15 Mensuration

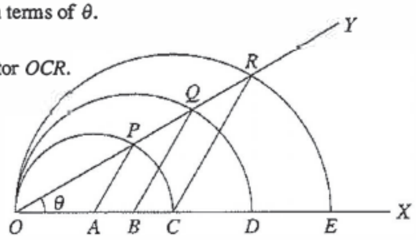
15A Lengths and areas of plane figures

15A.1 HKCEE MA 1980(1/1*/3) I 10 (To continue as **12A.1**.)

A, B and C are three points on the line OX such that $OA = 2$, $OB = 3$ and $OC = 4$. With A, B, C as centres and OA, OB, OC as radii, three semi circles are drawn as shown in the figure. A line OY cuts the three semi-circles at P, Q, R respectively.

(a) If $\angle YOX = \theta$, express $\angle PAX$, $\angle QBX$ and $\angle RCX$ in terms of θ.

(b) Find the following ratios:
area of sector OAP : area of sector OBQ : area of sector OCR.



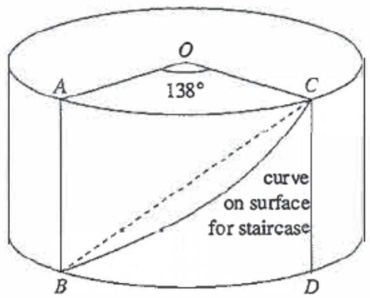
15A.2 (HKCEE MA 1981(1/2/3) – I – 12)

The figure shows a cylinder 10 metres high and 10 metres in radius used for storing coal gas. AB and CD are two vertical lines on the curved surface of the cylinder. The arc AC subtends an angle of 138° at the point O, which is the centre of the top of the cylinder.

(a) Inside the cylinder, a straight pipe runs from B to C. Calculate the length of the pipe BC correct to 3 significant figures.

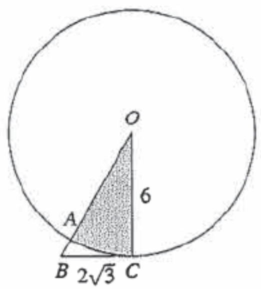
(b) Calculate the area of the curved surface $ABDC$ bounded by the minor arcs AC, BD and the lines AB, CD.

(c) A staircase from B to C is built along the shortest curve on the curved surface $ABDC$. Find the length of the curve.



15A.3 HKCEE MA 1982(1/2/3) – I – 4

In the figure, the circle, centre O and radius 6, touches the straight line BC at C. $BC = 2\sqrt{3}$. OAB is a straight line. Find the area of the shaded sector in terms of π.



15A.4 HKCEE MA 1982(1/2/3) – I – 9

(In this question, answers should be given in surd form.)

In Figures (1) and (2), $ABCDEF$ is a regular hexagon with $AB = 1$.

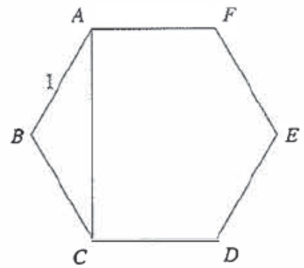


Figure (1)

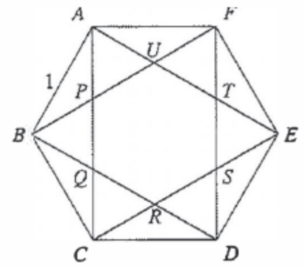


Figure (2)

(a) Calculate the area of the hexagon in Figure (1) and the length of its diagonal AC.

(b) In Figure (2), $PQRSTU$ is another regular hexagon formed by the diagonals of $ABCDEF$.

 (i) Calculate the length of PQ.

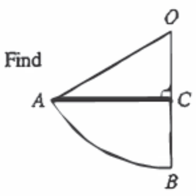
 (ii) Calculate the area of the hexagon $PQRSTU$.

15A.5 HKCEE MA 1983(A/B) I 5

In the figure, O is the centre of the sector OAB. $OA = 30$, $CB = 15$ and $AC \perp OB$. Find

(a) $\angle AOC$,

(b) the length of the arc AB in terms of π.



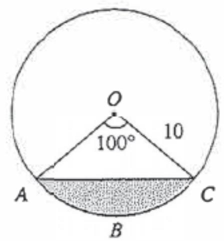
15A.6 HKCEE MA 1988 – I – 5

In the figure, ABC is a circle with centre O and radius 10. $\angle AOC = 100°$. Calculate, correct to 2 decimal places,

(a) the area of sector $OABC$,

(b) the area of $\triangle OAC$,

(c) the area of segment ABC.

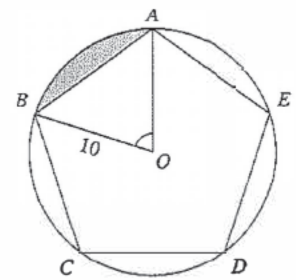


15A.7 HKCEE MA 1992 – I – 7

In the figure, $ABCDE$ is a regular pentagon inscribed in a circle with centre O and radius 10.

(a) Find $\angle AOB$ and the area of triangle OAB.

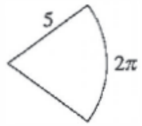
(b) Find the area of the shaded part in the figure.



Provided by dse.life

15A.8 HKCEE MA 1994 – I 2(d)

In the figure, find the area of the sector.

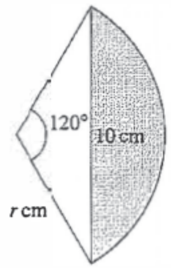


15A.9 HKCEE MA 1999 – I 9

The figure shows a sector.

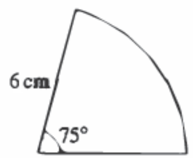
(a) Find r.

(b) Find the area of the shaded region.



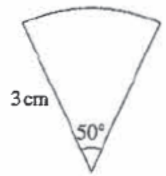
15A.10 HKCEE MA 2000 I 3

Find the area of the sector in the figure.



15A.11 HKCEE MA 2001 – I – 3

Find the perimeter of the sector in the figure.

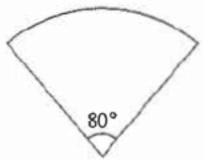


15A.12 HKCEE MA 2004 – I – 9

In the figure, the area of the sector is $162\pi\,\text{cm}^2$.

(a) Find the radius of the sector.

(b) Find the perimeter of the sector in terms of π.

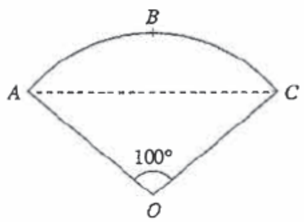


15A.13 HKCEE MA 2005 – I – 9

In the figure, $OABC$ is a sector with $\widehat{ABC} = 10\pi\,\text{cm}$.

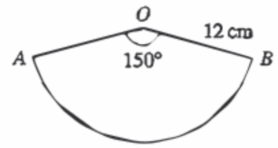
(a) Find OA.

(b) Find the area of segment ABC.



15A.14 HKCEE MA 2006 – I – 4

In the figure, the radius of the sector OAB is 12 cm. Find the length of $\widehat{AB}$ in terms of π.

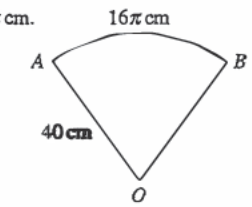


15A.15 HKCEE MA 2007 – I – 9

In the figure, the radius of the sector AOB is 40 cm. It is given that $\widehat{AB} = 16\pi$ cm.

(a) Find $\angle AOB$.

(b) Find the area of the sector AOB in terms of π.



15A.16 HKDSE MA 2015 – I – 9

The radius and the area of a sector are 12 cm and $30\pi\,\text{cm}^2$ respectively.

(a) Find the angle of the sector.

(b) Express the perimeter of the sector in terms of π.

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15B Volumes and surface areas of solids

15B.1 HKCEE MA 1983(A/B) – I – 8

The solid in Figure (1) is made up of two parts. The lower part is a right circular cylinder of height h cm and radius r cm; the upper part is a hemisphere of the same radius r cm. The two parts are of the same volume.

(a) Find the ratio $r : h$.

(b) Figure (2) shows a section of the solid through the axis of the cylinder. The perimeter of this section is 136 cm.

(i) Calculate r to 2 significant figures.

(ii) Calculate the total external surface area (including the base) of the solid in cm^2 to 1 significant figure.

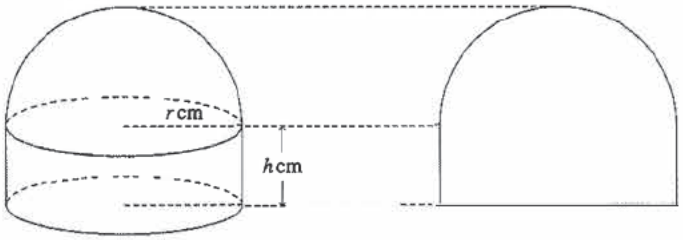


Figure (1) Figure (2)

15B.2 HKCEE MA 1984(A/B) I 12

In the figure, all vertical cross-sections of the solid that are parallel to $APBCQD$ are identical. $ABCD$, $BRTC$ and $ABRS$ are squares, each of side 20 cm. P is the mid point of AB. CQD is a circular arc with centre P and radius PC.

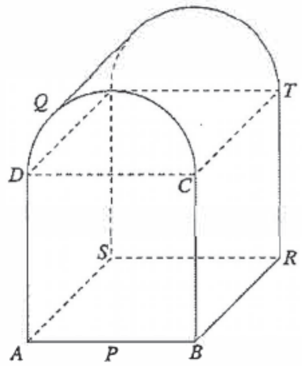
(In this question, give your answers correct to 1 decimal place.)

(a) Find $\angle CPD$.

(b) Find, in cm, the length of the arc CQD.

(c) Find, in cm^2, the area of the cross section $APBCQD$.

(d) Find, in cm^2, the total surface area of the solid.



15B.3 HKCEE MA 1985(A/B) I 11

Figure (1) shows a solid right circular cone. O is the vertex and P is a point on the circumference of the base. The area of the curved surface is 135π cm^2 and the radius of the base is 9 cm.

(a) (i) Find the length of OP.

(ii) Find the height of the cone.

(b) The cone in Figure (1) is cut into two portions by a plane parallel to its base. The upper portion is a cone of base radius 3 cm. The lower portion is a frustum of height x cm.

(i) Find the value of x.

(ii) A right cylindrical hole of radius 3 cm is drilled through the frustum (see Figure (2)). Find the volume of the solid which remains in the frustum. (Give your answer in terms of π.)

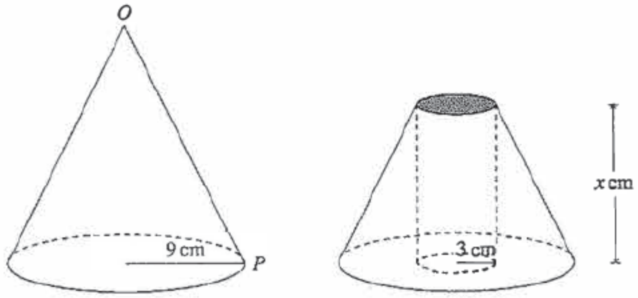


Figure (1) Figure (2)

15B.4 HKCEE MA 1986(A/B) I – 12

Figure (1) shows a solid consisting of a right circular cone and a hemisphere with a common base which is a circle of radius 6. The volume of the cone is equal to $\frac{4}{3}$ of the volume of the hemisphere.

(a) (i) Find the height of the cone.

(ii) Find the volume of the solid. (Leave your answer in terms of π.)

(b) (i) The solid is cut into two parts. The upper part is a right circular cone of height y and base radius x as shown in Figure (2). Find $\frac{x}{y}$.

(ii) If the two parts in (b)(i) are equal in volume, find y, correct to 1 decimal place.

