) In a game, two dice are thrown. In each throw, 2 points are gained if the sum of the numbers on the two dice is greater than 9 or the numbers on the two dice are equal; otherwise 1 point is lost. Using the result in (a)(ii), find the probability of

  (i) losing a total of 2 points in two throws,

  (ii) gaining a total of 1 point in two throws.

17C.5 HKCEE MA 1986(A/B) I 13

A box contains wooden blocks of 5 different shapes A, B, C, D and E. For each shape, there are 5 different colours red, orange, yellow, green and blue. For each colour of each shape, there is one block of each of the sizes L, M and S. (Hint: There are altogether 75 blocks in the box.)

(a) When a block is picked out randomly from the box, what is the probability that it is of

  (i) red colour?

  (ii) blue colour and shape C?

  (iii) size S, shape A or E but not yellow?

(b) Two blocks are drawn at random from the box, one after the other. The first block drawn is put back into the box before the second is drawn. Find the probability that

  (i) the first block drawn is of size L and the second block is of size S,

  (ii) one of the blocks drawn is of size L and the other of size S,

  (iii) the two blocks drawn are of different sizes.

17C.6 HKCEE MA 1987(A/B) – I – 13

P, Q and R are three bags. P contains 1 black ball, 2 green balls and 3 white balls; Q contains 4 green balls; R contains 5 white balls. A ball is drawn at random from P and is put into Q; then a ball is drawn at random from Q and is put into R. Find the probability that

(a) the black ball still remains in P,

(b) the black ball is in Q,

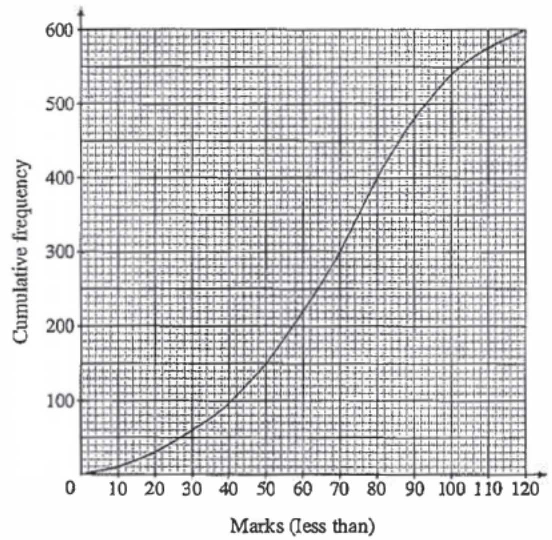
(c) the black ball is in R,

(d) all the balls in R are white.

17C.7 HKCEE MA 1988 – I – 11

(Continued from 18C.4.)

The figure shows the cumulative frequency curve of the marks of 600 students in a mathematics contest.



(a) From the curve, find

  (i) the median, and

  (ii) the interquartile range of the distribution of marks.

(b) A student with marks greater than or equal to 100 will be awarded a prize.

  (i) Find the number of students who will be awarded prizes.

  (ii) If one student is chosen at random from the 600 students, find the probability that the student is a prize-winner.

  (iii) If two students are chosen at random, find the probability that

    (1) both of them are prize-winners,

    (2) at least one of them is a prize winner.

17C.8 HKCEE MA 1989 – I – 13

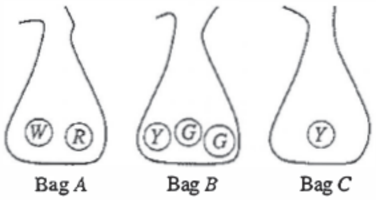
(a) Bag A contains a number of balls. Some are black and the rest are white. A ball is drawn at random from bag A. Let p be the probability that the ball drawn is black and q be the probability that the ball drawn is white. If $p = 3q$, find q.

(b) Bag C contains 10 balls of which n $(2 \le n \le 10)$ balls are black.
  (i) If two balls are drawn at random from bag C, find the probability, in terms of n, that both balls are black.
  (ii) If the probability obtained in (i) is greater that $\frac{1}{3}$, find the possible values of n.

(c) Bag M contains 1 red and 1 green ball. Bag N contains 3 red and 2 green balls. A ball is drawn at random from bag M and put into bag N; then a ball is drawn at random from bag N. Find the probability that the ball drawn from bag N is red.

17C.9 HKCEE MA 1990 – I – 13

The figure shows 3 bags A, B and C.
Bag A contains 1 white ball (W) and 1 red ball (R).
Bag B contains 1 yellow ball (Y) and 2 green balls (G).
Bag C contains only 1 yellow ball (Y).



(a) Peter choose one bag at random and then randomly draws one ball from the bag. Find the probability that
  (i) the ball drawn is green;
  (ii) the ball drawn is yellow.

(b) After Peter has drawn a ball in the way described in (a), he puts it back into the original bag. Next, Alice chooses one bag at random and then randomly draws one ball from the bag. Find the probability that
  (i) the balls drawn by Peter and Alice are both green;
  (ii) the balls drawn by Peter and Alice are both yellow and from the same bag.

17C.10 HKCEE MA 1991 I 10

The practical test for a driving licence consists of two independent parts, A and B. To pass the practical test, a candidate must pass in both parts. If a candidate fails in any one of these parts, the candidate may take that part again. Statistics shows that the passing percentages for Part A and Part B are 70% and 60% respectively.

(a) A candidate takes the practical test. Find the probabilities that the candidate
  (i) fails Part A on the first attempt and passes it on the second attempt,
  (ii) passes Part A in no more than two attempts,
  (iii) passes the practical test in no more than two attempts in each part.

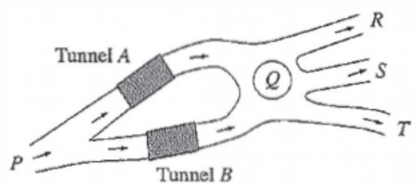
(b) In a sample of 10 000 candidates taking the practical test, how many of them would you expect to pass the practical test in no more than two attempts in each part?

17C.11 HKCEE MA 1992 – I – 10

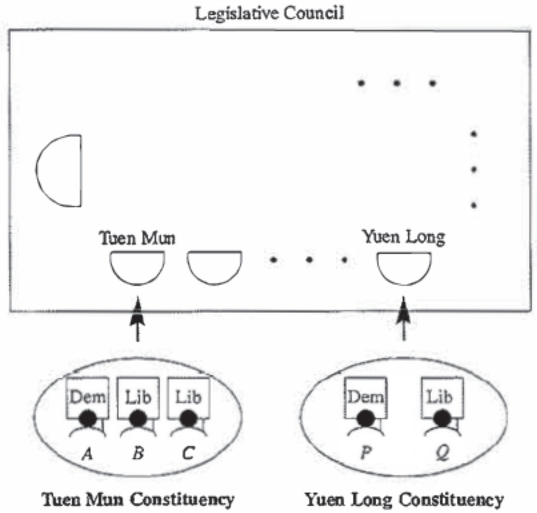
The figure shows a one way road network system from Town P to Towns R, S and T. Any car leaving Town P will pass though either Tunnel A or Tunnel B and arrive at Towns R, S or T via the roundabout Q. A survey shows that $\frac{2}{5}$ of the cars leaving P will pass through Tunnel A. The survey also shows that $\frac{1}{7}$ of all the cars passing through the roundabout Q will arrive at R, $\frac{2}{7}$ at S, and $\frac{4}{7}$ at T.

(a) Find the probabilities that a car leaving P will
  (i) pass through Tunnel B,
  (ii) not arrive at T,
  (iii) arrive at R through Tunnel B,
  (iv) pass through Tunnel A but not arrive at R.

(b) Two cars leave P. Find the probabilities that
  (i) one of them arrives at R and the other one at S,
  (ii) both of them arrive at S, one through Tunnel A and the other one through Tunnel B.



17C.12 HKCEE MA 1993 – I – 13



In a Legislative Council election, each registered voter in a constituency (i.e. district) could select only one candidate in that constituency and cast one vote for that candidate. The candidate who got the greatest number of valid votes won the election in that constituency.

In the Tuen Mun constituency, there were 3 candidates, A, B and C. A belonged to a political party called 'The Democrats'; B and C belonged to a political party called 'The Liberals'.

In the Yuen Long constituency, there were 2 candidates, P and Q. P belonged to 'The Democrats' and Q belonged to 'The Liberals'.

(a) A survey conducted before the election showed that the probabilities of winning the election for A, B and C were respectively 0.65, 0.25 and 0.1 while the probabilities of winning the election for P and Q were respectively 0.45 and 0.55. Calculate from the above data the following probabilities:
  (i) The elections in the Tuen Mun and Yuen Long constituencies would both be won by 'The Democrats'.
  (ii) The elections in the Tuen Mun and Yuen Long constituencies would both be won by the same party.

(b) After the election, it was found that in the Tuen Mun constituency there were 40 000 valid votes of which A got 70%, B got 20% and C got 10%; in the Yuen Long constituency, there were 20 000 valid votes of which P got 40% and Q got 60%. Suppose two votes were chosen at random (one after the other with replacement) from the 60 000 valid votes in the two constituencies. What would be the probability that
  (i) both votes came from the Tuen Mun constituency and were for 'The Democrats',
  (ii) both votes were for 'The Democrats',
  (iii) the votes were for different parties?

17C.13 HKCEE MA 1994 – I – 9

Siu Ming lives in Tuen Mun. He travels to school either by LRT (Light Railway Transit) or on foot. The probability of being late for school is $\frac{1}{7}$ if he travels by LRT and $\frac{1}{10}$ if he travels on foot.

(a) In a certain week, Siu Ming travels to school by LRT on Monday, Tuesday and Wednesday. Find the probability that
  (i) he will be late on all these three days;
  (ii) he will not be late on all these three days.

(b) In the same week, Siu Ming travels to school on foot on Thursday, Friday and Saturday. Find the probability that
  (i) he will be late on Thursday and Friday only in these three days;
  (ii) he will be late on any two of these three days.

(c) On Sunday, Siu Ming goes to school to take part in a basketball match. If he is equally likely to travel by LRT or on foot, find the probability that he will be late on that day.

