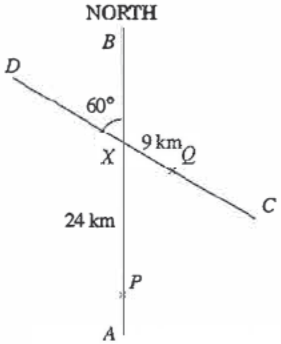
14 Applications of Trigonometry

14A Two-dimensional applications

14A.1 HKCEE MA 1981(2/3) I 11



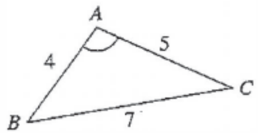
AB and CD are two straight roads intersecting at X. AB runs North and makes an angle of 60° with CD. At noon, two people P and Q are respectively 24 km and 9 km from X as shown in the figure. P walks at a speed of 4.5 km/h towards B and Q walks at a speed of 6 km/h towards D.

(a) Calculate the distance between P and Q at noon.

(b) What are the distances of P and Q from X at 4 p.m.?

(c) Calculate the bearing of Q from P at 4 p.m. to the nearest degree.

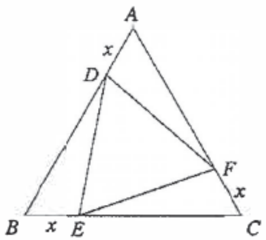
14A.2 HKCEE MA 1982(3) – I – 2



In the figure, $AB = 4$, $AC = 5$ and $BC = 7$.
Calculate $\angle A$ to the nearest degree.

14A.3 HKCEE MA 1985(A/B) I 13

(To continue as 7C.1.)



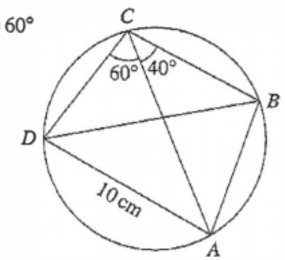
In the figure, ABC is an equilateral triangle. $AB = 2$. D, E, F are points on AB, BC, CA respectively such that $AD = BE = CF = x$.

(a) By using the cosine formula or otherwise, express DE^2 in terms of x.

(b) Show that the area of $\triangle DEF = \dfrac{\sqrt{3}}{4}(3x^2 - 6x + 4)$.

14A.4 HKCEE MA 1989 – I – 6

(Continued from 12A.7.)

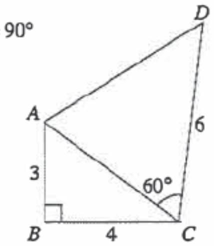


In the figure, $ABCD$ is a cyclic quadrilateral with $AD = 10$ cm, $\angle ACD = 60°$ and $\angle ACB = 40°$.

(a) Find $\angle ABD$ and $\angle BAD$.

(b) Find the length of BD in cm, correct to 2 decimal places.

14A.5 HKCEE MA 1997 I 5



In the figure, ABC is a right angled triangle. $AB = 3$, $BC = 4$, $CD = 6$, $\angle ABC = 90°$ and $\angle ACD = 60°$. Find

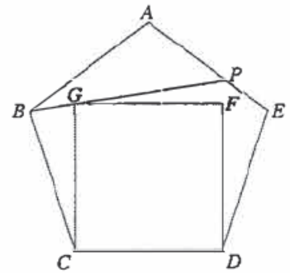
(a) AC,

(b) AD,

(c) the area of $\triangle ACD$.

14A.6 HKCEE MA 2000 I 13

(Continued from 11A.11.)

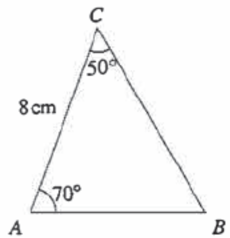


In the figure, $ABCDE$ is a regular pentagon and $CDFG$ is a square. BG produced meets AE at P.

(a) Find $\angle BCG$, $\angle ABP$ and $\angle APB$.