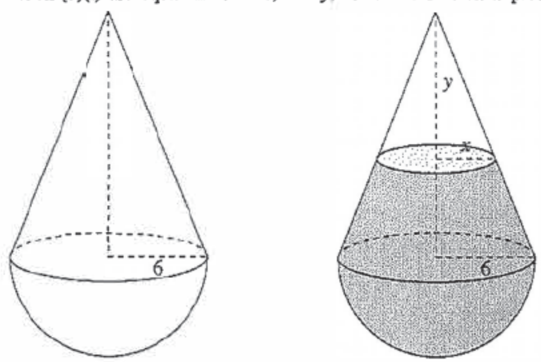


Figure (1) Figure (2)

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15B.5 HKCEE MA 1989 – I – 11

Figure (1) shows a rectangular swimming pool 50 m long and 20 m wide. The floor of the pool is an inclined plane. The depth of water is 10 m at one end and 2 m at the other.

(a) Find the volume of water in the pool in m^3.

(b) Water in the pool is now pumped out through a pipe of internal radius 0.125 m. Water flows in the pipe at a constant speed of 3 m/s.

(i) Find the volume of water, in m^3, REMAINING in the pool when the depth of water is 8 m at the deeper end.

(ii) Find the volume of water pumped out in 8 hours, correct to the nearest m^3.

(iii) Let h metres be the depth of water at the deeper end after 8 hours (see Figure (2)). Find the value of h, correct to 1 decimal place.

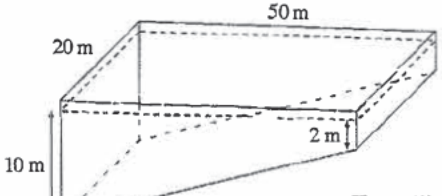


Figure (1)

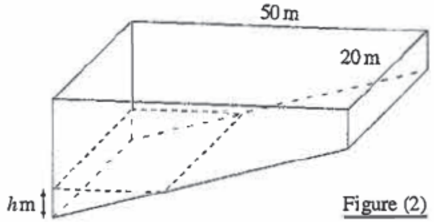


Figure (2)

15B.6 HKCEE MA 1990 – I – 11 (To continue as 4B.8.)

A solid right circular cylinder has radius r and height h. The volume of the cylinder is V and the total surface area is S.

(a) (i) Express S in terms of r and h.

(ii) Show that $S = 2\pi r^2 + \dfrac{2V}{r}$.

15B.7 HKCEE MA 1991 – I – 11

Figure (1) shows a metal bucket. Its slant height AB is 60 cm. The diameter AD of the base is 40 cm and the diameter BC of the open top is 80 cm. The curved surface of the bucket is formed by the thin metal sheet $ABB'A'$ shown in Figure (2), where $\overparen{ADA'}$ and $\overparen{BCB'}$ are arcs of concentric circles with centre O.

(a) Find OA and $\angle AOA'$.

(b) Find the area of the metal sheet $ABB'A'$, leaving your answer in terms of π.

(c) There is an ant at the point A on the outer curved surface of the bucket. Find the shortest distance for it to crawl along the outer **curved surface** of the bucket to reach the point C.

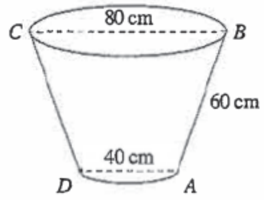


Figure (1)

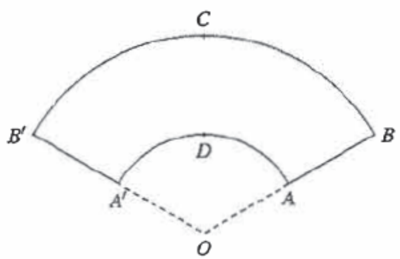


Figure (2)

15B.8 (HKCEE MA 1993 – I – 9)

The figure shows a right circular cylinder. O is the centre and r is the radius of its top face. A chord AB divides the area of the top face in the ratio 4 : 1 and subtends an angle α at O. C is a point on the minor arc AB.

(a) (i) Find the area of the sector $OACB$ in terms of r and α.

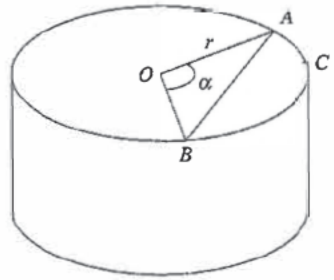
(ii) Find the area of the segment ACB in terms of r and α.

(iii) Show that $\sin\alpha = \left(\dfrac{\alpha}{180°} - \dfrac{2}{5}\right)\pi$.

(iv) *[Out of syllabus]*

(v) *[Out of syllabus: The result $\alpha \approx 121°$ is obtained.]*

(b) The cylinder is cut along AB into 2 parts by a plane perpendicular to its top face. Find the ratio of the curved surface areas of the two parts in the form $k : 1$, where $k > 1$.



15B.9 HKCEE MA 1994 – I – 10

Figure (1) shows the longitudinal section of a right cylindrical water tank of base radius 2 m and height 3 m. The tank is filled with water to a depth of 1.5 m.

(a) Express the volume of water in the tank in terms of π.

(b) If a solid sphere of radius 0.6 m is put into the tank and is completely submerged in water, the water level rises by h metres. Find h (see Figure (2)).

(c) A solid sphere of radius r m is put into the tank and is just submerged in water (see Figure (3)).

(i) Show that $2r^3 - 12r + 9 = 0$.

(ii) *[Out of syllabus]*

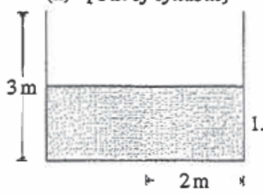


Figure (1)

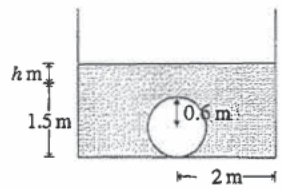


Figure (2)

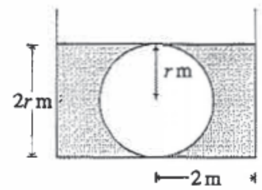


Figure (3)

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15B.10 HKCEE MA 1995 – I – 13

A right cylindrical vessel of base radius 4 cm and height 11 cm is placed on a horizontal table. A right conical vessel of base radius 6 cm and height 12 cm is placed, with its axis vertical, in the cylindrical vessel. The conical vessel is full of water and the cylindrical vessel is empty. Figure (1) shows the longitudinal sections of the two vessels where A is the vertex of the conical vessel.

(a) Find, in terms of π, the volume of water in the conical vessel.

(b) The vertex A is d cm from the base of the cylindrical vessel. Use similar triangles to find d.

(c) Suppose water leaks out from the conical vessel through a small hole at the vertex A into the cylindrical vessel.

 (i) Find, in terms of π, the volume of water that has leaked out when the water level in the cylindrical vessel reaches the vertex A.

 (ii) If $104\pi\,\text{cm}^3$ of water has leaked out and the water level in the cylindrical vessel is h cm above the vertex A (see Figure (2)), show that $h^3 - 192h + 672 = 0$.

 (iii) *[Out of syllabus]*

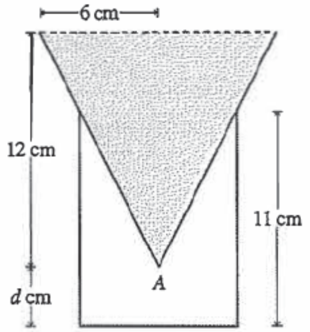


Figure (1)

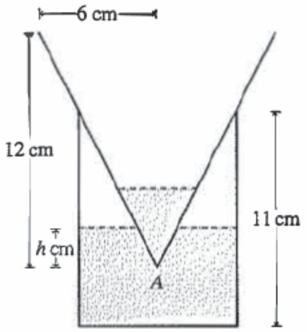


Figure (2)

15B.11 HKCEE MA 1996 I 8

Figure (1) shows a paper cup in the form of a right circular cone of base radius 5 cm and height 12 cm.

(a) Find the capacity of the paper cup.

(b) If the paper cup is cut along the slant side AB and unfolded to become a sector as shown in Figure (2), find

 (i) the area of the sector;

 (ii) the angle of the sector.

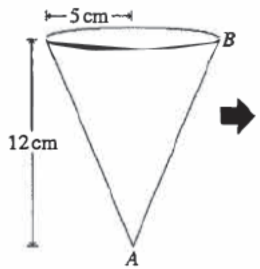


Figure (1)

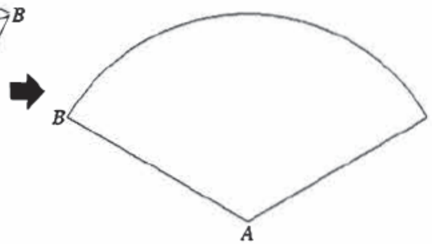


Figure (2)

15B.12 HKCEE MA 1997 I 12

Figure (1) shows a greenhouse $VABCD$ in the shape of a right pyramid with a square base of side 6 m. M is the mid-point of BC and VN is the height of the pyramid. Each of the triangular faces makes an angle θ with the square base.

(a) (i) Express VN and VM in terms of θ.

 (ii) Find the capacity and total surface area of the greenhouse (excluding the base) in terms of θ.

(b) Figure (2) shows another greenhouse in the shape of a right cylinder with base radius r m and height h m. It is known that both the base areas and the capacities of the two greenhouses are equal.

 (i) Express r in terms of π.

 (ii) Express h in terms of θ.

 (iii) If the total surface areas of the two greenhouses (excluding the bases) are equal, show that $3 + \sqrt{\pi}\tan\theta = \dfrac{3}{\cos\theta}$.

 (iv) *[Out of syllabus]*

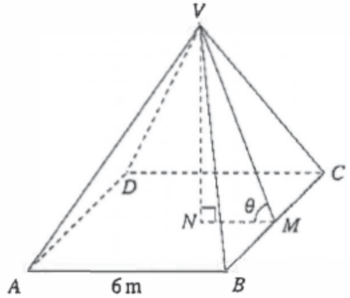


Figure (1)

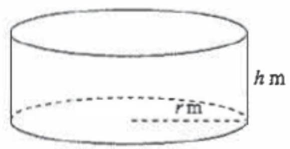
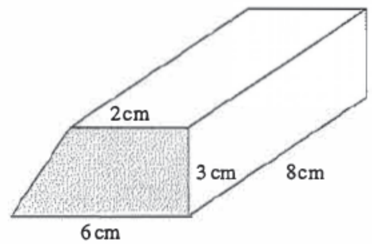


Figure (2)

15B.13 HKCEE MA 1998 – I – 1

The figure shows a right prism, the cross section of which is a trapezium. Find the volume of the prism.



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15B.14 HKCEE MA 1999 – I – 13

In Figure (1), a piece of wood in the form of an inverted right circular cone is cut into two portions by a plane parallel to its base. The upper portion is a frustum with height 10 cm, and the radii of the two parallel faces are 9 cm and 4 cm respectively. The pen stand shown in Figure (2) is made from the frustum by drilling a hole in the middle. The hole consists of a cylindrical upper part of radius 5 cm and a hemispherical lower part of the same radius. The depth of the hole is 9 cm.

(a) Find, in terms of π, the capacity of the hole.

(b) Find, in terms of π, the volume of wood in the pen-stand.

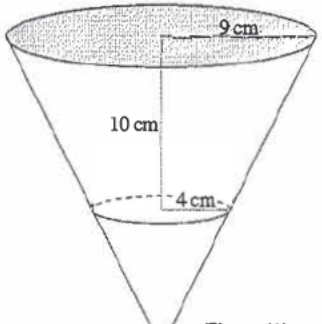


Figure (1)

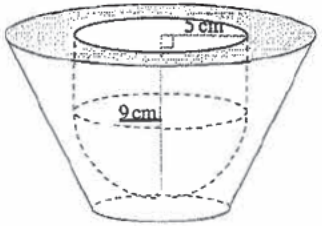


Figure (2)

15B.15 HKCEE MA 2002 – I – 15

(a) Figure (1) shows two vessels of the same height 24 cm, one in the form of a right circular cylinder of radius 6 cm and the other a right circular cone of radius 9 cm. The vessels are held vertically on two horizontal platforms, one of which is 5 cm higher than the other. To begin with, the cylinder is empty and the cone is full of water. Water is then transferred into the cylinder from the cone until the water in both vessels reaches the same horizontal level. Let h cm be the depth of water in the cylinder.