17C.14 HKCEE MA 1995 – I – 11

If Wai Ming studies in the evening for a test the next day, the probability of him passing the test is $\frac{4}{5}$. If he does not study in the evening for the test, he will certainly fail.

(a) **(You may use Figure (1) to help you answer this part.)**
  (i) If Wai Ming studies in the evening for a test the next day, find the probability p that he will fail the test.
  (ii) If Wai Ming does not study in the evening for a test the next day, find the probability q that he will pass the test and the probability r that he will fail the test.

(b) **(You may use Figure (1) and Figure (2) to help you answer this part.)**
There are four teams competing for the World Women's Volleyball Championship (WWVC) with two games in the semi finals: China against U.S.A. and Japan against Cuba.
The winner of each game will be competing in the final for the Championship. The four teams have an equal chance of beating their opponents.
  (i) Find the probability that China will win the Championship.
  (ii) The final of the WWVC will be shown on television on a Sunday evening and Wai Ming has a test the next day. Wai Ming will definitely watch the TV programme if China gets to the final and the probability of him studying afterwards for the test is $\frac{1}{3}$. If China fails to get to the final, he will not watch that programme at all and will study for the test.
    (1) Find the probability that Wai Ming will study for the test.
    (2) Find the probability that Wai Ming will pass the test.

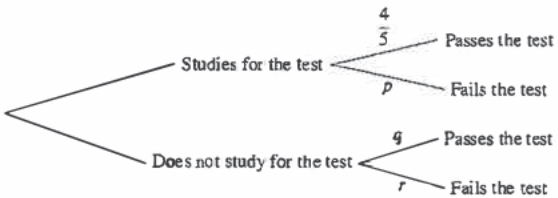


Figure (1)

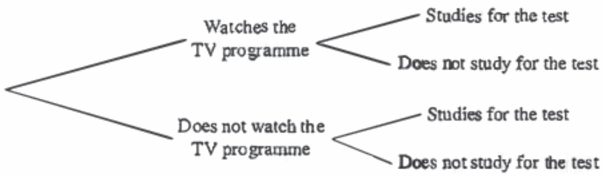


Figure (2)

17C.15 HKCEE MA 1997 – I – 14

In a small pond, there were exactly 40 *small* fish and 10 *large* fish. The ranges of their weights W g are shown in the table.
In the morning on a certain day, a man went fishing in the pond. He caught two fish and their total weight was T g. Suppose each fish was equally likely to be caught.

	Weight (W g)
Small fish	$0 < W \le 100$
Large fish	$500 \le W \le 600$

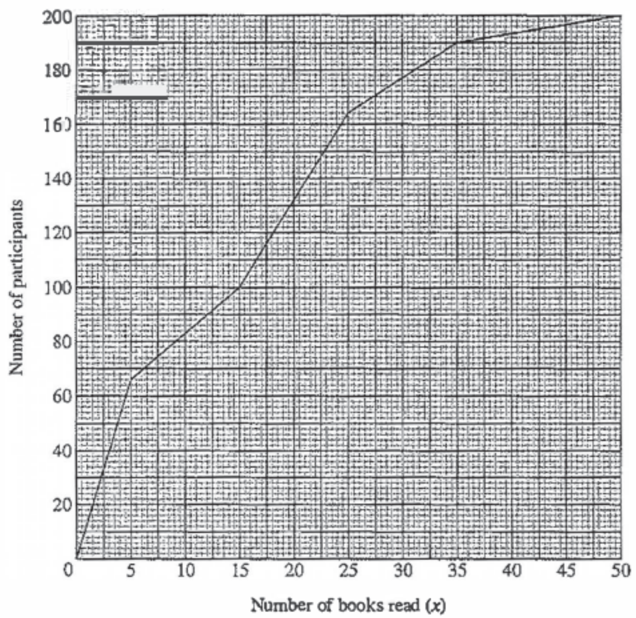
(a) Find the probability that
  (i) $0 < T \le 200$,
  (ii) $500 \le T \le 700$,
  (iii) $1000 \le T \le 1200$,
  (iv) $T > 1200$.

(b) Suppose the two fish he caught in the morning were returned alive to the pond. He went fishing again in the pond in the afternoon and also caught two fish.
  (i) If the total weight of the fish caught in the morning was 650 g, find the probability that the difference between the total weights of the fish caught in the morning and in the afternoon is more than 200 g.
  (ii) Find the probability that the difference between the total weights of the fish caught in the morning and in the afternoon is more than 200 g.

17C.16 HKCEE MA 2002 – I – 12

(Continued from 18C.11.)

The cumulative frequency polygon of the distribution of the numbers of books read by the participants



Two hundred students participated in a summer reading programme. The figure shows the cumulative frequency polygon of the distribution of the numbers of books read by the participants.

(a) The table below shows the frequency distribution of the numbers of books read by the participants. Using the graph in the figure, complete the table.

Number of books read (x)	Number of participants	Award
$0 < x \le 5$	66	Certificate
$5 < x \le 15$		Book coupon
$15 < x \le 25$	64	Bronze medal
$25 < x \le 35$		Silver medal
$35 < x \le 50$	10	Gold medal

(b) Using the graph in the figure, find the inter-quartile range of the distribution.

(c) Two participants were chosen randomly from those awarded with medals. Find the probability that

(i) they both won gold medals;

(ii) they won different medals.

17C.17 HKCEE MA 2003 – I – 16

John will participate in a contest to be held at a university. If John wins the contest, he will go to Canteen X for lunch. Otherwise, he will go to Canteen Y. Table (1) shows the types of set lunches and the prices served in the two canteens. He will choose one type of set lunch randomly.

Canteen	Set lunch	Price ($)
X	A	40
	B	50
Y	C	15
	D	20

Table (1)

Transportation	Cost of a single trip ($)
Bus	4.5
Train	7.5

Table (2)

(a) If the probability of John winning the contest is $\frac{1}{10}$, find the probability that he will spend $15 for lunch.

(b) If John takes a bus leaving at 8:00 a.m. to the university, his probability of winning the contest will be $\frac{1}{10}$. If he misses the bus, he will take a train leaving at 8:20 a.m. Owing to his nervousness, his probability of winning will be reduced to $\frac{2}{25}$.

(i) Suppose John misses the bus, find the probability that he will spend $15 for lunch.

(ii) Table (2) shows the cost of a single trip by bus or train.
It is known that the probability of John taking the bus is twice that of taking the train.

(1) Find the probability that John will spend $15 for lunch after the contest.

(2) If John goes home by train after lunch, find the probability that he will spend more than a total of $30 for the lunch and the transportation of the two trips.

17C.18 HKCEE MA 2005 – I – 11

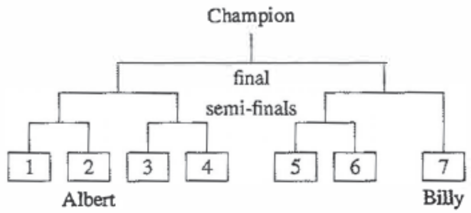
Seven players take part in a men's singles tennis knock out tournament. They are randomly assigned to the positions 1, 2, 3, 4, 5, 6 and 7. It is known that Albert and Billy are in positions 2 and 7 respectively. The winner of each game proceeds to the next round as shown in the figure and the loser is knocked out. Billy goes straight to the semi finals. In each game, each player has an equal chance of beating his opponent.

(a) Write down the probability that Albert will reach the semi-finals.

(b) Find the probability that Albert will be the champion.

(c) Find the probability that Albert will fail to reach the final.

(d) Find the probability that Albert will play against Billy in the final.



17C.19 HKCEE MA 2006 – I – 14

(Continued from 18C.14.)

The stem and leaf diagram below show the distributions of the scores (in marks) of the students of classes A and B in a test, where a, b, c and d are non-negative integers less than 10. It is given that each class consists of 25 students.

Class A

Stem (tens)	Leaf (units)
0	a 9
1	2 5 7 8 8
2	3 3 5 6 7 9
3	2 3 5 6 9 9 9
4	1 2 2 4 b

Class B

Stem (tens)	Leaf (units)
0	c 3 3 4 5
1	1 1 2 2 3 3 5 6 7 8
2	1 1 5 5 5 7 8
3	5 9
4	d

(a) (i) Find the inter-quartile range of the score distribution of the students of class A and the inter quartile range of the score distribution of the students of class B.

(ii) Using the results of (a)(i), state which one of the above score distributions is less dispersed. Explain your answer.

(b) The passing score of the test is 20 marks. From the 50 students, 3 students are randomly selected.

(i) Find the probability that exactly 2 of the selected students pass the test.

(ii) Find the probability that exactly 2 of the selected students pass the test and both of them are in the same class.

(iii) Given that exactly 2 of the selected students pass the test, find the probability that both of them are in the same class.

17C.20 HKCEE MA 2007 – I – 15

The following table shows the results of a survey about the sizes of shirts dressed by 80 students on a certain school day.

Student \ Size	Small	Medium	Large	Total
Boy	8	28	12	48
Girl	20	8	4	32

(a) On that school day, a student is randomly selected from the 80 students.
  (i) Find the probability that the selected student is a boy.
  (ii) Find the probability that the selected student is a boy and he dresses a shirt of large size.
  (iii) Find the probability that the selected student is a boy or the selected student dresses a shirt of large size.
  (iv) Given that the selected student is a boy, find the probability that he dresses a shirt of large size.

(b) On the school day, two students are randomly selected from the 80 students.
  (i) Find the probability that the two selected students both dress shirts of large size.
  (ii) Is the probability of dressing shirts of the same size by the two selected students greater than that of dressing different sizes? Explain your answer.

17C.21 HKCEE MA 2008 I – 14 (To continue as **18C.17**.)