(b) Using the fact that $\dfrac{AP}{\sin \angle ABP} = \dfrac{AB}{\sin \angle APB}$, or otherwise, determine which line segment, AP or PE, is longer.

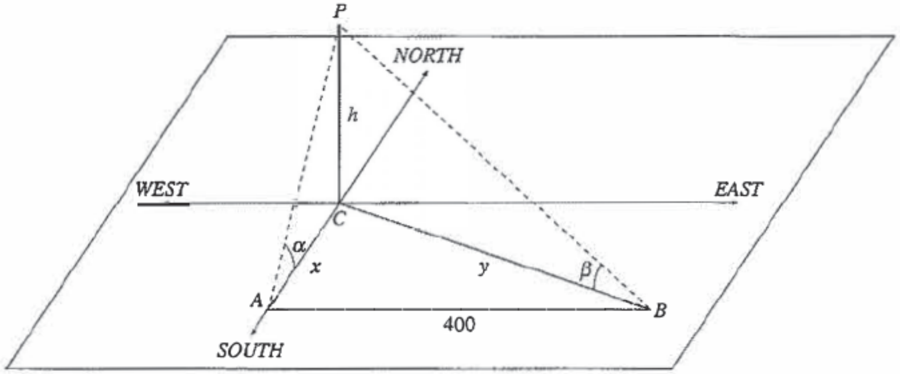
14A.7 HKCEE MA 2001 I 9



In the figure, find AB and the area of $\triangle ABC$.

14B Three-dimensional applications

14B.1 HKCEE MA 1980(1/1*/3) – I – 9



In the figure, PC represents a vertical object of height h metres. From a point A, south of C, the angle of elevation of P is α. From a point B, 400 metres east of A, the angle of elevation of P is β. AC and BC are x metres and y metres respectively.

(a) (i) Express x in terms of h and α.
  (ii) Express y in terms of h and β.

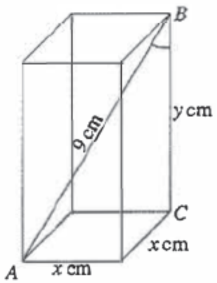
(b) If $\alpha = 60°$ and $\beta = 30°$, find the value of h correct to 3 significant figures.

14B.2 HKCEE MA 1982(1/2/3) I 8

The figure represents the framework of a cuboid made of iron wire. It has a square base of side x cm and a height of y cm. The length of the diagonal AB is 9 cm. The total length of wire used for the framework (including the diagonal AB) is 69 cm.

(a) Find all the values of x and y.

(b) Hence calculate $\angle ABC$ to the nearest degree for the case in which $y > x$.

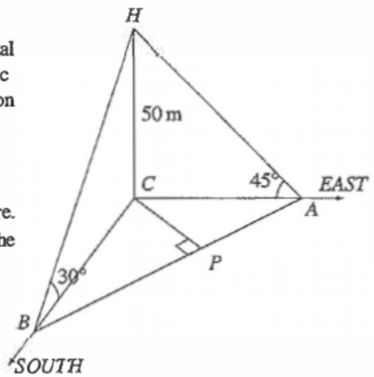


14B.3 HKCEE MA 1983(A/B) I – 13

In the figure, A, B and C are three points on the same horizontal ground. HC is a vertical tower 50 m high. A and B are respectively due east and due south of the tower. The angles of elevation of H observed from A and B are respectively 45° and 30°.

(a) Find the distance between A and B.

(b) P is a point on AB such that $CP \perp AB$.
  (i) Find the distance between C and P to the nearest metre.
  (ii) Find the angle of elevation of H observed from P to the nearest degree.



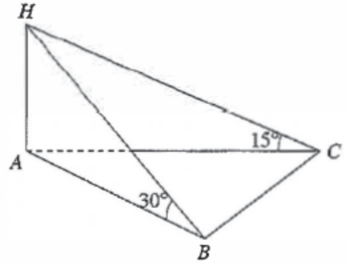
14B.4 HKCEE MA 1984(A/B) – I 13

In the figure, A, B and C lie in a horizontal plane. $AC = 20$ m. HA is a vertical pole. The angles of elevation of H from B and C are 30° and 15° respectively.