(i) Show that $h^3 + 15h^2 + 843h \quad 13699 = 0$.

(ii) *[Out of syllabus; the result $h = 11.8$ (cor. to 1 d.p.) is obtained.]*

(b) Figure (2) shows a set up which is modified from the one in Figure (1). The lower part of the cone is cut off and sealed to form a frustum of height 19 cm. The two vessels are then held vertically on the same horizontal platform. To begin with, the cylinder is empty and the frustum is full of water. Water is then transferred into the cylinder from the frustum until the water in both vessels reaches the same horizontal level. Find the depth of water in the cylinder.

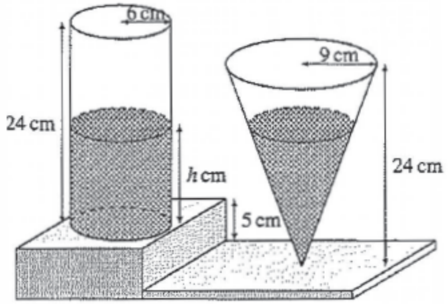


Figure (1)

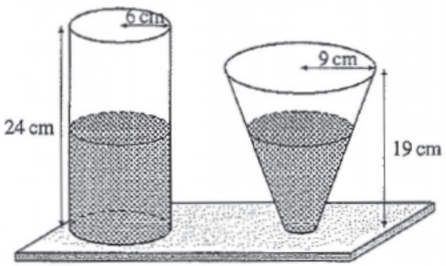
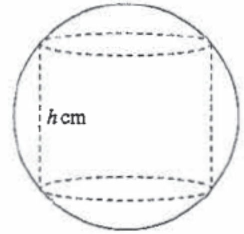


Figure (2)

15B.16 HKCEE MA 2004 – I – 14

In the figure, a solid right circular cylinder of height h cm and volume V cm^3 is inscribed in a thin hollow sphere of radius 12 cm.



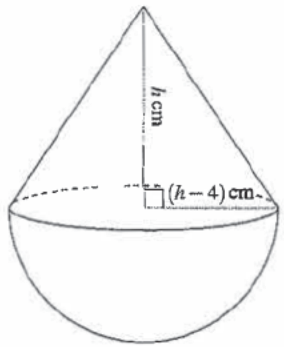
(a) Prove that $V = 144\pi h - \frac{\pi}{4}h^3$.

(b) ***[Out of syllabus]***

(c) If the volume of the cylinder is 286π cm^3, find the exact height(s) of the cylinder.

15B.17 HKCEE MA 2005 – I – 12

The figure shows a solid consisting of a right circular cone and a hemi sphere with a common base. The height and the base radius of the cone are h cm and $(h-4)$ cm respectively. It is known that the volume of the cone is equal to the volume of the hemisphere.



(a) Find h.

(b) Find the total surface area of the solid correct to the nearest cm^2.

(c) If the solid is cut into two identical parts, find the increase in the total surface area correct to the nearest cm^2.

15B.18 HKCEE MA 2009 – I – 13

(a) The height and the base radius of an inverted right circular conical container are 18 cm and 12 cm respectively.

(i) Find the capacity of the circular conical container in terms of π.

(ii) Figure (1) shows a frustum which is made by cutting off the lower part of the container. The height of the frustum is 6 cm. Find the volume of the frustum in terms of π.

(b) Figure (2) shows a vessel which is held vertically. The vessel consists of two parts with a common base: the upper part is the frustum shown in Figure (1) and the lower part is a right circular cylinder of height 10 cm. Some water is poured into the vessel. The vessel now contains 884π cm^3 of water.

(i) Find the depth of water in the vessel.

(ii) If a piece of metal of volume 1000 cm^3 is then put into the vessel and the metal is totally immersed in the water, will the water overflow? Explain your answer.

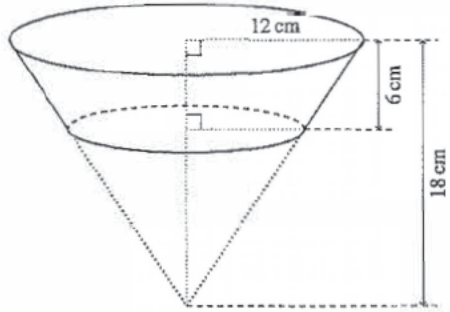


Figure (1)

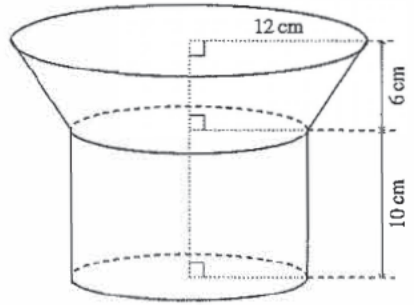


Figure (2)

Provided by dse.life

15B.19 HKCEE MA 2011 – I – 13

Figure (1) shows the thin paper sector $OXYZ$ of area $2880\pi\ \text{mm}^2$. By joining OX and OZ together, $OXYZ$ is folded to form an inverted right circular conical container as shown in Figure (2).

(a) Find the length of OX.

(b) Find the height of the container

(c) Suppose that the container is held vertically. If water of volume $150\ \text{cm}^3$ is poured into the container, will the water overflow? Explain your answer.

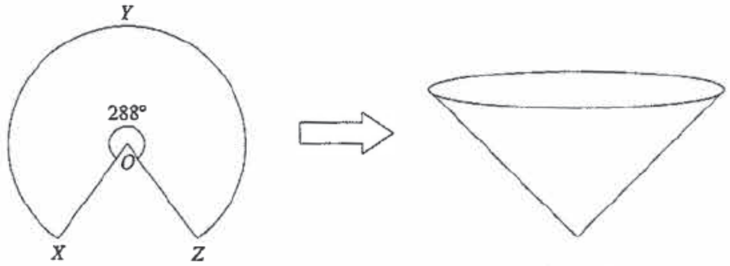


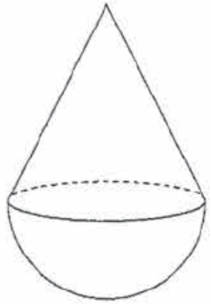
Figure (1) Figure (2)

15B.20 HKDSE MA SP – I – 6

The figure shows a solid consisting of a hemisphere of radius r cm joined to the bottom of a right circular cone of height 12 cm and base radius r cm. It is given that the volume of the circular cone is twice the volume of the hemisphere.

(a) Find r.

(b) Express the volume of the solid in terms of π.

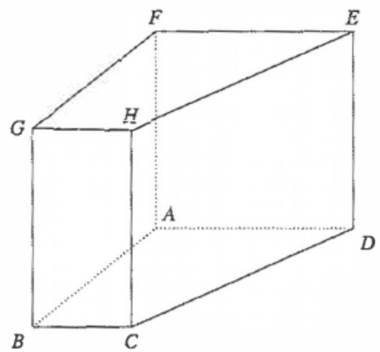


15B.21 HKDSE MA 2012 – I – 9

In the figure, the volume of the solid right prism $ABCDEFGH$ is $1020\ \text{cm}^3$. The base $ABCD$ of the prism is a trapezium, where AD is parallel to BC. It is given that $\angle BAD = 90°$, $AB = 12$ cm, $BC = 6$ cm and $DE = 10$ cm. Find

(a) the length of AD,

(b) the total surface area of the prism $ABCDEFGH$.



15B.22 HKDSE MA 2012 I – 12

Figure (1) shows a solid metal right circular cone of base radius 48 cm and height 96 cm.

(a) Find the volume of the circular cone in terms of π.

(b) A hemispherical vessel of radius 60 cm is held vertically on a horizontal surface. The vessel is fully filled with milk.

(i) Find the volume of the milk in the vessel in terms of π.

(ii) The circular cone is now held vertically in the vessel as shown in Figure (2). A craftsman claims that the volume of the milk remaining in the vessel is greater than $0.3\ \text{m}^3$. Do you agree? Explain your answer.

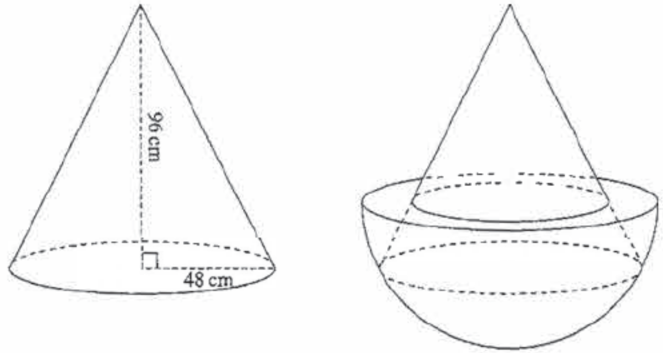


Figure (1) Figure (2)

15B.23 HKDSE MA 2020 – I – 12

The height and the base radius of a solid right circular cone are 36 cm and 15 cm respectively. The circular cone is divided into three parts by two planes which are parallel to its base. The heights of the three parts are equal. Express, in terms of π,

(a) the volume of the middle part of the circular cone; (3 marks)

(b) the curved surface area of the middle part of the circular cone. (3 marks)

Provided by dse.life

15C Similar plane figures and solids

15C.1 HKCEE MA 1981(1/2/3) I – 1

The capacities of two spherical tanks are in the ratio 27 : 64. If 72 kg of paint is required to paint the outer surface of the smaller tank, then how many kilograms of paint would be required to paint the outer surface of the bigger tank?

15C.2 HKCEE MA 1987(A/B) I 9

Figure (1) shows a test-tube consisting of a hollow cylindrical tube joined to a hemisphere bowl of the same radius. The height of the cylindrical tube is h cm and its radius is r cm. The capacity of the test-tube is $108\pi \text{cm}^3$. The capacity of the hemispherical part is $\frac{1}{6}$ of the whole test tube.

(a) (i) Find r and h.

(ii) The test tube is placed upright and water is poured into it until the water level is 4 cm beneath the rim as shown in Figure (2). Find the volume of the water. (Leave your answer in terms of π.)

(b) The water in the test-tube is poured into a right circular conical vessel placed upright as shown in Figure (3). If the depth of water is half the height of the vessel, find the capacity of the vessel. (Leave your answer in terms of π.)

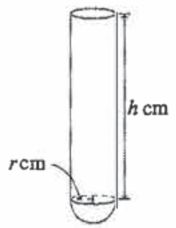


Figure (1)

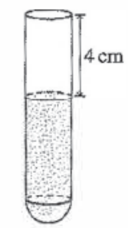


Figure (2)

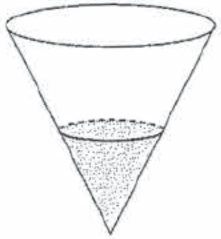


Figure (3)

15C.3 HKCEE MA 1992 – I – 12

Figure (1) shows a vertical cross-section of a separating funnel with a small tap at its vertex. The funnel is in the form of a right circular cone of base radius 9 cm and height 20 cm. It contains oil and water (which do not mix) of depths 5 cm and 10 cm respectively, with the water at the bottom.

(a) (i) Find the capacity of the separating funnel in terms of π.

(ii) Find the ratios volume of water : total volume of oil and water : capacity of the funnel.
Hence, or otherwise, find the ratios volume of water : volume of oil : capacity of the funnel.

(b) All the water in the funnel is drained through the tap into a glass tube of height 15 cm. The glass tube consists of a hollow cylindrical upper part of radius 3 cm and a hollow hemispherical lower part of the same radius, as shown in Figure (2). Find the depth of the water in the glass tube.

(c) After all the water has been drained into the glass tube, find the depth of the oil remaining in the funnel.

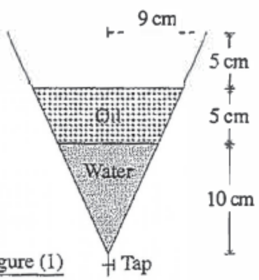


Figure (1)

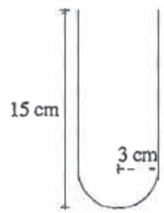


Figure (2)

15C.4 HKCEE MA 1994 – I – 2(e)

The ratio of the radii of two spheres is 2 : 3. Find the ratio of their volumes.