The stem-and-leaf diagram below shows the suggested bonuses (in dollars) of the 36 salesgirls of a boutique:

Stem (thousands)	Leaf (hundreds)
2	4 4 7
3	2 5 6 6 8
4	3 3 3 4 4 7 8 8 8
5	0 0 3 4 4 6
6	2 3 3 4 4 9 9
7	0 4 4 8
8	2 3

(a) The suggested bonus of each salesgirl of the boutique is based on her performance. The following table shows the relation between level of performance and suggested bonus:

Level of performance	Suggested bonus ($x)
Excellent	$x > 6500$
Good	$4500 < x \leq 6500$
Fair	$x \leq 4500$

  (i) From the 36 salesgirl, one of them is randomly selected. Given that the level of performance of the selected salesgirl is good, find the probability that her suggested bonus is less than $5 500.
  (ii) From the 36 salesgirls, two of them are randomly selected.
    (1) Find the probability that the level of performance of one selected salesgirl is excellent and that of the other is good.
    (2) Find the probability that the levels of performance of the two selected salesgirls are different.

17C.22 HKCEE MA 2009 – I – 14

The frequency distribution table shows the lifetime (in hours) of a batch of randomly chosen light bulbs of brand A and a batch of randomly chosen light bulbs of brand B.

Lifetime (x hours)	Frequency	
	Brand A	Brand B
$1000 \leq x < 1100$	8	4
$1100 \leq x < 1200$	50	12
$1200 \leq x < 1300$	42	40
$1300 \leq x < 1400$	10	36
$1400 \leq x < 1500$	10	28

(a) According to the above frequency distribution, which brand of light bulbs is likely to have a longer lifetime? Explain your answer.

(b) If the lifetime of a light bulb is not less than 1300 hours, then the light bulb is classified as *good*. Otherwise, it is classified as *acceptable*.
  (i) If a light bulb is randomly chosen from the batch of light bulbs of brand A, find the probability that the chosen light bulb is *acceptable*.
  (ii) If two light bulbs are randomly chosen from the batch of light bulbs of brand A, find the probability that at least one of the two chosen light bulbs is *good*.
  (iii) The following 2 methods describe how 2 light bulbs are chosen from the 2 batches of light bulbs.
  Method 1: One batch is randomly selected from the two batches of light bulbs and two light bulbs are then randomly chosen from the selected batch.
  Method 2: One light bulb is randomly chosen from each of the two batches of light bulbs.
  Which one of the above two methods should be adopted in order to have a greater chance of choosing at least one *good* light bulb? Explain your answer.

17C.23 HKCEE MA 2010 – I – 14

An athlete, Alice, of a school gets the following results (in seconds) in 10 practices of 1500 m race:

279, 280, 264, 267, 283, 281, 281, 266, 284, 265

(a) Two results are randomly selected from the above results.
  (i) Find the probability that both the best two results are not selected.
  (ii) Find the probability that only one of the best two results is selected.
  (iii) Find the probability that at most one of the best two results is selected.

(b) Another athlete, Betty, of the school gets the following results (in seconds) in 10 practices of 1500 m race: 272, 269, 275, 274, 273, 274, 270, 275, 266, 272
  Alice and Betty will represent the school to participate in the 1500 m race in the inter school athletic meet.
  (i) Which athlete is likely to get a better result? Explain your answer.
  (ii) The best record of the 1500 m race in the past inter school athletic meets is 267 seconds. Which athlete has a greater chance of breaking the record? Explain your answer.

17C.24 HKCEE MA 2011 I 1 4

In a bank, the queuing times (in minutes) of 12 customers are recorded as follows:

5.1, 5.2, 5.4, 6.1, 6.7, 7.1, 7.4, 7.7, 7.8, 8.4, 9.0, 10.1

It is found that if the queuing time of a customer in the bank is less than 8 minutes, then the probability that the customer makes a complaint is $\frac{1}{6}$. Otherwise, the probability that the customer makes a complaint is $\frac{1}{3}$.

(a) If a customer is randomly selected from the 12 customers, find the probability that the selected customer does not make a complaint.

(b) Two customers are now randomly selected from the 12 customers.

(i) If the queuing time of the selected customer is less than 8 minutes and the queuing time of the other customer is not less than 8 minutes, find the probability that both of them do not make complaints.

(ii) Find the probability that the queuing times of both of the selected customers are not less than 8 minutes and both of them do not make complaints.

(iii) Is the probability of not making complaints by the two selected customers greater than the probability of making complaints by both of them? Explain your answer.

17C.25 HKALE MS 1994 – 1 1

A day is regarded as humid if the relative humidity is over 80% and is regarded as dry otherwise. In city K, the probability of having a humid day is 0.7.

(a) Assume that whether a day is dry or humid is independent from day to day.

(i) Find the probability of having exactly 3 dry days in a week.

(ii) *[Out of syllabus]*

(iii) Today is dry. What is the probability of having two or more humid days before the next dry day?

(b) After some research, it is known that the relative humidity in city K depends solely on that of the previous day. Given a dry day, the probability that the following day is dry is 0.9 and given a humid day, the probability that the following day is humid is 0.8.

(i) If it is dry on March 19, what is the probability that it will be humid on March 20 and dry on March 21?

(ii) If it is dry on March 19, what is the probability that it will be dry on March 21?

(iii) Suppose it is dry on both March 19 and March 21. What is the probability that it is humid on March 20?

17C.26 HKALE MS 1995 – 1 1

Madam Wong purchases cartons of oranges from a supplier every day. Her buying policy is to randomly select five oranges from a carton and accept the carton if all five are not rotten. Under usual circumstances, 2% of the oranges are rotten.

(a) Find the probability that a carton of oranges will be rejected by Madam Wong.

(b) *[Out of syllabus]*

(c) Today, Madam Wong has a target of buying 20 acceptable cartons of oranges from the supplier. Instead of applying the stopping rule in (b), she will keep on inspecting the cartons until her target is achieved. Unfortunately, the supplier has a stock of 22 cartons only.

(i) Find the probability that she can achieve her target.

(ii) Assuming she can achieve her target, find the probability that she needs to inspect 20 cartons only.

(d) The supplier would like to import oranges of better quality so that each carton will have at least a 95% probability of being accepted by Madam Wong. If $r\%$ of these oranges are rotten, find the greatest acceptable value of r.

17C.27 HKALE MS 1998 3 (Continued from 18B.12.)

40 students participate in a 5-day summer camp. The stem-and-leaf diagram below shows the distribution of heights in cm of these students.

(a) Find the median of the distribution of heights.

(b) A student is to be selected randomly to hoist the school flag every day during the camp. Find the probability that

(i) the fourth day will be the first time that a student taller than 170 cm will be selected,

(ii) out of the 5 selected students, exactly 3 are taller than 170 cm.

Stem (tens)	Leaf (units)
13	8
14	1 5 6 9
15	0 1 3 4 4 4 5 5 6 7 8 8 9
16	1 1 2 3 3 4 5 6 7 7 8 8
17	0 2 2 3 4 5 6 7
18	1 4

17C.28 HKALE MS 1998 – 5

John and Mary invite 8 friends to their Christmas party.

(a) When playing a game, all of the 10 participants are arranged in a row. Find the number of arrangements that can be made if

(i) there is no restriction,

(ii) John and Mary are next to each other.

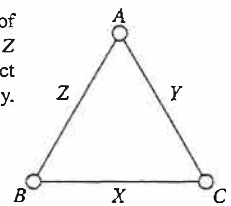
(b) By the end of the party, the participants are arranged in 2 rows of 5 in order to take a photograph. Find the number of arrangements that can be made if

(i) there is no restriction,

(ii) John and Mary are next to each other.

17C.29 HKALE MS 1999 7

Three control towers A, B and C are in telecommunication contact by means of three cables X, Y and Z as shown in the figure. A and B remain in contact only if Z is operative or if both cables X and Y are operative. Cables X, Y and Z are subject to failure in any one day with probabilities 0.015, 0.025 and 0.030 respectively. Such failures occurs independently.



(a) Find, to 4 significant figures, the probability that, on a particular day,

(i) both cables X and Z fail to operate,

(ii) all cables X, Y and Z fail to operate,

(iii) A and B will not be able to make contact.

(b) Given that cable X fails to operate on a particular day, what is the probability that A and B are not able to make contact?

(c) Given that A and B are not able to make contact on a particular day, what is the probability that cable X has failed?

17C.30 HKALE MS 2002 – 7

(Continued from **18B.13.**)

Twenty two students in a class attended an examination. The stem and leaf diagram below shows the distribution of the examination marks of these students.

Stem (tens)	Leaf (units)
3	5 7
4	2 4 6
5	0 3 4 4 4 5
6	1 2 5 5 8
7	3 8 9
8	4 8
9	5

(a) Find the mean of the examination marks.

(b) Two students left the class after the examination and their marks are deleted from the stem and leaf diagram. The mean of the remaining marks is then increased by 1.2 and there are two modes. Find the two deleted marks.

(c) Two students are randomly selected from the remaining 20 students. Find the probability that their marks are both higher than 75.

17C.31 HKALE MS 2003 11

In a game, two boxes A and B each contains n balls which are numbered $1, 2, \ldots, n$. A player is asked to draw a ball randomly from each box. If the number drawn from box A is greater than that from box B, the player wins a prize.

(a) Find the probability that the two numbers drawn are the same.

(b) Let p be the probability that a player wins the prize.

- (i) Find, in terms of p only, the probability that the number drawn from box B is greater than that from box A.
- (ii) Using the result of (i), express p in terms of n.
- (iii) If the above game is designed so that at least 46% of the players win the prize, find the least value of n.

(c) Two winners, John and Mary, are selected to play another game. They take turns to throw a fair six sided die. The first player who gets a number '6' wins the game. John will throw the die first.

- (i) Find the probability that John will win the game on his third throw.
- (ii) Find the probability that John will win the game.
- (iii) Given that Mary has won the game, find the probability that Mary did not win the game before her third throw.

17C.32 HKALE MS 2004 11

A manufacturer of brand C potato chips runs a promotion plan. Each packet of brand C potato chips contains either a red coupon or a blue coupon. Four red coupons can be exchanged for a toy. Five blue coupons can be exchanged for a lottery ticket. It is known that 30% of the packets contain red coupons and the rest contain blue coupons.

(a) Find the probability that a lottery ticket can be exchanged only when the 6th packet of brand C potato chips has been opened.

(b) A person buys 10 packets of brand C potato chips.