(In this question, give your answers correct to 2 decimal places.)

(a) (i) Find, in m, the length of the pole HA.
  (ii) Find, in m, the length of AB.

(b) If A, B and C lie on a circle with AC as diameter,
  (i) find, in m, the distance between B and C;
  (ii) find, in m^2, the area of $\triangle ABC$.

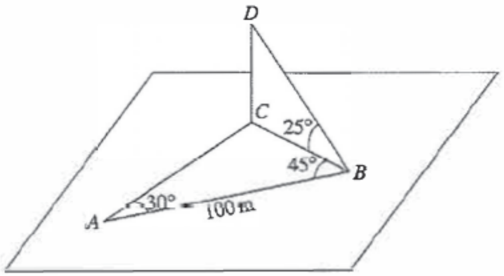


14B.5 HKCEE MA 1985(A/B) – I – 8

In the figure, A, B and C are three points in a horizontal plane. $AB = 100$ m, $\angle CAB = 30°$, $\angle ABC = 45°$.

(a) Find BC and AC, in metres, correct to 1 decimal place.

(b) D is a point vertically above C. From B, the angle of elevation of D is 25°.
  (i) Find CD, in metres, correct to 1 decimal place.
  (ii) X is a point on AB such that $CX \perp AB$.
    (1) Find CX, in metres, correct to 1 decimal place.
    (2) Find the angle of elevation of D from X, correct to the nearest degree.

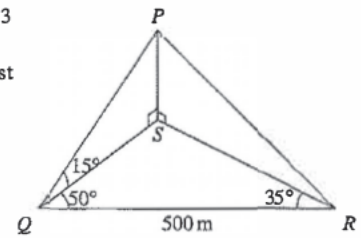


14B.6 HKCEE MA 1986(A/B) I 10

In the figure, Q, R and S are three points on the same horizontal plane. $QR = 500$ m, $\angle SQR = 50°$ and $\angle QRS = 35°$. P is a point vertically above S. The angle of elevation of P from Q is 15°.

(a) Find the distance, in metres, from P to the plane, correct to 3 significant figures.

(b) Find the angle of elevation of P from R, correct to the nearest degree.



14B.7 HKCEE MA 1987(A/B) I – 11

In this question, you should give your answers in cm or degrees, correct to 3 decimal places.

The figure shows a solid in which $ABCD$, $DCFE$ and $ABFE$ are rectangles. DG is the perpendicular from D to AE. $AB = 3$ cm, $AD = 3$ cm and $DE = 2$ cm. $\angle ADE = 80°$.

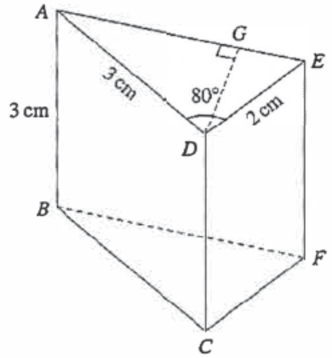
(a) Find AE.

(b) Find $\angle DAE$.

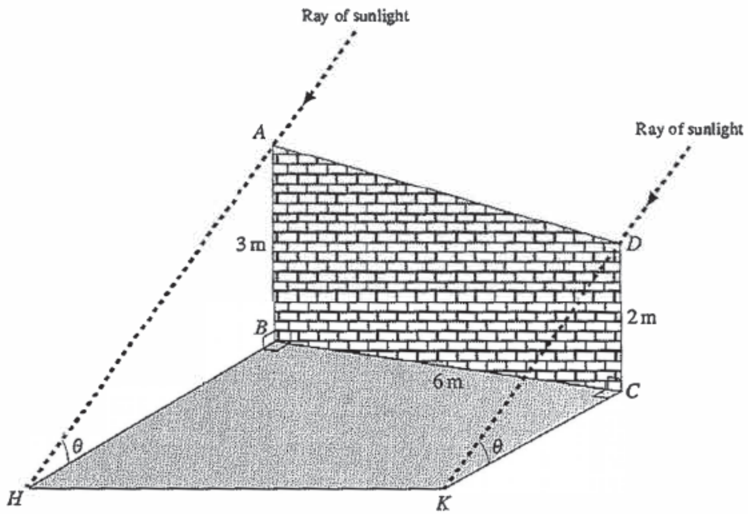
(c) Find DG.