15C.5 HKCEE MA 1997 – I – 7

(To continue as 8C.8.)

The ratio of the volumes of two similar solid circular cones is 8 : 27.

(a) Find the ratio of the height of the smaller cone to the height of the larger cone.

15C.6 HKCEE MA 2000 – I 8

On a map of scale 1 : 5000, the area of the passenger terminal of the Hong Kong International Airport is 220cm^2. What is the actual area, in m^2, occupied by the terminal on the ground?

15C.7 HKCEE MA 2002 – I – 6

The radius of a circle is 8 cm. A new circle is formed by increasing the radius by 10%.

(a) Find the area of the new circle in terms of π.

(b) Find the percentage increase in the area of the circle.

15C.8 HKCEE MA 2002 I – 11

(Continued from 8C.13.)

The area of a paper bookmark is A cm^2 and its perimeter is P cm. A is a function of P. It is known that A is the sum of two parts, one part varies as P and the other part varies as the square of P. When $P = 24$, $A = 36$ and when $P = 18$, $A = 9$.

(a) Express A in terms of P.

(b) (i) The best-selling paper bookmark has an area of 54cm^2. Find the perimeter of this bookmark.

(ii) The manufacturer of the bookmarks wants to produce a gold miniature similar in shape to the best selling paper bookmark. If the gold miniature has an area of 8cm^2, find its perimeter.

15C.9 HKCEE MA 2003 I – 13

Sector OCD is a thin metal sheet. The sheet $ABCD$ is formed by cutting away sector OBA from sector OCD as shown in Figure (1).
It is known that $\angle COD = x°$, $AD = BC = 24$ cm, $OA = OB = 56$ cm and $\overset{\frown}{CD} = 30\pi$ cm.

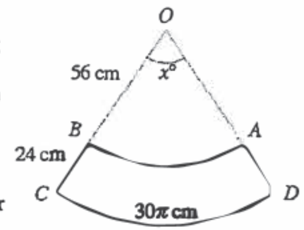


Figure (1)

(a) (i) Find x.

(ii) Find, in terms of π, the area of $ABCD$.

(b) Figure (2) shows another thin metal sheet $EFGH$ which is similar to $ABCD$. It is known that $FG = 18$ cm.

(i) Find, in terms of π, the area of $EFGH$.

(ii) By joining EH and FG together, $EFGH$ is then folded to form a hollow frustum of base radius r cm as shown in Figure (3). Find r.

Figure (2)

Figure (3)

Provided by dse.life

15C.10 HKCEE MA 2006 – I 13

In Figure (1), the frustum of height 8 cm is made by cutting off a right circular cone of base radius 3 cm from a solid right circular cone of base radius 6 cm. Figure (2) shows the solid X formed by fixing the frustum onto a solid hemisphere of radius 6 cm. The solid Y in Figure (3) is similar to X. The ratio of the surface area of X to the surface area of Y is $4:9$.

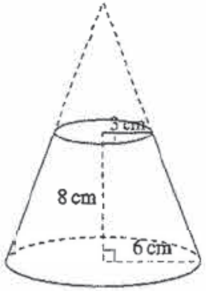
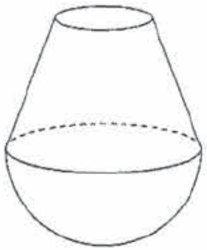
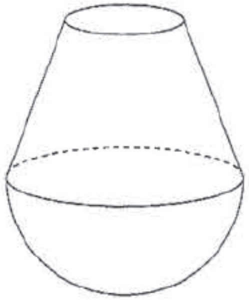


Figure (1)



Solid X

Figure (2)

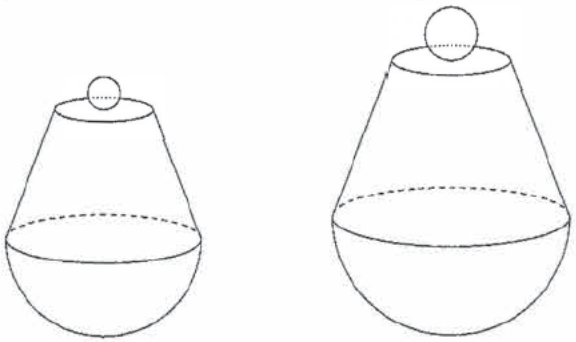


Solid Y

Figure (3)

(a) Find the volume of X and the volume of Y. Give your answers in terms of π.

(b) In Figure (4), the solid X' is formed by fixing a solid sphere of radius 1 cm onto the centre of the top circular surface of X while another solid Y' is formed by fixing a solid sphere of radius 2 cm onto the centre of the top circular surface of Y. Are X' and Y' similar? Explain your answer.

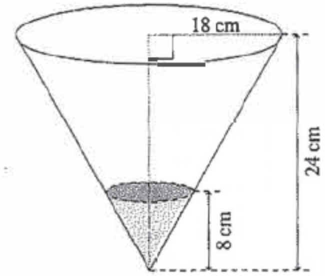


Solid X' Solid Y'

Figure (4)

15C.11 HKCEE MA 2007 – I 11

The figure shows an inverted right circular conical vessel which is held vertically. The height and the base radius of the vessel are 24 cm and 18 cm respectively. The vessel contains some water and the depth of the water is 8 cm.



(a) Find the volume of water contained in the vessel in terms of π.

(b) (i) Find the area of the wet curved surface of the vessel in terms of π.

(ii) Another inverted right circular conical vessel with height 36 cm and base radius 27 cm is held vertically. This bigger vessel and the vessel shown in the figure contain the same volume of water. Find the area of the wet curved surface of the bigger vessel in terms of π.

15C.12 HKCEE MA 2008 – I – 13

In Figure (1), sector $OABC$ is a thin metal sheet. By joining OA and OC together, $OABC$ is folded to form a right ciruclar cone X as shown in Figure (2). It is given that $OA = 20$ cm.

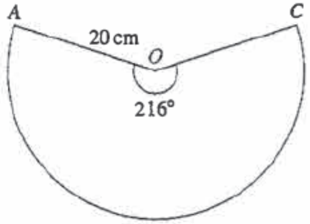


Figure (1)

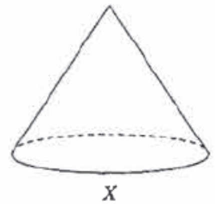


Figure (2)

(a) Find the base radius and the height of X.

(b) Find the volume of X in terms of π.

(c) In Figure (3), sector $PDEF$ is another thin metal sheet. By joining PD and PF together, $PDEF$ is folded to form another right circular cone Y as shown in Figure (4). It is given that $PD = 10$ cm. Are X and Y similar? Explain your answer.

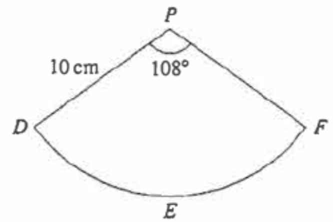


Figure (3)

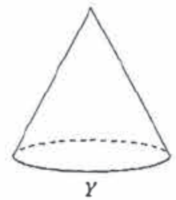


Figure (4)

Provided by dse.life

15C.13 HKCEE MA 2010 – I – 13

In Figure (1), $ABCDEF$ is a wooden block in the form of a right prism. It is given that $AB = AC = 17$ cm, $BC = 16$ cm and $CD = 20$ cm.

(a) Find the area of $\triangle ABC$.

(b) Find the volume of the wooden block $ABCDEF$.

(c) The plane $PQRS$ which is parallel to the face $BCDF$ cuts the wooden block $ABCDEF$ into two blocks $APQRES$ and $BCQPSFDR$ as shown in Figure (2). It is given that $PQ = 4$ cm.

(i) Find the volume of the wooden block $APQRES$.

(ii) Are the wooden blocks $APQRES$ and $ABCDEF$ similar? Explain your answer.

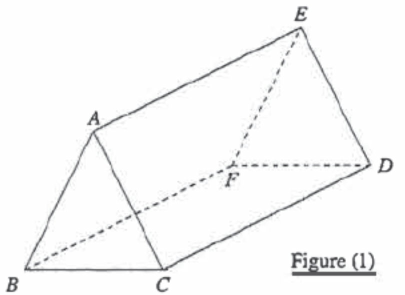


Figure (1)

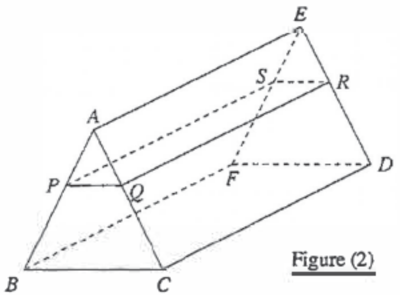


Figure (2)

15C.14 HKDSE MA 2012 – I – 11 (Continued from 8C.23.)

Let \$$C$ be the cost of painting a can of surface area A m^2. It is given that C is the sum of two parts, one part is a constant and the other part varies as A. When $A = 2$, $C = 62$; when $A = 6$, $C = 74$.

(a) Find the cost of painting a can of surface area 13 m^2.

(b) There is a larger can which is similar to the can described in (a). If the volume of the larger can is 8 times that of the can described in (a), find the cost of painting the larger can.

15C.15 HKDSE MA 2013 – I – 13

In a workshop, 2 identical solid metal right circular cylinders of base radius R cm are melted and recast into 27 smaller identical solid right circular cylinders of base radius r cm and height 10 cm. It is given that the base area of a larger circular cylinder is 9 times that of a smaller one.

(a) Find

(i) $r : R$,

(ii) the height of a larger circular cylinder.

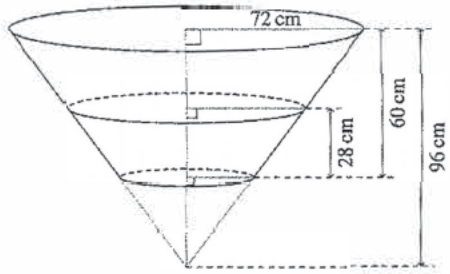
(b) A craftsman claims that a smaller circular cylinder and a larger circular cylinder are similar. Do you agree? Explain your answer.

15C.16 HKDSE MA 2014 – I – 14

The figure shows a vessel in the form of a frustum which is made by cutting off the lower part of an inverted right circular cone of base radius 72 cm and height 96 cm. The height of the vessel is 60 cm. The vessel is placed on a horizontal table. Some water is now poured into the vessel. John finds that the depth of water in the vessel is 28 cm.

(a) Find the area of the wet curved surface of the vessel in terms of π.

(b) John claims that the volume of water in the vessel is greater than 0.1 m^3. Do you agree? Explain your answer.



15C.17 HKDSE MA 2016 – I – 11

An inverted right circular conical vessel contains some milk. The vessel is held vertically. The depth of milk in the vessel is 12 cm. Peter then pours 444π cm^3 of milk into the vessel without overflowing. He now finds that the depth of milk in the vessel is 16 cm.

(a) Express the final volume of milk in the vessel in terms of π.

(b) Peter claims that the final area of the wet curved surface of the vessel is at least 800 cm^2. Do you agree? Explain your answer.

15C.18 HKDSE MA 2017 – I – 12

A solid metal right prism of base area 84 cm^2 and height 20 cm is melted and recast into two similar solid right pyramids. The bases of the two pyramids are squares. The ratio of the base area of the smaller pyramid to the base area of the larger pyramid is 4 : 9.

(a) Find the volume of the larger pyramid.

(b) If the height of the larger pyramid is 12 cm, find the total surface area of the smaller pyramid.

15C.19 HKDSE MA 2018 – I – 14

A right circular cylindrical container of base radius 8 cm and height 64 cm and an inverted right circular conical vessel of base radius 20 cm and height 60 cm are held vertically. The container is fully filled with water. The water in the container is now poured into the vessel.

(a) Find the volume of water in the vessel in terms of π.

(b) Find the depth of water in the vessel.

(c) If a solid metal sphere of radius 14 cm is then put into the vessel and the sphere is totally immersed in the water, will the water overflow? Explain your answer.