- (i) Find the probability that at least 1 toy can be exchanged.
- (ii) Find the probability that exactly 1 toy and exactly 1 lottery ticket can be exchanged.
- (iii) Given that at least 1 toy can be exchanged, find the probability that exactly 1 lottery ticket can also be exchanged.

(c) Two persons buy 10 packets of brand C potato chips each. Assume that they do not share coupons or exchange coupons with each other.

- (i) Find the probability that they can each get at least 1 toy.
- (ii) Find the probability that one of them can get at least 1 toy and the other can get 2 lottery tickets.

17C.33 HKALE MS 2005 – 6

Mrs. Wong has 12 bottles of fruit juice in her kitchen: 1 bottle of grape juice, 6 bottles of apple juice and 5 bottles of orange juice. She randomly chooses 4 bottles to serve her friends, Ann, Billy, Christine and Donald.

(a) Find the probability that exactly 2 bottles of orange juice are chosen by Mrs. Wong.

(b) Suppose that each of the four friends randomly selects a bottle of fruit juice from the 4 bottles offered by Mrs. Wong.

- (i) If only 2 of the bottles of fruit juice offered by Mrs. Wong are orange juice, find the probability that both Ann and Billy select orange juice.
- (ii) Find the probability that fewer than 4 of the bottles of fruit juice offered by Mrs. Wong are orange juice and both Ann and Billy select orange juice.

17C.34 HKALE MS 2010 5

(Continued from **18B.14.**)

The following stem-and-leaf diagram shows the distribution of the test scores of 21 students taking a statistics course. Let $\bar{x}$ be the mean of these 21 scores.

It is known that if the smallest value of these 21 scores is removed, the range is decreased by 27 and the mean is increased by 2.

Stem (Tens)	Leaf (Units)
2	a
3	
4	9
5	0 0 1 3 7 7
6	0 2 3 5 5 5 9
7	0 3 4 9
8	2 b

(a) Find the values of a, b and $\bar{x}$.

(b) The teacher wants to select 6 students to participate in a competition by first excluding the student with the lowest score. If the students are randomly selected, find the probability that there will be

- (i) no students with score higher than 70 begin selected;
- (ii) at least 2 students with scores higher than 70 being selected.

17C.35 HKALE MS 2012 6

(Continued from **18C.35.**)

An educational psychologist adopts the Internet Addiction Test to measure the students' level of Internet addiction. The scores of a random sample of 30 students are presented in the following stem and leaf diagram. Let σ be the standard deviation of the scores. It is known that the mean of the scores is 71 and the range of the scores is 56.

Stem (tens)	Leaf (units)
3	a
4	
5	2 4 6 8
6	0 1 3 5 6 7 8 8 9
7	1 2 2 4 5 5 6 8
8	0 2 3 5 8
9	0 2 b

(a) Find the values of a, b and σ.

(b) The psychologist classifies those scoring between 73 and 100 as excessive Internet users. If 4 students are selected randomly from the excessive Internet users among the students, find the probability that 3 of them will have scores higher than 80.

(c) *[Out of syllabus]*

17C.36 HKALE MS 2013 – 11

According to the school regulation, air conditioners can only be switched on if the temperature at 8 am exceeds 26°C. From past experience, the probability that the temperature at 8 am does NOT exceed 26°C is q ($q > 0$). Assume that there are five school days in a week. For two consecutive school days, the probability that the air conditioners are switched on for not more than one day is $\frac{7}{16}$.

(a) (i) Show that the probability that the air-conditioners are switched on for not more than one day on two consecutive school days is $2q - q^2$.
(ii) Find the value of q.

(b) The air conditioners are said to be *fully engaged* in a week if the air conditioners are switched on for all five school days in a week.
(i) Find the probability that the fifth week is the second week that the air conditioners are *fully engaged*.
(ii) ***[Out of syllabus]***

(c) On a certain day, the temperature at 8 am exceeds 26°C and all the 5 classrooms on the first floor are reserved for class activities after school. There are 2 air-conditioners in each classroom. The number of air conditioners being switched off in the classroom after school depends on the number of students staying in the classroom. Assume that the number of students in each classroom is independent.

Case	I	II	III
Number of air conditioners being switched off	2	1	0
Probability	0.25	0.3	0.45

(i) What is the probability that all air-conditioners are switched off on the first floor after school?
(ii) Find the probability that there are exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air conditioner being switched off on the first floor after school.
(iii) Given that there are 6 air-conditioners being switched off on the first floor after school, find the probability that at least 1 classroom has no air conditioners being switched off.

17C.37 HKDSE MA 2013 – I – 10

(Continued from 18C.41.)

The ages of the members of Committee A are shown as follows:

17	18	21	21	22	22	23	23	23	31
31	34	35	36	47	47	58	68	69	69

(a) Write down the median and the mode of the ages of the members of Committee A.

(b) The stem-and leaf diagram shows the distribution of the ages of the members of Committee B. It is given that the range of this distribution is 47.
(i) Find a and b.
(ii) From each committee, a member is randomly selected as the representative of that committee. The two representatives can join a competition when the difference of their ages exceeds 40. Find the probability that these two representatives can join the competition.

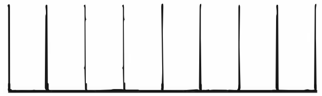
Stem (tens)	Leaf (units)
2	a 5 6 7
3	3 3 8
4	3
5	1 2 9
6	7 b

17C.38 HKDSE MA 2014 – I – 19

Ada and Billy play a game consisting of two rounds. In the first round, Ada and Billy take turns to throw a fair die. The player who first gets a number '3' wins the first round. Ada and Billy play the first round until one of them wins. Ada throws the die first.

(a) Find the probability that Ada wins the first round of the game.

(b) In the second round of the game, balls are dropped one by one into a device containing eight tubes arranged side by side (see the figure). When a ball is dropped into the device, it falls randomly into one of the tubes. Each tube can hold at most three balls.



The player of this round adopts one of the following two options.

Option 1: Two balls are dropped one by one into the device. If the two balls fall into the same tube, then the player gets 10 tokens. If the two balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.

Option 2: Three balls are dropped one by one into the device. If the three balls fall into the same tube, then the player gets 50 tokens. If the three balls fall into three adjacent tubes, then the player gets 10 tokens. If the three balls fall into two adjacent tubes, then the player gets 5 tokens. Otherwise, the player gets no tokens.

(i) If the player of the second round adopts Option 1, find the expected number of tokens got.
(ii) Which option should the player of the second round adopt in order to maximise the expected number of tokens got? Explain your answer.
(iii) Only the winner of the first round plays the second round. It is given that the player of the second round adopts the option which can maximise the expected number of tokens got. Billy claims that the probability of Ada getting no tokens in the game exceeds 0.9. Is the claim correct? Explain your answer.

17 Probability

17A Counting principles

17A.1 HKALE MS 1995 – 3

(a) No of ways $= C_6^{18}C_6^{12}C_6^{6} = 17153136$

(b) Required p $= \dfrac{(C_5^{15}C_1^{3})(C_5^{10}C_1^{2})(C_5^{5}C_1^{1})}{17153136} = \dfrac{4040536}{17153136} = \dfrac{9}{34}$

17A.2 HKALE MS 1999 – 6

(a) No of ways $= P_2^5 C_{10}^{16} = 160160$

(b) 10, 11, 12, 13, 14, 15, 16

17A.3 HKALE MS 2011 – 5

(Each route is a 8-step route consisting of 3 N's and 5 E's, such as NNNEEEEE or NNENEEEE.)

(a) *Method 1*

The routes are all possible routes subtracted by the routes going through T_1.

$\therefore$ No of routes $= C_3^8 - C_3^7 = 21$

Method 2

The routes are all the routes that start from the junction 1N from A.

$\therefore$ No of routes $= C_2^7 = 21$

(b) No of ways $= C_3^8 \quad C_2^5 \times C_1^3 = 26$

17A.4 HKDSE MA 2018 – I – 15

(a) Required no $= 8! = 40320$

(b) Required no $= P_2^4 \times 6! = 8640$

17A.5 HKDSE MA 2019 – I – 15

Required no $= C_5^{21+11} \quad C_5^{11} = 200914$

17B Probability (short questions)

17B.1 HKCEE MA 1981(1/3) – I – 3

Method 1 Required p $= \dfrac{1}{C_2^{40}} = \dfrac{1}{780}$

Method 2 Required p $= \dfrac{2}{40} \times \dfrac{1}{39} = \dfrac{1}{780}$

17B.2 HKCEE MA 1982(1/3) – I – 6

(a) Required p $= \dfrac{3}{36} = \dfrac{1}{12}$

(b) Required p $= \dfrac{3}{36} = \dfrac{1}{12}$

(c) Required p $= 1 \quad \dfrac{1}{12} - \dfrac{1}{12} = \dfrac{5}{6}$

17B.3 HKCEE MA 1996 – I – 7

(a) Area $= \pi(12)^2 \quad \pi(2)^2 = 140\pi$ (cm^2)

(b) (i) Required p $= \dfrac{140\pi}{144\pi} \times \dfrac{140\pi}{144\pi} = \dfrac{1225}{1296}$

(ii) Required p $= \dfrac{140\pi}{144\pi} \times \dfrac{4\pi}{144\pi} \times 2 = \dfrac{35}{648}$

17B.4 HKCEE MA 1998 – I – 11

(a) Required p $= \dfrac{8}{14} \times \dfrac{7}{13} = \dfrac{4}{13}$

(b) Required p $= \dfrac{4}{13} + \dfrac{4}{14} \times \dfrac{3}{13} + \dfrac{2}{14} \times \dfrac{1}{13} = \dfrac{5}{13}$

17B.5 HKCEE MA 1999 – I – 12

(a) Required p $= 75\% \times 20\% = 0.15$

(b) Required p $= \dfrac{2}{5} \times \dfrac{3}{4} + \dfrac{3}{5} \times \dfrac{2}{4} = \dfrac{3}{5}$

17B.6 HKCEE MA 2000 – I – 12

(a) Required p $= \dfrac{9}{900} = \dfrac{1}{100}$

(b) Required p $= \dfrac{9 \times 9 \times 9}{900} = \dfrac{81}{100}$

(c) Required p $= 1 \quad \dfrac{1}{100} \quad \dfrac{81}{100} = \dfrac{9}{50}$

17B.7 HKCEE MA 2004 – I – 8

(a) Required p $= \dfrac{5}{9}$

(b) Required p $= 1 - P(\text{both odd}) = 1 - \left(\dfrac{5}{9}\right)^2 = \dfrac{22}{27}$

17B.8 HKCEE MA 2006 – I – 8

(a) Sum $\quad 11 \times 10 \Rightarrow k \quad 24$

(b) Required p $= \dfrac{4}{10} = \dfrac{2}{5}$

17B.9 HKCEE MA 2008 – I – 5

Favourable outcomes: 4&6, 3&7

$\therefore$ Required p $= \dfrac{2}{3 \times 2} = \dfrac{1}{3}$

17B.10 HKCEE MA 2009 – I – 5

Required p $= \dfrac{7+21}{7+21+30+53+57+32} = \dfrac{7}{50}$

17B.11 HKALE MS 1994 – 1

(a) BB, BG, GB, GG

(b) (i) BB, BG, GB

(ii) Required p $= \dfrac{1}{3}$

17B.12 HKALE MS 1994 – 3

The shortest paths must consist of 6 steps, among which 2 are 'up', 2 are 'forward' and 2 are 'right'.