(d) Find BD.

(e) Find the angle between the line BD and the face $ABFE$.



14B.8 HKCEE MA 1988 – I – 13



In the figure, $ABCD$ is a wall in the shape of a trapezium with AB and DC vertical. Rays of sunlight coming from the back of the wall cast a shadow $HBCK$ on the horizontal ground such that the edges HB and KC of the shadow are perpendicular to BC. Suppose the angle of elevation of the sun is θ, $AB = 3$ m, $CD = 2$ m and $BC = 6$ m.

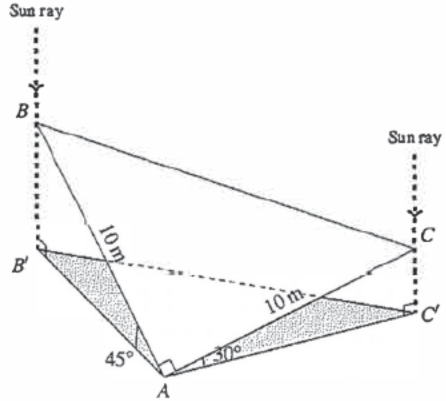
(a) Express HB and KC in terms of θ.

(b) (i) Find the area S_1 of the wall.

(ii) Find, in terms of θ, the area S_2 of the shadow. Hence show that $\dfrac{S_1}{S_2} = \tan\theta$.

(c) If $\theta = 30°$, find the length of the edge HK, leaving your answer in surd form.

14B.9 HKCEE MA 1989 – I – 10



Answers in this question should be given correct to at least 3 significant figures or in surd form.

In the figure, a triangular board ABC, right angled at A with $AB = AC = 10$ m, is placed with the vertex A on the horizontal ground. AB and AC make angles of 45° and 30° with the horizontal respectively. The sun casts a shadow $AB'C'$ of the board on the ground such that B' and C' are vertically below B and C respectively.

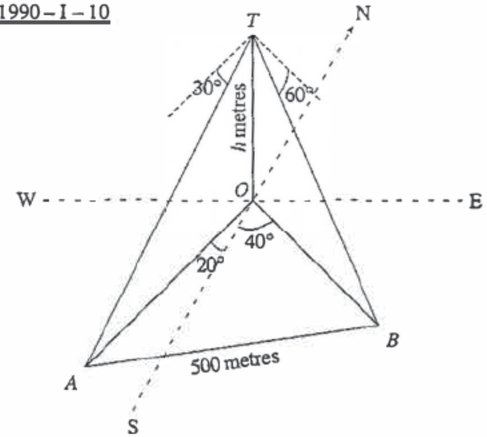
(a) Find the lengths of AB' and AC'.

(b) Find the lengths of BC, BB' and CC'.

(c) Using the results of (b), or otherwise, find the length of $B'C'$.

(d) Find $\angle B'AC'$. Hence find the area of the shadow.

14B.10 HKCEE MA 1990 – I – 10



In the figure, OT represents a vertical tower of height h metres. From the top T of the tower, two landmarks A and B, 500 metres apart on the same horizontal ground, are observed to have angles of depression 30° and 60° respectively. The bearings of A and B from the tower OT are S20°W and S40°E respectively.

(a) Find the lengths of OA and OB in terms of h.

(b) Express the length of AB in terms of h. Hence, or otherwise, find the value of h.