15C.20 HKDSE MA 2019 – I – 9

The sum of the volumes of two spheres is 324π cm^3. The radius of the larger sphere is equal to the diameter of the smaller sphere. Express, in terms of π,

(a) the volume of the larger sphere;

(b) the sum of the surface areas of the two spheres.

Provided by dse.life

15 Mensuration

15A Lengths and areas of plane figures

15A.1 <u>HKCEE MA 1980(1/1*/3) I – 10</u>

(a) $\angle PAX = 2\theta$ ($\angle$ at centre twice $\angle$ at $\odot^{ce}$)
Similarly, $\angle QBX = \angle RCX = 2\theta$

(b) Areas of sector $OAP : OBQ : OCR = (OA : OB : OC)^2$
$= 4 : 9 : 16$

(c) $\cos \angle RCX = \frac{CD}{CR} = \frac{2}{4} = \frac{1}{2} \Rightarrow 2\theta = 60^\circ \Rightarrow \theta = 30^\circ$

15A.2 <u>(HKCEE MA 1981(1/2/3) – I – 12)</u>

(a) $AC = 10\sin(138^\circ \div 2) \times 2 = 18.6716$ (m)
$\therefore BC = \sqrt{AB^2 + AC^2} = 21.2$ (m, 3 s.f.)

(b) Area of $ABDC = \frac{138^\circ}{\cancel{360^\circ}} \times$ C.S.A. of cylinder
$= \frac{138^\circ}{\cancel{360^\circ}} \times 2\pi(10)(10) = 241$ (cm^2, 3 s.f.)

(c) (Imagine the curved $ABDC$ is straightened.)
Length of curve $= \sqrt{AB^2 + (\widehat{AC})^2} = 26.1$ m (3 s.f.)

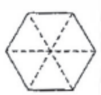
15A.3 <u>HKCEE MA 1982(1/2/3) – I – 4</u>

$\angle BOC \quad \tan^{-1}\frac{2\sqrt{3}}{6} = 30^\circ$

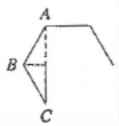
$\therefore$ Areaw $= \frac{30^\circ}{\cancel{360^\circ}} \times \pi(6)^2 = 3\pi$

15A.4 <u>HKCEE MA 1982(1/2/3) – I – 9</u>

(a) Divide the hexagon into 6 equal parts.
Area of hexagon $= 6 \times \frac{1}{2} \times 1 \times 1 \times \sin 60^\circ$
$\frac{3\sqrt{3}}{2}$



$AC = 2 \times AB\sin 60^\circ = \sqrt{3}$



(b) (i) In $\triangle ABC$, $\angle BAC = 30^\circ$
Similarly, $\angle ABF = 30^\circ$
$\Rightarrow AP = BP$ and, similarly, $BQ = QC$
Besides, $\angle PBQ = 120^\circ - 30^\circ - 30^\circ = 60^\circ$
Hence, $\triangle BPQ$ is equilateral. $\Rightarrow AP = PQ = QC$
$\Rightarrow PQ = \frac{1}{3}AC = \frac{\sqrt{3}}{3}$

(ii) Area $= 6 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{3}\right)^2 \sin 60^\circ = \frac{\sqrt{3}}{2}$

15A.5 <u>HKCEE MA 1983(A/B) – I – 5</u>

(a) $OC = OB - CB = 15$
$\angle AOC = \cos^{-1}\frac{OC}{OA} = 60^\circ$

(b) $\widehat{AB} = \frac{60^\circ}{\cancel{360^\circ}} \times 2\pi(30) = 20\pi$

15A.6 <u>HKCEE MA 1988 – I – 5</u>

(a) Area of $OABC = \frac{100^\circ}{\cancel{360^\circ}} \times \pi(10)^2 = 87.27$ (2 d.p.)

(b) Area of $\triangle OAC = \frac{1}{2}(10)^2 \sin 100^\circ = 49.24$ (2 d.p.)

(c) Area of $ABC = 87.27 - 49.24 = 38.03$ (2 d.p.)

15A.7 <u>HKCEE MA 1992 – I – 7</u>

(a) $\angle AOB = 360^\circ \div 5 = 72^\circ$
Area of $\triangle OAB = \frac{1}{2}(10)^2 \sin 72^\circ = 47.533 = 47.6$ (3 s.f.)

(b) Shaded area $= \frac{72^\circ}{\cancel{360^\circ}} \times \pi(10)^2 - 47.553 = 15.3$ (3 s.f.)

15A.8 <u>HKCEE MA 1994 – I 2(d)</u>

<u>Method 1</u>

$\angle$ subtended $= \frac{2\pi}{2\pi(5)} \times 360^\circ = 72^\circ$

$\therefore$ Area of sector $\frac{72^\circ}{360^\circ} \times \pi(5)^2 = 5\pi = 15.7$ (3 s.f.)

<u>Method 2</u>

Area of sector $=$ Area of circle $\times \frac{\text{Arc length}}{\text{Circumference}}$
$= \pi(5)^2 \times \frac{2\pi}{2\pi(5)} = 5\pi = 15.7$ (3 s.f.)

15A.9 <u>HKCEE MA 1999 – I – 9</u>

(a) $r\sin 60^\circ = 5 \Rightarrow r = \frac{10}{\sqrt{3}} = 5.77$ (3 s.f.)

(b) Area $= \frac{120^\circ}{\cancel{360^\circ}} \times \pi r^2 - \frac{1}{2}r^2 \sin 120^\circ = 20.5$ (cm^2, 3 s.f.)

15A.10 <u>HKCEE MA 2000 – I – 3</u>

Area $= \frac{75^\circ}{\cancel{360^\circ}} \times \pi(6)^2 = 2.20$ (cm^2, 3 s.f.)

15A.11 <u>HKCEE MA 2001 – I – 3</u>

Perimeter $= \frac{50^\circ}{360^\circ} \times 2\pi(3) + 3 + 3 = 8.62$ (cm, 3 s.f.)

15A.12 <u>HKCEE MA 2004 – I – 9</u>

(a) Let r cm be the radius.
$\frac{80^\circ}{360^\circ} \times \pi r^2 = 162\pi \Rightarrow r = 27$
$\therefore$ The radius is 27 cm.

(b) Perimeter $= \frac{80^\circ}{\cancel{360^\circ}} \times 2\pi(27) + 27 \times 2 = 91.7$ (cm, 3 s.f.)

15A.13 <u>HKCEE MA 2005 – I – 9</u>

(a) $\frac{100^\circ}{\cancel{360^\circ}} \times 2\pi(OA) = 10\pi \Rightarrow OA = 18$ (cm)

(b) Area $=$ Area of sector OAC Area of $\triangle OAC$
$= \frac{100^\circ}{\cancel{360^\circ}} \times \pi(18)^2 \quad \frac{1}{2}(18)^2 \sin 100^\circ$
$= 123$ (cm^2, 3 s.f.)

15A.14 <u>HKCEE MA 2006 – I – 4</u>

$\widehat{AB} = \frac{150^\circ}{\cancel{360^\circ}} \times 2\pi(12) = 10\pi$ (cm)

15A.15 <u>HKCEE MA 2007 – I – 9</u>

(a) $\frac{\angle AOB}{\cancel{360^\circ}} \times 2\pi(40) = 16\pi \Rightarrow \angle AOB = 72^\circ$

(b) Area $= \frac{72^\circ}{360^\circ} \times \pi(40)^2 = 320\pi$ (cm^2)

15A.16 <u>HKDSE MA 2015 – I – 9</u>

(a) $\frac{\text{Angle}}{\cancel{360^\circ}} \times \pi(12)^2 = 30\pi \Rightarrow$ Angle $= 75^\circ$

(b) Perimeter $= \frac{75^\circ}{\cancel{360^\circ}} \times 2\pi(12) + 12 \times 2 = 5\pi + 24$ (cm)

15B Volumes and surface areas of solids

15B.1 <u>HKCEE MA 1983(A/B) – I – 8</u>

(a) Volume of cylinder $=$ Volume of hemisphere
$\pi r^2 h = \frac{4}{3}\pi r^3 \div 2$
$h = \frac{2}{3}r \Rightarrow r : h = 3 : 2$

(b) (i) $\because h = \frac{2}{3}r$
$\therefore 136 = \frac{1}{2}(2\pi r) + h + (2r) + h$
$136 = \pi r + 2r + 2\left(\frac{2}{3}r\right)$
$r = 136 \div \left(\pi + \frac{10}{3}\right) = 21$ (2 s.f.)

(ii) Total external s.a.
$= 4\pi r^2 \div 2 + 2\pi rh + \pi r^2$
$= 2\pi(21)^2 + 2\pi(21)\left(\frac{2}{3} \cdot 21\right) + \pi(21)^2$
$= 6000$ (cm^2, 1 s.f.)

15B.2 <u>HKCEE MA 1984(A/B) – I – 12</u>

(a) Suppose E is the mid-point of CD.
$\angle CPD = 2\angle CPE$
$= 2\tan^{-1}\frac{CE}{PE}$
$= 2\tan^{-1}\frac{20 \div 2}{20} = 53.1301^\circ = 53.1^\circ$ (1 d.p.)

(b) $PC = \sqrt{PE^2 + CE^2} = \sqrt{500}$
$\therefore \widehat{CQD} = \frac{53.1301^\circ}{360^\circ} \times 2\pi(\sqrt{500})$
$= 20.73495 = 20.7$ (cm, 1 d.p.)

(c) Area of $APBCQD$
$=$ Area of sector $PCD + 2 \times$ Area of $\triangle PBC$
$= \frac{53.1301^\circ}{360^\circ} \times \pi(\sqrt{500})^2 + 2 \times \frac{20 \times 10}{2}$
$= 431.8$ (cm^2, 1 d.p.)

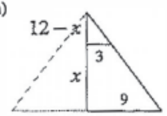
(d) T.S.A. $= BR \times$ Perimeter of $APBCQD$
$= 20 \times (20 \times 3 + \widehat{CQD}) = 1614.7$ (cm^2, 1 d.p.)

15B.3 <u>HKCEE MA 1985(A/B) – I 11</u>

(a) (i) $135\pi = \pi\, 9 \cdot OP \Rightarrow OP = 15$ (cm)

(ii) Height $= \sqrt{15^2 - 9^2} = 12$ (cm)

(b) (i) By $\sim \triangle$s, $\frac{12 - x}{x} = \frac{3}{9} = \frac{1}{3}$
$3(12 - x) = x$
$x = 8$



(ii) *<u>Method 1</u>*
Volume remained
$=$ Big cone $-$ Small cone $-$ Cylinder
$= \frac{1}{3}\pi(9)^2(12) - \frac{1}{3}\pi(3)^2(12 - 8) - \pi(3)^2(8)$
$= 240\pi$ (cm^3)

<u>Method 2</u>
Vol of frustum $=$ Vol of big cone $\times \left[1 \quad \left(\frac{3}{9}\right)^3\right]$
$= \frac{1}{3}\pi(9)^2(12) \times \left(1 \quad \frac{1}{27}\right)$
$= 324\pi \times \frac{26}{27} = 312\pi$ (cm^3)
$\therefore$ Vol remained $= 312\pi - \pi(3)^2(8) = 240\pi$ (cm^3)

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15B.4 HKCEE MA 1986(A/B) – I – 12

(a) (i) Let h be the height of the cone.

$\frac{1}{3}\pi(6)^2(h)=\frac{4}{3}\cdot\left[\frac{4}{3}\pi(6)^3\div 2\right]$

$12\pi h=\frac{4}{3}(144\pi) \Rightarrow h=16$

$\therefore$ The height of the cone is 16.

(ii) $\text{Vol}=144\pi+144\pi\times\frac{4}{3}=336\pi$

(b) (i) By $\sim\triangle$s, $\frac{x}{y}=\frac{6}{16}=\frac{3}{8}$

(ii) $\frac{1}{3}\pi x^2 y=336\pi\div 2$

$\frac{1}{3}\pi\left(\frac{3}{8}y\right)^2 y=168\pi$

$\frac{3}{64}\pi y^3=168\pi \Rightarrow y=15.3$ (1 d.p.)