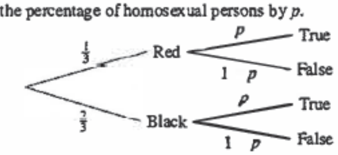
(a) No of ways $= C_2^6 C_2^4 C_2^2 = 90$

(b) (i) No of ways $= C_1^3 C_1^2 C_1^1 = 6$

(ii) Required p $\quad \dfrac{6}{90} = \dfrac{1}{15}$

17B.13 HKALE MS 1994 – 7

(a) Let the percentage of homosexual persons by p.

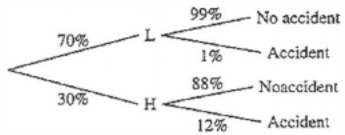


$\dfrac{790}{1200} = \dfrac{1}{3}(p) + \dfrac{2}{3}(1-p) \Rightarrow p = 0.025$

$\therefore$ 2.5% are homosexual.

(b) $P(\text{Homo}|\text{True}) = \dfrac{\frac{1}{3}(p)}{\frac{790}{1200}} = 1.27\%$ (3 s.f.)

17B.14 HKALE MS 1995 – 5



$P(\text{No accident}) = 70\% \times 99\% + 30\% \times 88\% = 0.957$

(a) $P(\text{H}|\text{No accident}) = \dfrac{30\% \times 88\%}{0.957} = 0.276$ (3 s.f.)

(b) *Method 1*

$P(\text{L}|\text{No accident}) = \dfrac{70\% \times 99\%}{0.957} = 0.724$ (3 s.f.)

Method 2

$P(\text{L}|\text{No accident}) = 1 - 0.276 = 0.724$ (3 s.f.)

17B.15 HKALE MS 1996 – 6

(a) $P(\text{Accepted}) = P(\text{A})P(\text{Accepted}|\text{A}) + P(\text{B}P(\text{Accepted}|\text{B})$

$= \dfrac{1}{2} \times (1-4\%)^2 + \dfrac{1}{2} \times (1 \quad 1\%)^2 = 0.95085$

(b) $P(\text{A}|\text{Accepted}) = \dfrac{\frac{1}{2} \times (1-4\%)^2}{0.98085} = 0.485$ (3 s.f.)

17B.16 HKALE MS 1997 – 7

(a) $P(\text{Not working}) = 0.15 \times 0.24 = 0.036$

(b) $P(\text{Working}) = 1 - 0.036 = 0.964$

$\Rightarrow$ Required p $= \dfrac{0.85}{0.964} = 0.882$ (3 s.f.)

(c) Required p $= (0.964)^9(0.036) = 0.0259$ (3 s.f.)

17B.17 HKALE MS 1998 – 6

(a) (i) Required p $= \dfrac{1}{8} \times (1 \quad 0.8\%) = 0.124$

(ii) Required p $= \dfrac{1}{8}(1 \quad 0.8\%) + \dfrac{2}{8}(1 \quad 0.2\%) + \dfrac{5}{8}$

$= 0.9985$

(b) Required p $= \dfrac{\frac{1}{8}(0.8\%)}{1 - 0.9985} \quad \dfrac{2}{3}$

17B.18 HKALE MS 1999 – 5

(a) Required p $= C_5^{10}(60\%)^5(1-60\%)^5 = 0.200658$

$= 0.201$ (3 s.f.)

(b) Expected no $= 10 \times 60\% = 6$

(c) Required p $= (1 - 0.200658)^2(0\ 200658) = 0.128$ (3 s.f.)

17B.19 HKALE MS 2000 – 6

(a) Required p $= \dfrac{C_3^4}{C_3^6} = \dfrac{1}{5}$

(b) Required p $= \dfrac{C_2^4 C_1^2}{C_3^6} = \dfrac{3}{5}$

17B.20 HKALE MS 2000 – 8

(a) Required p $= 0.3 \times \dfrac{5}{9} + 0.7 \times \dfrac{5}{6} = 0.75$

(b) Let a be the probability of generating game A.

$a \times \dfrac{5}{9} + (1 \quad a) \times \dfrac{5}{6} = \dfrac{2}{3} \Rightarrow a = 0.6$

$\therefore P(\text{Game } A) = 0.6,\ P(\text{Game } B) = 0.4$

17B.21 HKALE MS 2001 – 6

(a) Required p $\quad \dfrac{C_2^9}{C_3^{10}} \quad \dfrac{3}{10}$

(b) Required p $= \dfrac{C_2^6}{C_3^{10}} = \dfrac{1}{8}$

(c) *Method 1*

~~Required~~ p $= \dfrac{C_3^8 + C_1^2 C_2^8}{C_3^{10}} = \dfrac{14}{15}$

Method 2

Required p $= 1 - P(\text{2 heaviest selected}) = 1 - \dfrac{C_1^8}{C_3^{10}} = \dfrac{14}{15}$

17B.22 HKALE MS 2001 – 7

(a) Required p $= \dfrac{58\% \times 39\%}{65\% \times 48\% + 58\% \times 39\% + 50\% \times 13\%}$

$= 0.375$

(b) Required p $= (0.375)^2(1 \quad 0.375)^2 = 0.0343$ (3 s.f.)

17B.23 HKALE MS 2002 – 5

(a) Required p $= \dfrac{1}{3} \times \dfrac{6}{8} + \dfrac{1}{3} \times \dfrac{4}{7} + \dfrac{1}{3} \times \dfrac{2}{7} = \dfrac{15}{28}$

(b) Required p $= \dfrac{\frac{1}{3} \times \frac{6}{8}}{\frac{15}{28}} = \dfrac{7}{15}$

17B.24 HKA LBMS 2002 – 8

(a) 3W, 3Y, 2R1W, 2R1Y, 1R2W, 1R2Y, 2W1Y, 1W2Y, 1R1W1Y

(b) Required p $= \dfrac{C_1^2 C_2^{11}}{C_3^{13}} = \dfrac{5}{13}$

(c) *Method 1*

Required p $= \dfrac{P(\text{1R1W1Y})}{P(\text{Exactly 1R})} = \dfrac{C_1^2 C_1^5 C_1^6}{C_1^2 C_2^{11}} = \dfrac{6}{11}$

Method 2

Required p $= P(\text{Exactly 1W1Y after 1R is selected})$

$= \dfrac{C_1^5 C_1^6}{C_2^{11}} = \dfrac{6}{11}$

17B.25 HKA MS 2003 – 12

(a) Required p $= 1 - P(\text{No boy}) - P(\text{No girl})$

$= 1 - \dfrac{C_7^{17}}{C_7^{30}} - \dfrac{C_7^{13}}{C_7^{30}} = \dfrac{38743}{39150} = 0.990$

(b) Required p $= \dfrac{P(\text{4 or 5 or 6 girls})}{\text{Prob in (a)}}$

$= \dfrac{\dfrac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30}}}{\dfrac{38743}{39150}} = 0.657$

17B.26 HKA LBMS 2004 – 6

(a) Required p $= 1 - P(\text{all 3 same}) = 1 - \left(\dfrac{1}{10}\right)^2 = \dfrac{99}{100}$

(b) Required p $= \dfrac{3!}{10^3} = \dfrac{3}{500}$

(c) Required p $= C_2^3 \left(\dfrac{4}{10}\right)^2 \left(\dfrac{6}{10}\right) = \dfrac{36}{125}$

17B.27 HKA LBMS 2004 10

(a) Required p $= 7.5\% \times 94\% + (1 - 7.5\%) \times 14\% = 0.2$

(b) Required p $= \dfrac{(1-7.5\%) \times 14\%}{0.2} = 0.6475$

17B.28 HKA LBMS 2007 – 6

(a) *Method 1* Required p $= \dfrac{C_1^1 C_2^4}{C_3^{10}} \times \dfrac{1}{3} = \dfrac{1}{60}$

Method 2 Required p $= \dfrac{1}{10} \times \dfrac{4}{9} \times \dfrac{3}{8} = \dfrac{1}{60}$

(b) *Method 1* Required p $= \dfrac{1}{60} + \dfrac{C_1^5 C_2^5}{C_1^{10} C_2^9} = \dfrac{7}{45}$

Method 2 Required p $= \dfrac{1}{60} + \dfrac{5}{10} \times \dfrac{5}{9} \times \dfrac{4}{8} = \dfrac{7}{45}$

17B.29 HKA LBMS 2009 – 5

(a) Required p $= 1 - C_1^{15}(36\%)(64\%)^{14} - C_2^{15}(36\%)^2(64\%)^{13}$
$C_3^{15}(36\%)^3(64\%)^{12} = 0.847$

(b) Required p

$= \dfrac{C_1^5(36\%)(64\%)^4 \times C_1^5(36\%)(64\%)^4 \times C_2^5(36\%)^2(64\%)^3 \times 3}{C_4^{15}(36\%)^4(64\%)^{11}}$

$= \dfrac{50}{91} = 0.549$

17B.30 HKA LBMS 2011 – 4

(a) Required p

$= 0.75^2(1 - 0.55^2) + 0.75(1 - 0.75)(1 - 0.55)^2 \cdot$

$= 0.468$

(b) Required p $= \dfrac{0.75^2 \times 0.55(1 \quad 0.55) \cdot 2}{1 - (1 - 0.55)^2} = 0.349$

17B.31 HKA LE MS 2011 – 5

(Each route is a 8-step route consisting of 3 N's and 5 E's, such as NNNEEEEE or NNENEEEE.)