15B.5 HKCEE MA 1989 – I – 11

(a) Vol of water $=\frac{(10+2)\times 50}{2}\times 20=6000\ (\text{m}^3)$

(b) (i) (The cross-section would change from a trapezium to a triangle.)

Vol of water remaining $=\frac{8\times 50}{2}\times 20=4000\ (\text{m}^3)$

(ii) Vol of water through pipe in 1 second

$=\pi(0.125)^2(3)=0.046875\pi\ (\text{m}^3)$

$\therefore$ Vol of water pumped in 8 hours

$=0.046875\pi\times 8\times 60\times 60$

$=1350\pi=4241$ (m^3, nearest m^3)

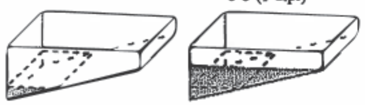
(iii) Vol of water remaining after 8 hours

$=6000-1350\pi=1758.8499\ (\text{m}^3)$

Since the cross-section right-angled $\triangle$s are similar,

$\frac{1758.8499}{4000}=\left(\frac{h\text{ m}}{8\text{ m}}\right)^2 \Rightarrow h=\sqrt{\frac{1758.8499}{4000}}\times 8$

$=5.3$ (1 d.p.)



15B.6 HKCEE MA 1990 – I – 11

(a) (i) $S=2\pi r^2+2\pi rh$

(ii) $V=\pi r^2 h \Rightarrow h=\frac{V}{\pi r^2}$

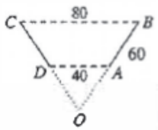
$\therefore S=2\pi r^2+2\pi r\left(\frac{V}{\pi r^2}\right)=2\pi r^2+\frac{2V}{r}$

15B.7 HKCEE MA 1991 – I – 11

(a) By $\sim\triangle$s, $\frac{OA}{OA+60}=\frac{40}{80}=\frac{1}{2}$

$2OA=OA+60$

$OA=60$ (cm)



In Figure (2), $\widehat{ADA'}$ = Base $\odot^{ce}$ of bucket

$=2\pi(40\div 2)=40\pi$ (cm)

$\therefore \frac{\angle AOA'}{360^\circ}\times 2\pi(OA)=40\pi$

$\angle AOA'=\frac{40\pi}{120\pi}\cdot 360^\circ=120^\circ$

(b) Area of $ABB'A'=\frac{120^\circ}{360^\circ}\left[\pi(60+60)^2-\pi(60)^2\right]$

$=3600\pi\ (\text{cm}^2)$

(c) The shortest path is AC in Figure (2).

Method 1

Since $OA=OB$ and $\angle AOC=120^\circ\div 2=60^\circ$, $\triangle OBC$ is equilateral.

$\therefore$ Required path $=OC\sin 60^\circ=120\cdot\frac{\sqrt{3}}{2}=60\sqrt{3}$ (cm)

Method 2

Required path $=\sqrt{OA^2+OC^2-2OA\cdot OC\cos\angle AOC}$

$=\sqrt{60^2+120^2-2\cdot 60\cdot 120\cos 60^\circ}$

$=\sqrt{10800}$ (cm)

15B.8 (HKCEE MA 1993 – I – 9)

(a) (i) Area of sector $OACB=\frac{\alpha}{360^\circ}\times\pi r^2$

(ii) Area of segment $ACB=\frac{\alpha}{360^\circ}\times\pi r^2-\frac{1}{2}r^2\sin\alpha$

(iii) $\because$ A of segment $ACB=\frac{1}{5}$(A of circle)

$\therefore \left(\frac{\alpha\pi}{360^\circ}-\frac{\sin\alpha}{2}\right)r^2=\frac{1}{5}(\pi r^2)$

$\frac{\alpha\pi}{360^\circ}-\frac{\sin\alpha}{2}=\frac{1}{5}$

$\sin\alpha=\left(\frac{\alpha}{180^\circ}-\frac{2}{5}\right)\pi$

(b) Required ratio $=\frac{\text{major }\widehat{AB}}{\text{minor }\widehat{AB}}=\frac{360^\circ-\alpha}{\alpha}=1.98:1$ (3 s.f.)

15B.9 HKCEE MA 1994 – I – 10

(a) Vol of water $=\pi(2)^2(1.5)=6\pi\ (\text{m}^3)$

(b) $\pi(2)^2 h=\frac{4}{3}\pi(0.6)^3$

$h=\frac{\frac{4}{3}\pi(0.6)^3}{4\pi}=0.072$

(c) (i) $\cancel{\pi}\cancel{(2)^2}(2r-1.5)=\frac{\cancel{4}}{3}\cancel{\pi}r^3$

$2r-1.5=\frac{1}{3}r^3 \Rightarrow 2r^3-12r+9=0$

15B.10 HKCEE MA 1995 – I – 13

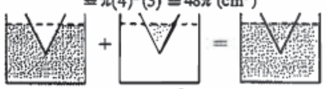
(a) Vol of water $=\frac{1}{3}\pi(6)^2(12)=144\pi\ (\text{cm}^3)$

(b) Consider (the cross-section of) the entire conical vessel and (the cross-section of) the part of the conical vessel inside the cylindrical vessel.

$\frac{6}{12}=\frac{4}{11-d} \Rightarrow 11-d=8 \Rightarrow d=3$

(c) (i) Vol leaked = Vol of water in cylindrical vessel

$=\pi(4)^2(3)=48\pi\ (\text{cm}^3)$

(ii) 

$104\pi+\frac{1}{3}\pi\left(\frac{h}{2}\right)^2(h)=\pi(4)^2(3+h)$

$1248+h^3=192(3+h)$

$h^3-192h+672=0$

15B.11 HKCEE MA 1996 – I – 8

(a) Cap of cup $=\frac{1}{3}\pi(5)^2(12)=100\pi=314$ (cm^3, 3 s.f.)

(b) (i) Area of sector = C.S.A. of cone

$=\pi(5)\sqrt{5^2+12^2}$

$=\pi(5)(13)=65\pi=204$ (cm^2, 3 s.f.)

(ii) $\frac{\angle\text{ of sector}}{360^\circ}\times\pi(13)^2=65\pi$

$\angle$ of sector $=138^\circ$ (3 s.f.)

15B.12 HKCEE MA 1997 – I – 12

(a) (i) In $\triangle VMN$, $NM=6\div 2=3$ (m)

$VN=NM\tan\theta=3\tan\theta$ (m)

$VM=\frac{NM}{\cos\theta}=\frac{3}{\cos\theta}$ (m)

(ii) $\text{Cap}=\frac{1}{3}\times 6\times 6\times 3\tan\theta=36\tan\theta\ (\text{m}^3)$

$\text{T.S.A.}=4\times\frac{6\times\frac{3}{\cos\theta}}{2}=\frac{36}{\cos\theta}\ (\text{m}^2)$

(b) (i) $6\times 6=\pi r^2 \Rightarrow r=\frac{6}{\sqrt{\pi}}$

(ii) $\pi r^2 h=36\tan\theta \Rightarrow (36)h=36\tan\theta \Rightarrow h=\tan\theta$

(iii) $2\pi rh+\pi r^2=\frac{36}{\cos\theta}$

$2\pi\left(\frac{6}{\sqrt{\pi}}\right)(\tan\theta)+(36)=\frac{36}{\cos\theta}$

$12\sqrt{\pi}\tan\theta+36=\frac{36}{\cos\theta}$

$\sqrt{\pi}\tan\theta+3=\frac{3}{\cos\theta}$

15B.13 HKCEE MA 1998 – I – 1

Volume $=\frac{(2+6)\times 3}{2}\times 8=96\ (\text{cm}^3)$

15B.14 HKCEE MA 1999 – I – 13

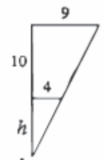
(a) Capacity of hole $=\frac{4}{3}\pi(5)^3\times\frac{1}{2}+\pi(5)^2(9-5)$

Capacity of hole $=\frac{550}{3}\pi\ (\text{cm}^3)$

(b) By $\sim\triangle$s, $\frac{h}{h+10}=\frac{4}{9}$

$9h=4h+40$

$h=8$



$\therefore$ Vol of frustum $=\frac{1}{3}\pi(9)^2(10+8)-\frac{1}{3}\pi(4)^2(8)$

$=\frac{1330}{3}\pi\ (\text{cm}^3)$

$\therefore$ Vol of wood $=\frac{1330}{3}\pi-\frac{550}{3}\pi=260\pi\ (\text{cm}^3)$

15B.15 HKCEE MA 2002 – I – 15

(a) (i) Total vol of water $=\frac{1}{3}\pi(9)^2(24)=648\pi\ (\text{cm}^3)$

Vol of water remained in cone $=648\pi\times\left(\frac{h+5}{24}\right)^3$

$=\frac{3}{64}\pi(h+5)^3$

$\therefore \frac{3}{64}\pi(h+5)^3=648\pi-\pi(6)^2 h$

$(h+5)^3=13824-768h$

$h^3+15h^2+75h+125=13824-768h$

$h^3+15h^2+843h-13699=0$

(b) (The final situation in Figure (2) is the same as Figure (1) with the lowest 5 cm removed.)

Depth of water = 11.8 cm

15B.16 HKCEE MA 2004 – I – 14

(a) Base radius of cylinder $=\sqrt{12^2-\left(\frac{h}{2}\right)^2}=\sqrt{144-\frac{h^2}{4}}$

$\therefore V=\pi\left(144-\frac{h^2}{4}\right)(h)=144\pi h-\frac{\pi}{4}h^3$

(c) $144\pi h-\frac{\pi}{4}h^3=286\pi \Rightarrow h^3-576h+1144=0$

Since $(2)^3-576(2)+1144=0$, $h-2$ is a factor.

$(h-2)(h^2+2h-572)=0$

$h=2$ or $\frac{-2\pm\sqrt{4+2288}}{2}$

$=2$ or $\sqrt{573}-1$ or $-\sqrt{573}-1$ (rj.)

Hence, the height is 2 cm or $(\sqrt{573}-1)$ cm.

15B.17 HKCEE MA 2005 – I – 12

(a) $\frac{1}{3}\pi(h-4)^2 h=\frac{4}{3}\pi(h-4)^3\div 2$

$h=2(h-4) \Rightarrow h=8$

(b) $\text{T.S.A.}=\pi(h-4)\sqrt{h^2+(h-4)^2}+4\pi(h-4)^2\div 2$

$=\pi(8)\sqrt{8^2+4^2}+2\pi(4)^2$

$=325$ (cm^2, nearest cm^2)

(c) Increase $=2\times(\triangle+\text{semi-circle})$

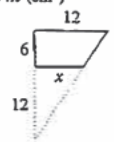
$=2\times\left[\frac{8\times 8}{2}+\frac{\pi(4)^2}{2}\right]=114$ (cm^2, nrst cm^2)

15B.18 HKCEE MA 2009 – I – 13

(a) (i) Capacity $=\frac{1}{3}\pi(12)^2(18)=864\pi\ (\text{cm}^3)$

(ii) By $\sim\triangle$s, $\frac{x}{12}=\frac{18-6}{18}$

$x=8$



$\therefore$ Vol of frustum $=864\pi-\frac{1}{3}\pi(8)^2(12)$

$=608\pi\ (\text{cm}^3)$

(b) (i) Cap of cylinder $=\pi(8)^2(10)=640\pi\ (\text{cm}^3)$

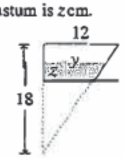
$\therefore$ Vol of water in the frustum part

$=884\pi-640\pi=244\pi\ (\text{cm}^3)$

Suppose the depth of water in the frustum is z cm.

By $\sim\triangle$s, $\frac{z+12}{18}=\frac{y}{12}$

$y=\frac{2}{3}(z+12)$



$\therefore 244\pi=\frac{1}{3}y^2(z+12)-256\pi$

$500\pi=\frac{1}{3}\pi\left(\frac{2}{3}(z+12)\right)^2(z+12)$

$500=\frac{4}{27}(z+12)^3$

$(z+12)^3=3375 \Rightarrow z+12=15 \Rightarrow z=3$

Hence, depth of water in vessel is $10+3=13$ (cm).

(ii) Cap of vessel $=640\pi+608\pi=1248\pi=3920\ (\text{cm}^3)$

Vol of water + metal $=884\pi+1000=3777\ (\text{cm}^3)$

< Cap of vessel

$\therefore$ NO.

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15B.19 HKCEE MA 2011 – I – 13

(a) $\frac{288°}{\cancel{360°}}\times\pi(OX)^2 = 2880\pi \Rightarrow OX = 60$ (mm)

(b) $\overset{\frown}{XYZ} = \frac{288°}{\cancel{360°}}\times 2\pi(60) = 96\pi$ (mm)

$\therefore$ Base radius of container $= \frac{96\pi}{2\pi} = 48$ (mm)

$\Rightarrow$ Height of container $= \sqrt{60^2-48^2} = 36$ (mm)

(c) Cap of container $= \frac{1}{3}\pi(48)^2(36) = 86859$ mm^3

$= 86.859$ cm$^3 < 150$ cm^3

$\therefore$ YES.

15B.20 HKDSE MA SP – I – 6

(a) $\frac{1}{3}\pi r^2(12) = 2\times\left(\frac{4}{3}\pi r^3\times\frac{1}{2}\right)$

$4\pi r^2 = \frac{4}{3}\pi r^3 \Rightarrow r = 3$

(b) Volume $= \frac{2}{3}\pi(3)^3\times 3 = 54\pi$ (cm^3)

15B.21 HKDSE MA 2012 – I – 9

(a) Base area $= \frac{\text{Volume}}{\text{Height}} = \frac{1020}{10} = 102$ (cm^2)

$\frac{(6+AD)\times 12}{2} = 102 \Rightarrow AD = 11$ (cm)

(b) Perimeter of base $= 11+12+6+\sqrt{(11-6)^2+12^2}$

$= 42$ (cm)

$\therefore$ T.S.A. $= 2\times102+42\times10 = 624$ (cm^2)

15B.22 HKDSE MA 2012 – I – 12

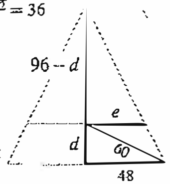
(a) Vol of cone $= \frac{1}{3}\pi(48)^2(96) = 73728\pi$ (cm^3)

(b) (i) Vol of milk $= \frac{4}{3}\pi(60)^3\div 2 = 144000\pi$ (cm^3)

(ii) In the figure, $d = \sqrt{60^2-48^2} = 36$

$\frac{e}{48} = \frac{96-d}{96}$

$e = \frac{60}{96}\times 48 = 30$



Method 1

$\therefore$ Vol of part of cone in milk

$= 73728\pi - \frac{1}{3}\pi(30)^2(60)$

$= 73728\pi - 18000\pi = 55728\pi$ (cm^3)

$\therefore$ Vol of milk remaining $= 144000\pi - 55728\pi$

Method 2

$\because \frac{\text{Height of cone outside milk}}{\text{Height of the whole cone}} = \frac{96-36}{96} = \frac{5}{8}$

$\therefore$ Vol of part of cone in milk

$= 73728\pi\times\left[1-\left(\frac{5}{8}\right)^3\right] = 55728\pi$ (cm^3)

Hence

$\therefore$ Vol of milk remaining $= 144000\pi - 55728\pi$

$= 88272\pi$ cm^3

$= 277000$ cm^3

$= 0.277$ m$^3 < 0.3$ m^3

$\therefore$ The craftsman is disagreed.

*15B.23 in the end of 15C

15C Similar plane figures and solids

15C.1 HKCEE MA 1981(1/2/3) – I – 1

$\frac{\text{S.A. of bigger tank}}{\text{S.A. of smaller tank}} = \left(\sqrt[3]{\frac{64}{27}}\right)^2$

$\frac{\text{Paint for bigger tank}}{72\text{ kg}} = \left(\frac{4}{3}\right)^2$

Paint for bigger tank $= \frac{16}{9}\times 72 = 128$ (kg)

15C.2 HKCEE MA 1987(A/B) – I – 9

(a) (i) $108\pi =$ Vol of hemisphere $\times 6$

$108\pi = \left[\frac{4}{3}\pi(r)^3\div 2\right]\times 6$

$108\pi = 4\pi r^3 \Rightarrow r = 3$

Vol of cylindrical part $= \frac{5}{6}(108\pi)$

$\pi(3)^2(h) = 90\pi \Rightarrow h = 10$

(ii) Vol of water $= 108\pi -$ Vol of empty space

$= 108\pi - \pi(3)^2(4) = 69\pi$ (cm^3)

(b) $\because \frac{\text{Height of vessel}}{\text{Depth of water}} = 2$

$\therefore \frac{\text{Cap of vessel}}{\text{Vol of water}} = 2^3 = 8$

Cap of vessel $= 8\times 69\pi = 552\pi$ (cm^3)

15C.3 HKCEE MA 1992 – I – 12

(a) (i) Cap of funnel $= \frac{1}{3}\pi(9)^2(10+5+5) = 540\pi$ (cm^3)

(ii) V of water : Total v of water & oil : Cap of funnel

$= (10)^3 : (10+5)^3 : (10+5+5)^3$

$= 8 : 27 : 81$

$\therefore$ V of water : V of oil : Cap of funnel

$= 8 : (27-8) : 81 = 8 : 19 : 81$

(b) V of water $= 540\pi\times\frac{8}{81} = \frac{160}{3}\pi$ (cm^3)

In the tube,

V of water in lower part $= \frac{4}{3}\pi(3)^3\div 2 = 18\pi$ (cm^3)

$\Rightarrow$ V of water in upper part $= \frac{160}{3}\pi - 18\pi = \frac{106}{3}\pi$ (cm^3)

$\therefore$ Depth of water $= \frac{\frac{106}{3}\pi}{\pi(3)^2}+3 = \frac{187}{27}$ (cm)

(c) $\because \frac{\text{Vol of oil}}{\text{Cap of funnel}} = \frac{19}{81}$

$\therefore \frac{\text{Depth of oil}}{\text{Height of funnel}} = \sqrt[3]{\frac{19}{81}}$

$\Rightarrow$ Depth of oil $= \sqrt[3]{\frac{19}{81}}\times 20 = 9.69$ (cm, 3 s.f.)

15C.4 HKCEE MA 1994 – I – 2(e)

Ratio of volumes $= \left(\frac{2}{3}\right)^3 = 8 : 27$

15C.5 HKCEE MA 1997 – I – 7

(a) Required ratio $= \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

15C.6 HKCEE MA 2000 – I – 8

Actual area $= 220\text{ cm}^2\times(5000)^2$

$= 5\,500\,000\,000\text{ cm}^2 = 550000\text{ m}^2$

15C.7 HKCEE MA 2002 – I – 6

Method 1

(a) New radius $= 8\times(1+10\%) = 8.8$ (cm)

$\therefore$ New area $= \pi(8.8)^2 = 77.44\pi$ (cm^2)

(b) Original area $= \pi(8)^2 = 64\pi$ (cm^2)

% increase $= \frac{77.44\pi-64\pi}{64\pi}\times 100\% = 21\%$

Method 2

(a) Original area $= \pi(8)^2 = 64\pi$ (cm^2)

$\therefore$ New area $= 64\pi\times(1+10\%)^2 = 77.44\pi$ (cm^2)

(b) % increase $= \frac{(1+10\%)^2-1}{1}\times 100\% = 21\%$

15C.8 HKCEE MA 2002 – I – 11

(a) Let $A = hP+kP^2$.

$\begin{cases}36 = 24h+576k\\ 9 = 18h+324k\end{cases} \Rightarrow \begin{cases}h = -\frac{5}{2}\\ k = \frac{1}{6}\end{cases} \Rightarrow A = \frac{-5}{2}P+\frac{1}{6}P^2$

(b) (i) $54 = -\frac{5}{2}P+\frac{1}{6}P^2$

$P^2-15P-324 = 0 \Rightarrow P = 27$ or -12 (rejected)

$\therefore$ The perimeter is 27 cm.

(ii) $\frac{\text{Area of miniature}}{\text{Area of original}} = \frac{8}{54} = \frac{4}{27}$

$\Rightarrow \frac{\text{Perimeter of miniature}}{\text{Perimeter of original}} = \frac{2}{\sqrt{27}}$

$\therefore$ Perimeter of miniature $= \frac{2}{\sqrt{27}}\times 27$

$= 2\sqrt{27}$ (cm) $(= 6\sqrt{3}$ cm)

15C.9 HKCEE MA 2003 – I – 13

(a) (i) $\frac{x°}{360°}\times 2\pi(56+24) = 30\pi \Rightarrow x = 67.5$

(ii) Area of $ABCD = \frac{67.5°}{\cancel{360°}}\times[\pi(80)^2-\pi(56)^2]$

$= 612\pi$ (cm^2)

(b) (i) Area of $EFGH = 612\pi\times\left(\frac{18}{24}\right)^2 = 344.25\pi$ (cm^2)

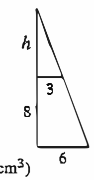
(ii) Base $\odot^{ce} = 30\pi\times\frac{18}{24} = 22.5\pi$ (cm)

$\Rightarrow r = \frac{22.5\pi}{2\pi} = 11.25$

15C.10 HKCEE MA 2006 – I – 13

(a) By $\sim\triangle$s, $\frac{h}{h+8} = \frac{3}{6} = \frac{1}{2}$

$2h = h+8 \Rightarrow h = 8$



$\therefore$ Vol of frustum

$= \frac{1}{3}\pi(6)^2(8+8)-\frac{1}{3}\pi(3)^2(8)$

$= 192\pi-24\pi = 168\pi$ (cm^3)

$\Rightarrow$ Vol of $X = 168\pi+\frac{4}{3}\pi(6)^3\div 2 = 312\pi$ (cm^3)

$\frac{\text{Vol of }Y}{\text{Vol of }X} = \left(\sqrt{\frac{\text{S.A. of }Y}{\text{S.A. of }X}}\right)^3 = \left(\sqrt{\frac{9}{4}}\right)^3 = \frac{27}{8}$

$\Rightarrow$ Vol of $Y = \frac{27}{8}(312\pi) = 1053\pi$ (cm^3)

(b) $\because$ Ratio of S.A. of spheres $= (1:2)^2 = 1:4$

$\neq 4:9$

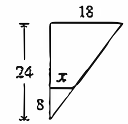
$\therefore$ NO

15C.11 HKCEE MA 2007 – I – 11

(a) *Method 1*

By $\sim\triangle$s, $\frac{x}{18} = \frac{8}{24} = \frac{1}{3}$

$x = 6$



$\therefore$ Vol of water $= \frac{1}{3}\pi(6)^2(8)$

$= 96\pi$ (cm^3)

Method 2

Vol of water $= \left(\frac{8}{24}\right)^3$ Vol of vessel

$= \frac{1}{27}\times\frac{1}{3}\pi(18)^2(24) = 96\pi$ (cm^3

(b) (i) *Method 1*

Area of wet surface $= \pi(6)\sqrt{6^2+8^2} = 60\pi$ (cm^2)

Method 2

Area of wet surface $= \left(\frac{8}{24}\right)^2$ C.S.A. of vessel

$= \frac{1}{9}\times\pi(18)\sqrt{18^2+24^2}$

$= 60\pi$ (cm^2)

(ii) Ratio of heights $= 24 : 36 = 2 : 3$

Ratio of base radii $= 18 : 27 = 2 : 3$

The two vessels are similar.

$\therefore$ Area of wet surface is also 60π cm^2.

15C.12 HKCEE MA 2008 – I – 13

(a) $\overset{\frown}{ABC} = \frac{216°}{\cancel{360°}}\times 2\pi(20) = 24\pi$ (cm)

$\therefore$ Base radius of $X = \frac{24\pi}{2\pi} = 12$ (cm)

Height $= \sqrt{20^2-12^2} = 16$ (cm)

(b) Vol of $X = \frac{1}{3}\pi(12)^2(16) = 768\pi$ (cm^3)

(c) *Method 1*

Base radius of $Y = \frac{\frac{108°}{360°}\times 2\pi(10)}{2\pi} = 3$ (cm)

$\because \begin{cases}\frac{\text{Slant height of }X}{\text{Slant height of }Y} = \frac{20}{10} = 2\\ \frac{\text{Base radius of }X}{\text{Base radius of }Y} = \frac{12}{3} = 4 \neq \frac{\text{Slant height of }X}{\text{Slant height of }Y}\end{cases}$

$\therefore$ NO.