(a) *Method 1*

The routes are all possible routes subtracted by the routes going through T_1.

$\therefore$ No of routes $= C_3^8 - C_3^7 = 21$

Method 2

The routes are all the routes that start from the junction 1N from A.

$\therefore$ No of routes $= C_2^7 = 21$

(b) No of ways $= C_3^8 - C_2^5 \times C_1^3 = 26$

(c) Required p $= \dfrac{C_2^7 - C_1^4 \times C_1^3}{C_3^8} = \dfrac{9}{56}$

17B.32 HKA LE MS 2013 – 4

(a) Required p $= \left(\dfrac{1}{4}\right)^4 \times 4 + \dfrac{4!}{4^4} = \dfrac{7}{64}$

(b) Required p $= \dfrac{\frac{4!}{4^4}}{\frac{4}{4} \times \frac{3}{4}} = \dfrac{1}{8}$

17B.33 HKDSE MA SP – I – 16

(a) Required p $= \dfrac{C_2^5 \times C_2^4}{C_4^9} = \dfrac{10}{21}$

(b) Required p $= 1 \quad \dfrac{10}{21} = \dfrac{11}{21}$

17B.34 HKDSE MA PP – I – 13

(a) Number of students $= 6 \div \dfrac{3}{20} = 40$

$\Rightarrow k = 40 - 6 - 11 \quad 5 \quad 10 = 8$

17B.35 HKDSE MA PP – I 16

(a) Required p $= \dfrac{C_4^{18}}{C_4^{30}} = \dfrac{68}{609}$

(b) Required p $= 1 - \dfrac{68}{609} - \dfrac{C_4^{12}}{C_4^{30}} = \dfrac{530}{609}$

17B.36 HKDSE MA 20 1 2 – I – 16

(a) Required p $= \dfrac{C_4^8 \times (C_1^2)^4}{C_4^{16}} = \dfrac{8}{13}$

(b) Required p $= 1 - \dfrac{8}{13} = \dfrac{5}{13}$

17B.37 HKDSE MA 2013 – I – 16

(a) Required p $= \dfrac{C_4^5 C_2^{11} + C_5^5 C_1^{11}}{C_6^{16}} = \dfrac{1}{28}$

(b) Required p $= 1 - \dfrac{1}{28} = \dfrac{27}{28}$

17B.38 HKDSE MA 2015 – I – 3

Required p $= \dfrac{1+2+3}{4 \times 5} = \dfrac{3}{10}$

17B.39 HKDSE MA 2015 – I – 16

(a) Required p $= \dfrac{C_2^5 C2^9}{C_4^{14}} = \dfrac{360}{1001}$

(b) Required p $= 1 - \dfrac{C_4^9}{C_4^{14}} - \dfrac{C_1^5 C_3^9}{C_4^{14}} \quad \dfrac{5}{11}$

17B.40 HKDSE MA 2016 – I – 9

(a) $x = 2 + 4 = 6$

$y = 37 - 15 = 22$

$z = 37 + 3 = 40$

(b) Required p $= \dfrac{22 \quad 6}{40} = \dfrac{2}{5}$

17B.41 HKDSE MA 2016 – I – 15

Required p $= \dfrac{P_4^6 P_5^5}{(4+5)!} = \dfrac{5}{42}$

17B.42 HKDSE MA 2017 – I – 7

(a) $x = 360° \times \dfrac{1}{9} = 40°$

(b) No of students $= 180 \div \dfrac{360° - 90° - 158° \quad 40°}{360°} = 900$

17B.43 HKDSE MA 2017 – I – 11

(a) $(80 + b) - 61 = 22 \;\Rightarrow\; b = 3$

$\dfrac{61 + \cdots + (70 + a) + \cdots + 83}{15} = 70 \;\Rightarrow\; a = 2$

$\therefore$ Median = \$69, SD = \$7.33

(b) Required p $= \dfrac{6}{15} = \dfrac{2}{5}$

17B.44 HKDSE MA 2017 – I – 17

(a) Required p $= \dfrac{C_1^{7+8}}{C_5^{4+7+8}} = \dfrac{5}{3876}$

(b) Required p $= \dfrac{C_3^4 C_2^{15}}{C_5^{19}} = \dfrac{35}{969}$

(c) Required p $= 1 - \dfrac{5}{3876} - \dfrac{35}{969} = \dfrac{3731}{3876}$

17B.45 HKDSE MA 2018 – I – 4

$\dfrac{8}{n+5+8} = \dfrac{2}{5} \;\Rightarrow\; n = 7$

17B.46 HKDSE MA 2019 – I – 8

(a) 2

(b) Mean $= 2 \times \dfrac{144°}{360°} + 3 \times \dfrac{54°}{360°} + 5 \times \dfrac{72°}{360°} + 7 \times \dfrac{90°}{360°} = 4$

(c) Required p $= \dfrac{72 + 90}{360} = \dfrac{9}{20}$

17B.47 HKDSE MA 2020 – I – 15

15a	The required probability $= \dfrac{C_4^7 + C_4^7}{C_4^{4+7+9}}$ $= \dfrac{161}{3876}$
b	The required probability $= 1 - \dfrac{161}{3876}$ $= \dfrac{3715}{3876}$

17C Probability (structural questions)

17C.1 HKCEE MA 1980(1/3) I – 14

(a) Required p $= \dfrac{9}{10} \times \dfrac{2}{3} = \dfrac{3}{5}$

(b) Required p $= \dfrac{1}{10} \times \dfrac{9}{10} \times \dfrac{2}{3} + \dfrac{9}{10} \times \dfrac{1}{3} \times \dfrac{2}{3} = \dfrac{13}{50}$

(c) Required p $= 1 - \left(1 \quad \dfrac{3}{5}\right)^2 = \dfrac{21}{25}$

17C.2 HKCEE MA 1983(A /B) – I – 11

(a) Required p $= 0.6^3 = 0.216$

(b) Required p $= (1 - 0.6)^3 = 0.064$

(c) Required p $= C_1^3(0.6)(0.4)^2 = 0.288$

(d) *Method 1*

Required p $= 1 \quad 0.216 - 0.064 \quad 0.288 = 0.432$

Method 2 Required p $= C_2^3(0.6)^2(0.4) = 0.432$

17C.3 HKCEE MA 1984(A/B) – I – 11

(a) (i) Required p $= \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$

(ii) Required p $= \dfrac{1}{9} \times 3 = \dfrac{1}{3}$

(iii) Required p $= 1 - \dfrac{1}{3} = \dfrac{2}{3}$

(b) (i) Required p $= \left(\dfrac{2}{7}\right)^2 = \dfrac{4}{49}$

(ii) Required p $= \dfrac{4}{49} + \left(\dfrac{2}{7}\right)^2 + \left(\dfrac{3}{7}\right)^2 = \dfrac{17}{49}$

(iii) Required p $= 1 - \dfrac{17}{49} = \dfrac{32}{49}$

17C.4 HKCEE MA 1985(A /B) I 10

(a) (i) Required p $= \dfrac{3+2+1}{36} = \dfrac{6}{36} = \dfrac{1}{6}$

(ii) Required p $= \dfrac{6+4}{36} = \dfrac{5}{18}$

(b) (i) Required p $= \left(1 - \dfrac{5}{18}\right)^2 = \dfrac{169}{324}$

(ii) Required p $= 2 \times \dfrac{5}{18} \times \dfrac{13}{18} = \dfrac{65}{162}$

17C.5 HKCEE MA 1986(A/B) – I – 13

(a) (i) Required p $= \dfrac{1}{5}$

(ii) Required p $= \dfrac{3}{75} = \dfrac{1}{25}$

(iii) Required p $= \dfrac{2 \times 4}{75} = \dfrac{8}{75}$

(b) (i) Required p $= \dfrac{1}{3} \times \dfrac{1}{3} = \dfrac{1}{9}$

(ii) Required p $= \dfrac{1}{9} \times 2! = \dfrac{2}{9}$

(iii) Required p $= 1 - \dfrac{1}{9} \times 3 = \dfrac{2}{3}$

17C.6 HKCEE MA 1987(A /B) – I – 13

(a) Required p $= \dfrac{5}{6}$

(b) Required p $= \dfrac{1}{6} \times \dfrac{4}{5} = \dfrac{2}{15}$

(c) Required p $= \dfrac{1}{6} \times \dfrac{1}{5} = \dfrac{1}{30}$

(d) Required p $= \dfrac{3}{6} \times \dfrac{1}{5} = \dfrac{1}{10}$

17C.7 HKCEE MA 1988 – I – 11

(a) (i) Median = 70 marks

(ii) IQR = 86 − 50 = 36 (marks)

(b) (i) Number of students = 600 − 540 = 60

(ii) Required p $= \frac{60}{600} = \frac{1}{10}$

(iii) (1) Required p $= \frac{C^{60}_2}{C^{600}_2} = \underline{\frac{59}{5990}}$

(2) Required p $= 1 - \frac{C^{540}_2}{C^{600}_2} = \underline{\frac{1139}{5990}}$

17C.8 HKCEE MA 1989 – I – 13

(a) $\begin{cases} p = 3q \\ p + q = 1 \end{cases} \Rightarrow q = 0.25$

(b) (i) Required p $= \frac{n}{10} \times \frac{n-1}{9} = \frac{n(n-1)}{90}$

(ii) $\frac{n(n-1)}{90} > \frac{1}{3} \Rightarrow n^2 - n - 30 > 0$

$\Rightarrow n < -5$ or $n > 6$

$\therefore$ Possible n's = 7, 8, 9, 10

(c) Required p $= \frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{3}{6} = \frac{7}{12}$