Method 2

Base $\odot^{ce}$ of $X = \frac{216°}{\cancel{360°}}\times 2\pi(20) = 24\pi$ (cm^2)

Base $\odot^{ce}$ of $Y = \frac{108°}{360°}\times 2\pi(10) = 6\pi$ (cm^2)

$\because \begin{cases}\frac{\text{Slant height of }X}{\text{Slant height of }Y} = \frac{20}{10} = 2\\ \frac{\text{Base }\odot^{ce}\text{ of }X}{\text{Base }\odot^{ce}\text{ of }Y} = \frac{24\pi}{6\pi} = 4 \neq \frac{\text{Slant height of }X}{\text{Slant height of }Y}\end{cases}$

$\therefore$ NO.

Method 3

C.S.A. of $X = \frac{216°}{360°}\times\pi(20)^2 = 240\pi$ (cm^2)

C.S.A. of $Y = \frac{\cancel{108°}}{360°}\times\pi(10)^2 = 30\pi$ (cm^2)

$\because \begin{cases}\frac{\text{Slant height of }X}{\text{Slant height of }Y} = \frac{20}{10} = 2\\ \frac{\text{C.S.A. of }X}{\text{C.S.A. of }Y} = \frac{240\pi}{30\pi} = 8 \neq \left(\frac{\text{Slant height of }X}{\text{Slant height of }Y}\right)^2\end{cases}$

$\therefore$ NO.

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Method 4

$\because$ Reflex $\angle AOC \neq \angle DPF$

$\therefore$ The sectors are not similar.

$\Rightarrow \dfrac{\text{Area of sector } OABC}{\text{Area of sector } PDEF} \neq \left(\dfrac{OA}{PD}\right)^2$

$\Rightarrow \dfrac{\text{C.S.A. of } X}{\text{C.S.A. of } Y} \neq \left(\dfrac{\text{Slant height of } X}{\text{Slant height of } Y}\right)^2$

$\therefore$ NO.

15C.13 HKCEE MA 2010 – I – 13

(a) Area of $\triangle ABC = \dfrac{16 \times \sqrt{17^2 - (16 \div 2)^2}}{2} = 120\ (\text{cm}^2)$

(b) Vol of $ABCDEF = 120 \times 20 = 2400\ (\text{cm}^3)$

(c) (i) $\dfrac{\text{Area of } \triangle APQ}{\text{Area of } \triangle ABC} = \left(\dfrac{PQ}{BC}\right)^2 = \dfrac{1}{16}$

$\therefore$ Vol of $APQRES = 2400 \times \dfrac{1}{16} = 150\ (\text{cm}^3)$

(ii) *Method 1*

$\because \dfrac{PQ}{BC} = \dfrac{1}{4}$, but $\dfrac{AE}{AE} = 1 \neq \dfrac{PQ}{BC}$

$\therefore$ NO.

Method 2

$\dfrac{PQ}{BC} = \dfrac{1}{4}$

$\because \dfrac{\text{Vol of } APQRES}{\text{Vol of } ABCDEF} = \dfrac{150}{2400} = \dfrac{1}{16} \neq \left(\dfrac{PQ}{BC}\right)^3$

$\therefore$ NO.

15C.14 HKDSE MA 2012 – I – 11

(a) Let $C = h + kA$.

$\begin{cases} 62 = h + 2k \\ 74 = h + 6k \end{cases} \Rightarrow \begin{cases} h = 56 \\ k = 3 \end{cases} \Rightarrow C = 56 + 3A$

$\therefore$ When $A = 13$, cost $= 56 + 3(13) = (\$)95$

(b) Volume is 8 times. $\Rightarrow$ Area is $(\sqrt[3]{8})^2 = 4$ times.

$\therefore$ Cost $= 56 + 3(13 \times 4) = (\$)212$

15C.15 HKDSE MA 2013 – I – 13

(a) (i) $\dfrac{r}{R} = \sqrt{\dfrac{1}{9}} = \dfrac{1}{3}$

(ii) Let H cm be the height of a larger cylinder.

$2 \times \pi R^2 H = 27 \times \pi r^2 (10)$

$2 \times (9\pi r^2) H = 27 \times \pi r^2 (10)$

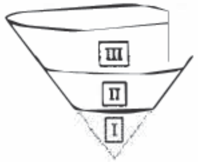
$18H = 270 \Rightarrow H = 15$

Hence, the height is 15 cm.

(b) $\because \dfrac{\text{Height of smaller cylinder}}{\text{Height of larger cylinder}} = \dfrac{10}{15} = \dfrac{2}{3} \neq \dfrac{r}{R}$

$\therefore$ NO.

15C.16 HKDSE MA 2014 – I – 14



(a) *Method 1*

C.S.A. of entire cone $= \pi(72)\sqrt{72^2 + 96^2} = 8640\pi\ (\text{cm}^2)$

With the label in the figure.

C.S.A. of 'I' : C.S.A. of 'I+II' : C.S.A. of 'I+II+III'

$= ((96-60) : (96 \quad 60+28) : 96)^2$

$= (9 : 16 : 24)^2 = 81 : 256 : 576$

$\therefore$ Area of wet curved surface $= 8640\pi \times \dfrac{256-81}{576}$

$= 2625\pi\ (\text{cm}^2)$

Method 2

With the label in the figure,

$\dfrac{\text{Base radius of 'I'}}{96-60} \quad \dfrac{\text{Base radius of 'I+II'}}{96 \quad 60+28} = \dfrac{72}{96}$

$\Rightarrow$ Base r of 'I' $= 27$ (cm), Base r of 'I+II' $= 48$ (cm)

$\therefore$ Area of wet curved surface $= \pi(48)\sqrt{48^2 + 64^2}$

$- \pi(27)\sqrt{27^2 + 36^2}$

$= 2625\pi\ (\text{cm}^2)$

(b) *Method 1*

Vol of entire cone $= \dfrac{1}{3}\pi(72)^2(96) = 165888\pi\ (\text{cm}^3)$

$\therefore$ Vol of water $= 165888\pi \times \dfrac{16^3 - 9^3}{24^3}$

$= 40404\pi\ (\text{cm}^3)$

$= 126933\ \text{cm}^3 = 0.127\ \text{m}^3 > 0.1\ \text{m}^3$

.. YES.

Method 2

Vol of water $= \dfrac{1}{3}\pi(48)^2(64) \quad \dfrac{1}{3}\pi(27)^2(36)$

$= 40404\pi\ (\text{cm}^3)$

$= 126933\ \text{cm}^3 = 0.127\ \text{m}^3 > 0.1\ \text{m}^3$

$\therefore$ YES.

15C.17 HKDSE MA 2016 – I – 11

(a) Let V cm^3 be the final volume of milk.

$\dfrac{\text{Initial volume of milk}}{\text{Final volume of milk}} = \left(\dfrac{\text{Initial depth of milk}}{\text{Final depth of milk}}\right)^3$

$\Rightarrow \dfrac{V \quad 444\pi}{V} = \left(\dfrac{12}{16}\right)^3 = \dfrac{27}{64} \Rightarrow V = 768\pi$

$\therefore$ The final volume of milk is 768π cm^3.

(b) Let r cm be the final radius of the milk surface.

$\dfrac{1}{3}\pi r^2(16) = 768\pi \Rightarrow r = 12$

$\therefore$ Final area of wet surface $= \pi(12)\sqrt{(12)^2 + 16^2}$

$= 240\pi$

$= 754.0\ (\text{cm}^2) < 800\ \text{cm}^2$

NO.

15C.18 HKDSE MA 2017 – I – 12

(a) Volume of metal $= 84 \times 20 = 1680\ (\text{cm}^3)$

$\dfrac{\text{Vol of smaller pyramid}}{\text{Vol of larger pyramid}} = \left(\sqrt{\dfrac{4}{9}}\right)^3 = \dfrac{8}{27}$

$\therefore$ Vol of larger pyramid $= 1680 \times \dfrac{27}{8+27} = 1296\ (\text{cm}^3)$

(b) For the larger pyramid,

Base area $= \dfrac{3 \times \text{Volume}}{\text{Height}} = \dfrac{3 \times 1296}{12} = 324\ (\text{cm}^2)$

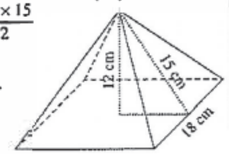
$\Rightarrow$ Length of one side of base $= \sqrt{324} = 18$ (cm)

$\Rightarrow$ Height of each lateral-face $\triangle = \sqrt{12^2 + (18 \div 2)^2}$

$= 15$ (cm)

$\therefore$ T.S.A. $= 324 + 4 \times \dfrac{18 \times 15}{2}$

$= 864\ (\text{cm}^2)$



Hence, for the smaller prd.

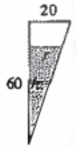
T.S.A. $= 864 \times \dfrac{9}{4}$

$= 384\ (\text{cm}^2)$

15C.19 HKDSE MA 2018 – I – 14

(a) Vol of water $= \pi(8)^2(64) = 4096\pi\ (\text{cm}^3)$

(b) *Method 1*



By $\sim \triangle$s, $\dfrac{h}{60} = \dfrac{r}{20}$

$h = 3r$

$\therefore \dfrac{1}{3}\pi r^2 h = 4096\pi$

$\dfrac{1}{3}\pi r^2(3r) = 4096\pi$

$r^3 = 4096 \Rightarrow r = 16$

Hence, the depth of water is $3(16) = 48$ (cm).

Method 2

Cap of vessel $= \dfrac{1}{3}\pi(20)^2(60) = 8000\pi\ (\text{cm}^3)$

$\therefore$ Depth of water $= \sqrt[3]{\dfrac{4096\pi}{8000\pi}} \times$ Height of vessel

$= \dfrac{4}{5} \times 60 = 48$ (cm)

(c) Vol of sphere $= \dfrac{4}{3}\pi(14)^3 = 3658\dfrac{2}{3}\pi\ (\text{cm}^3)$

Vol of empty space in vessel

$= \dfrac{1}{3}\pi(20)^2(60) - 4096\pi = 3904\pi >$ Vol of sphere

$\therefore$ NO.

15C.20 HKDSE MA 2019 – I – 9

(a) *Method 1*

Let the radii of the smaller and larger spheres be r cm and $2r$ cm respectively.

$\dfrac{4}{3}\pi(r)^3 + \dfrac{4}{3}\pi(2r)^3 = 324\pi$

$r^3 + 8r^3 = 243 \Rightarrow r^3 \quad 27 \Rightarrow r = 3$

$\therefore$ Vol of larger sphere $= \dfrac{4}{3}\pi(2 \times 3)^3 = 288\pi\ (\text{cm}^3)$

Method 2

$\dfrac{\text{V of larger sphere}}{\text{V of smaller sphere}} = \left(\dfrac{\text{R of larger sphere}}{\text{R of smaller sphere}}\right) = 8$

$\therefore$ V of larger sphere $= 324\pi \times \dfrac{8}{1+8} = 288\pi\ (\text{cm}^3)$

(b) R of larger sphere $= \sqrt[3]{288\pi \div \dfrac{4\pi}{3}} = 6$ (cm)

$\therefore$ Sum of S.A. $= 4\pi(6)^2 + 4\pi(6 \div 2)^2 = 180\pi\ (\text{cm}^2)$

****15B.23 HKDSE MA 2020 - I - 12**

12a	The required volume $= \dfrac{\pi}{3}(15)^2(36)\left[\left(\dfrac{2}{3}\right)^3 - \left(\dfrac{1}{3}\right)^3\right]$ 700π cm^3
b	The required curved surface area $\quad \pi(15)\sqrt{15^2 + 36^2}\left[\left(\dfrac{2}{3}\right)^2 - \left(\dfrac{1}{3}\right)^2\right]$ 195π cm^2

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