17C.9 HKCEE MA 1990 – I – 13

(a) (i) Required p $= \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

(ii) Required p $= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} = \frac{4}{9}$

(b) (i) Required p $= \frac{2}{9} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{81}$

(ii) Required p $= \left(\frac{1}{3} \times \frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 = \frac{10}{81}$

17C.10 HKCEE MA 1991 – I – 10

(a) (i) Required p = (1 − 70%)(70%) = 0.21

(ii) Required p = 70% + 0.21 = 0.91

(iii) Required p = 0.91 × [60% + (1 − 60%)(60%)]
= 0.7644

(b) Expected number = 10000 × 0.7644 = 7644

17C.11 HKCEE MA 1992 – I – 10

(a) (i) Required p $= 1 - \frac{2}{5} = \frac{3}{5}$

(ii) Required p $= 1 - \frac{4}{7} = \frac{3}{7}$

(iii) Required p $= \frac{3}{5} \times \frac{1}{7} = \frac{3}{35}$

(iv) Required p $= \frac{2}{5}\left(1 - \frac{1}{7}\right) = \frac{12}{35}$

(b) (i) Required p $= \frac{1}{7} \times \frac{2}{7} \times 2 = \frac{4}{49}$

(ii) Required p $= \left(\frac{2}{5} \times \frac{2}{7}\right) \times \left(\frac{3}{5} \times \frac{2}{7}\right) \times 2 = \frac{48}{1225}$

17C.12 HKCEE MA 1993 – I – 13

(a) (i) Required p = 0.65 × 0.45 = 0.2925

(ii) Required p = 0.2925 + (0.25 + 0.1) × 0.55 = 0.485

(b) (i) Required p $= \left(\frac{40000 \times 70\%}{60000}\right)^2 = \underline{\frac{49}{225}}$

(ii) Req. p $= \left(\frac{40000 \times 70\% + 20000 \times 40\%}{60000}\right)^2 = \frac{9}{25}$

(iii) Required p $= 1 - \frac{9}{25} - \left(\frac{60000 - 36000}{60000}\right)^2 = \frac{12}{25}$

17C.13 HKCEE MA 1994 – I – 9

(a) (i) Required p $= \left(\frac{1}{7}\right)^3 = \frac{1}{343}$

(ii) Required p $= \left(\frac{6}{7}\right)^3 = \frac{216}{343}$

(b) (i) Required p $= \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right) = \frac{9}{1000}$

(ii) Required p $= \frac{9}{1000} \times 3 = \underline{\frac{27}{1000}}$

(c) Required p $= \frac{1}{2} \times \frac{1}{7} + \frac{1}{2} \times \frac{1}{10} = \underline{\frac{17}{140}}$

17C.14 HKCEE MA 1995 – I – 11

(a) (i) $p = 1 - \frac{4}{5} = \frac{1}{5}$

(ii) $q = 0,\ r = 1$

(b) (i) Required p $= \frac{1}{2} \times \frac{1}{2}$

(ii) (1) Required p $= \frac{1}{2} \times \frac{1}{3} + \frac{1}{2} = \frac{2}{3}$

(2) Required p $= \frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$

17C.15 HKCEE MA 1997 – I – 14

(a) (i) Required p $= \frac{C^{40}_2}{C^{50}_2} = \underline{\frac{156}{245}}$

(ii) Required p $= \frac{C^{10}_1 C^{40}_1}{C^{50}_2} = \frac{16}{49}$

(iii) Required p $= \frac{C^{10}_2}{C^{50}_2} = \underline{\frac{9}{245}}$

(iv) Required p = 0

(b) (i) Required p $= \frac{156}{245} + \frac{9}{245} = \frac{33}{49}$

(ii) Required p $= 1 - \left(\frac{156}{245}\right)^2 - \left(\frac{16}{49}\right)^2 - \left(\frac{9}{245}\right)^2$
= 0.487

17C.16 HKCEE MA 2002 – I – 12

(a)

$0 < x \le 5$	66	Certificate
$5 < x \le 15$	34	Book coupon
$15 < x \le 25$	64	Bronze medal
$25 < x \le 35$	26	Silver medal
$35 < x \le 50$	10	Gold medal

(b) IQR = 23 − 4 = 19

(c) Number of medallists = 200 − 100 = 100

(i) Required p $= \frac{C^{10}_2}{C^{100}_2} = \underline{\frac{1}{110}}$

(ii) Required p $= 1 - \frac{1}{110} - \frac{C^{26}_2}{C^{100}_2} - \frac{C^{64}_2}{C^{100}_2} = \frac{1282}{2475}$

17C.17 HKCEE MA 2003 – I – 16

(a) Required p $= \frac{9}{10} \times \frac{1}{2} = \frac{9}{20}$

(b) (i) Required p $= \frac{23}{25} \times \frac{1}{2} = \frac{23}{50}$

(ii) (1) Required p $= \frac{2}{3} \times \frac{9}{20} + \frac{1}{3} \times \frac{23}{50} = \frac{34}{75}$

(2) Required p $= 1 - \frac{34}{75} = \frac{41}{75}$

17C.18 HKCEE MA 2005 – I – 11

(a) Required p $= \frac{1}{2}$

(b) Required p $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(c) Required p $= 1 - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$

(d) Required p $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

17C.19 HKCEE MA 2006 – I – 14

(a) (i) Class A: IQR = 39 − 18 = 21 (marks)
Class B: IQR = 25 − 11 = 14 (marks)

(ii) $\because$ IQR of B < IQR of A
$\therefore$ Class B is less dispersed.

(b) (i) Required p $= \frac{C^{18+10}_2 C^{22}_1}{C^{50}_3} = \frac{297}{700}$

(ii) Required p $= \frac{(C^{18}_2 + C^{10}_2)C^{22}_1}{C^{50}_3} = \frac{1089}{4900}$

(iii) Required p $= \frac{\frac{1089}{4900}}{\frac{297}{4900}} = \frac{11}{21}$

17C.20 HKCEE MA 2007 – I – 15

(a) (i) Required p $= \frac{48}{80} = \frac{3}{5}$

(ii) Required p $= \frac{12}{80} = \frac{3}{20}$

(iii) Required p $= \frac{48+4}{80} = \frac{13}{20}$

(iv) Required p $= \frac{12}{48} = \frac{1}{4}$

(b) (i) Required p $= \frac{C^{16}_2}{C^{80}_2} = \frac{3}{79}$

(ii) $\because$ P(same size) $= \frac{C^{28}_2}{C^{80}_2} + \frac{C^{36}_2}{C^{80}_2} + \frac{3}{79} = \underline{\frac{141}{395}} < \frac{1}{2}$

$\therefore$ NO

17C.21 HKCEE MA 2008 – I – 14

(a) (i) Required p $= \frac{9}{15} = \frac{3}{5}$

(ii) (1) Required p $= \frac{8 \times 15}{C^{36}_2} = \frac{4}{21}$

(2) Required p $= 1 - \frac{C^{8}_2}{C^{36}_2} - \frac{C^{15}_2}{C^{36}_2} - \frac{C^{13}_2}{C^{36}_2} = \underline{\frac{419}{630}}$

17C.22 HKCEE MA 2009 – I – 14

(a) For Brand A,

mean $= \frac{1050 \cdot 8 + 1150 \cdot 50 + 1250 \cdot 42 + 1350 \cdot 10 + 1450 \cdot 10}{120}$
= 1220 (h)

For Brand B,

mean $= \frac{1050 \cdot 4 + 1150 \cdot 12 + 1250 \cdot 40 + 1350 \cdot 36 + 1450 \cdot 28}{120}$
= 1310 (h) > 1220 (h)

$\therefore$ Brand B

(b) (i) Required p $= \frac{8 + 50 + 42}{120} = \frac{5}{6}$

(ii) Required p $= 1 - \frac{5}{6} \times \frac{99}{119} = \underline{\frac{73}{238}}$

(iii) Method 1: required p $= \frac{1}{2} \cdot \underline{\frac{73}{238}} + \frac{1}{2}\left(1 - \frac{C^{56}_2}{C^{120}_2}\right)$
$= \underline{\frac{779}{1428}}$

Method 2: required p $= 1 - \frac{5}{6} \times \underline{\frac{56}{120}} = \frac{11}{18} > \underline{\frac{779}{1428}}$

$\therefore$ Method 2

17C.23 HKCEE MA 2010 – I – 14

(a) (i) Required p $= \frac{C^{8}_2}{C^{10}_2} = \frac{28}{45}$

(ii) Required p $= \frac{C^{2}_1 C^{8}_1}{C^{10}_2} = \frac{16}{45}$

(iii) *Method 1* Required p $= \frac{28}{45} + \frac{16}{45} = \frac{44}{45}$

Method 2 Required p $= 1 - \frac{1}{C^{10}_2} = \frac{44}{45}$

(b) (i) Alice's mean = 275 s, Betty's mean = 272 s < 275 s
$\therefore$ Betty

(ii) Alice got 3 results < 267 s but Betty only got 1.
$\therefore$ Alice

17C.24 HKCEE MA 2011 – I – 14

(a) Required p $= \frac{9}{12}\left(1 - \frac{1}{6}\right) + \frac{3}{12}\left(1 - \frac{1}{3}\right) = \frac{19}{24}$

(b) (i) Required p $= \frac{5}{6} \times \frac{2}{3} = \frac{5}{9}$

(ii) Required p $= \left(\frac{3}{12} \cdot \frac{2}{3}\right) \times \left(\frac{2}{11} \cdot \frac{2}{3}\right) = \frac{2}{99}$

(iii) P(both not making complaints)
$= \left(\frac{9}{12} \cdot \frac{5}{6}\right) \cdot \left(\frac{8}{11} \cdot \frac{5}{6}\right) + 2\left(\frac{3}{12} \cdot \frac{9}{11} \cdot \frac{5}{9}\right) + \frac{2}{99}$
$= \frac{62}{99} > \frac{1}{2} \Rightarrow$ YES

17C.25 HKALE MS 1994 – 11

(a) (i) Required p $= C^{7}_3 (30\%)^3 (70\%)^4 = 0.227$

(iii) Required p
$= 1 - P$(next day dry) $- P$(next day humid, then dry)
= 1 − 30% − (70%)(30%) = 0.49

(b) (i) Required p = (1 − 0.9)(1 − 0.8) = 0.02

(ii) Required p = P(20 dry, 21 dry) + P(20 hmd. 21 dry)
= 0.02 + (0.9)(0.9) = 0.83

(iii) Required p $= \frac{0.02}{\underline{0.83}} = 0.0241$

17C.26 HKALE MS 1995 – 11

(a) Required p $= 1 - (1 - 2\%)^5 = 0.096079 = 0.0961$ (3 s.f.)

(c) (i) *Method 1*

Required p
$= P$(22 good) $+ P$(21 good 1 bad) $+ P$(20 good 2 bad)
$= (0.903921)^{22} + C^{22}_1 (0.903921)^{21}(0.096079)$
$+ C^{22}_2 (0.903921)^{20}(0.096079)^2$
= 0.64455 = 0.645 (3 s.f.)

Method 2

Required p = P(1st 20 accepted)
$+P$(1 rejected in 1st 20, 21st accepted)
$+P$(2 rejected in 1st 21, 22nd accepted)
$= (0.903921)^{20}$
$+ C^{20}_1 (0.096079)(0.903921)^{20}$
$+ C^{21}_2 (0.096079)^2 (0.903921)^{20}$
= 0.64455 = 0.645 (3 s.f.)

(ii) Required p $= \frac{(0.903921)^{20}}{0.64455} = 0.206$

(d) $(1 - r\%)^5 \ge 0.95 \Rightarrow 1 - r\% \ge \sqrt[5]{0.95} \Rightarrow r \le 1.02$
Hence, the greatest acceptable value of r is 1.02.

17C.27 <u>HKALE MS 1998 – 3</u>

(a) Median $= (161+162) \div 2 = 161.5$ (cm)

(b) (i) Required p $= \left(\frac{31}{40}\right)^3 \left(\frac{9}{40}\right) = 0.105$

(ii) Required p $= C_3^5 \left(\frac{31}{40}\right)^2 \left(\frac{9}{40}\right)^3 = 0.0684$

17C.28 <u>HKALE MS 1998 – 5</u>

(a) (i) No of arrangements $= 10! = 3628800$

(ii) No of arrangements $= 9! \times 2! = 725760$

(b) (i) No of arrangements $= 10! = 3628800$

(ii) <u>*Method 1*</u>

No of arrangements $= (9! - 8!) \times 2! = 645120$

<u>*Method 2*</u>

No of arrangements $= C_3^8 \times 4! \times 2! \times 5! \times 2! = 645120$

<u>*Method 3*</u>

No of arrangements $= 8! \times 2 \times 8 = 645120$

17C.29 <u>HKALE MS 1999 – 7</u>

(a) (i) Required p $= 0.015 \times 0.030 = 0.00045$

(ii) Required p $= 0.015 \times 0.025 \times 0.030 = 0.00001125$

(iii) Required p $= 0.00045 + 0.025 \times 0.030 - 0.00001125$
$= 0.00118875 = 0.001189$ (4 s.f.)

(b) Required p $= 0.030$

(c) Requi redp $= \frac{0.015 \times 0.030}{0.00118875} = 0.379$

17C.30 <u>HKALE MS 2002 – 7</u>

(a) Mean $= 61$

(b) Si ncethere are two modes, one deleted mark is 54.
The other mark $= 61 \times 22 - (61 + 1.2) \times 20 \quad 54 = 44$

(c) Required p $= \frac{C_2^5}{C_2^{20}} = \frac{1}{19}$

17C.31 <u>HKALE MS 2003 – 11</u>

(a) Required p $= \frac{1}{n}$

(b) (i) Required p $= p$

(ii) $p + p + \frac{1}{n} = 1 \Rightarrow p = \left(1 - \frac{1}{n}\right) \div 2 = \frac{1}{2} - \frac{1}{2n}$

(iii) $\frac{1}{2} - \frac{1}{2n} \geq 0.46 \Rightarrow n \geq 12.5 \Rightarrow$ Least $n = 13$

(c) (i) Required p $= \left(\frac{5}{6}\right)^4 \frac{1}{6} = \frac{625}{7776}$

(ii) Required p $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots$
$\frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$

(iii) Required p $= \frac{\left(1 - \frac{6}{11}\right) - \left(\frac{5}{6}\right)\frac{1}{6} \quad \left(\frac{5}{6}\right)^3 \frac{1}{6}}{1 - \frac{6}{11}} = \frac{625}{1296}$

17C.32 <u>HKALE MS 2004 – 11</u>

(a) Required p $= C_4^5 (70\%)^4 (30\%) \times 0.7 = 0.252105$

(b) (i) Required p $= 1 - (0.7)^{10} - C_1^{10}(0.7)^9(0.3)$
$- C_2^{10}(0.7)^8(0.3)^2 - C_3^{10}(0.7)^7(0.3)^3$
$= 0.350389 = 0.350$ (3 s.f.)

(i i) Required p $= C_4^{10}(0.7)^6(0.3)^4 + C_5^{10}(0.7)^5(0.3)^5$
$= 0.303040 = 0.303$ (3 s.f.)

(iii) Required p $= \frac{0.303040}{0.350389} = 0.865$

(c) (i) Required p $= (0.350389)^2 = 0.123$

(ii) Required p $= (0.350389)(0.7)^{10} \times 2 = 0.0198$

17C.33 <u>HKALE MS 2005 6</u>

(a) Required p $= \frac{C_2^5 C_2^7}{C_4^{12}} = \frac{14}{33}$

(b) (i) <u>*Method 1*</u> Required p $= \frac{P_2^2 \times P_2^2}{P_4^4} = \frac{1}{6}$

<u>*Method 2*</u> Required p $= \frac{C_2^2}{C_2^4} = \frac{1}{6}$

(ii) <u>*Method 1*</u>

Required p $= \frac{14}{33} \times \frac{1}{6} + \frac{C_3^5 C_1^7}{C_4^{12}} \times \frac{P_2^3 \times P_2^2}{P_4^4} = \frac{14}{99}$

<u>*Method 2*</u> Required p $= \frac{14}{33} \times \frac{1}{6} + \frac{C_3^5 C_1^7}{C_4^{12}} \times \frac{C_2^3}{C_2^4} = \frac{14}{99}$

17C.34 <u>HKALE MS 2010 – 5</u>

(a) $49 - (20 + a) = 27 \Rightarrow a = 2$

$\frac{49 + \quad + (80 + b)}{20} \quad \frac{22 + 49 + \cdots + (80 + b)}{21} = 2$

$\frac{1274 + b}{20} - \frac{1296 + b}{21} = 2$

$b = 6$

$\bar{x} = (1296 + 6) \div 21 = 62$

(b) (i) Required p $= \frac{C_6^{15}}{C_6^{20}} \quad \frac{1001}{7752}$

(ii) Required p $= 1 - \frac{1001}{7752} - \frac{C_1^5 C_5^{15}}{C_6^{20}} = \frac{937}{1938}$

17C.35 <u>HKALE MS 2012 – 6</u>

(a) $\frac{(30 + a) + 52 + \cdots + 92 + (90 + b)}{30} = 71$

$2120 + a + b = 2130$

$a + b = 10$

$(90 + b) - (30 + a) = 56 \Rightarrow a - b = 4$

Solving, $a = 7$, $b = 3$

$\Rightarrow \sigma = 12.7$

(b) Required p $= \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143}$

17C.36 <u>HKALE MS 2013 – 11</u>

(a) (i) Required p $= 1 - (1 - q)^2 = 2q - q^2$

(ii) $2q - q^2 = \frac{7}{16} \Rightarrow q = 0.25$ or 1.75 (rejected)

(b) (i) P(a week is *fully engaged*) $= (1 - q)^5 = 0.75^5$

$\therefore$ Required p $= C_1^4 (0.75^5)(1 - 0.75^5)^3 \times 0.75^5$
$= 0.0999$

(c) (i) Required p $= 0.25^5 = \frac{1}{1024}$

(ii) Required p $= C_2^5 (0.45)^2 \left(0.25^3 + C_1^3 (0.25)^2 (0.3)\right)$
$= 0.1455$

(iii) P(6 a/c switched off) $= C_1^5 (0.25)(0.3)^4$
$+ C_2^5 (0.25)^2 C_2^3 (0.3)^2 (0.45) + C_3^5 (0.25)^3 (0.45)^2$
$= 0.117703125$

$\therefore$ Required p $= 1 - \frac{C_1^5 (0.25)(0.3)^4}{0.117703125} = 0.914$

17C.37 <u>HKDSE MA 2013 – I – 10</u>

(a) Median $= 31$

Mode $= 23$

(b) (i) $(60 + b) - (20 + a) = 47 \Rightarrow b - a = 7$

$\because 0 \leq a \leq 5$ and $7 \leq b \leq 9$

$\therefore (a, b) = (0, 7), (1, 8)$ or $(2, 9)$

(ii) Required p $= \frac{3 + 3 + 3 + 3 + 2 + 9 + 9}{20 \times 13} = \frac{8}{65}$

17C.38 <u>HKDSE MA 2014 – I – 19</u>

(a) Required p $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^4 \frac{1}{6} + \left(\frac{5}{6}\right)^6 \frac{1}{6} + \ldots$
$= \frac{\frac{1}{6}}{1 - \left(\frac{5}{6}\right)^2} = \frac{6}{11}$

(b) (i) Expected no $= 10 \times \frac{1}{8} + 5 \times \frac{7 \cdot 2!}{8^2} = \frac{75}{32}$

(ii) Expected no of tokens with Option 2

$= 50 \times \frac{1}{8^2} + 10 \times \frac{6 \cdot 3!}{8^3} + 5 \times \frac{7 \times 2 \times C_2^3}{8^3} = \frac{485}{256}$

$< \frac{75}{32}$

Option 1

(iii) P(Ada getting no tokens) $= 1 - \frac{1}{6} \times \left(\frac{1}{8} + \frac{7 \cdot 2!}{8^2}\right)$
$= \frac{13}{16} < 0.9$

$\therefore$ NO