(c) Find $\angle OAB$, correct to the nearest degree. Hence write down

(i) the bearing of B from A,

(ii) the bearing of A from B.

14B.11 HKCEE MA 1992 – I – 15

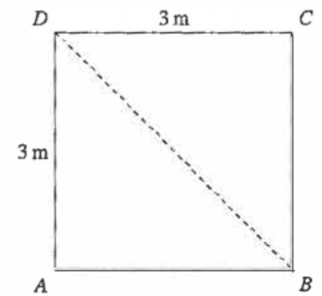


Figure (1)

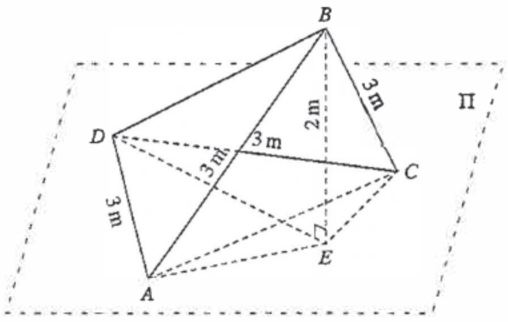


Figure (2)

In Figure (1), $ABCD$ is a thin square metal sheet of side three metres. The metal sheet is folded along BD and the edges AD and CD of the folded metal sheet are placed on a horizontal plane Π with B two metres vertically above the plane Π. E is the foot of the perpendicular from B to the plane Π. (See Figure (2).)

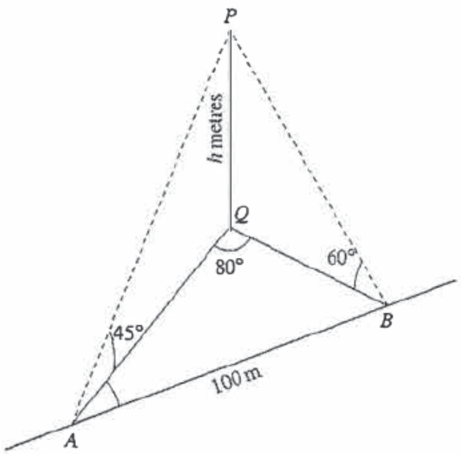
(a) Find the lengths of BD, ED and AE, leaving your answers in surd form.

(b) Find $\angle ADE$.

(c) Find the angle between BD and the plane Π.

(d) Find the angle between the planes ABD and CBD.

14B.12 HKCEE MA 1993 – I – 12



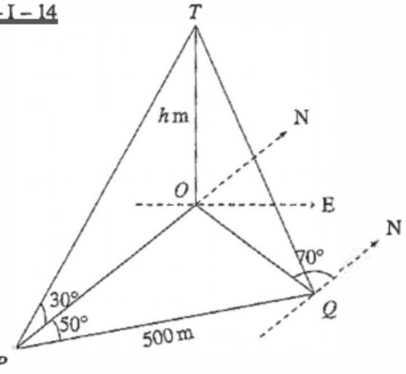
In the figure, PQ is a vertical television tower h metres high. A and B are two points 100 m apart on a straight road in front of the tower with A, B and Q on the same horizontal ground and $\angle AQB = 80°$. The angles of elevation of P from A and B are 45° and 60° respectively.

(a) (i) Express the lengths of AQ and BQ in terms of h.

(ii) Find h and $\angle QAB$.

(b) A person walks from A along the road towards B. At a certain point R between A and B, the person finds that the angle of elevation of P is 50°. How far away is R from A?

14B.13 HKCEE MA 1994 – I – 14



In the figure, OT is a vertical tower of height h metres and O, P and Q are points on the same horizontal plane. When a man is at P, he finds that the tower is due north and that the angle of elevation of the top T of the tower is 30°. When he walks a distance of 500 metres in the direction N50°E to Q, he finds that the bearing of the tower is N70°W.

(a) Find OQ and OP.

(b) Find h.

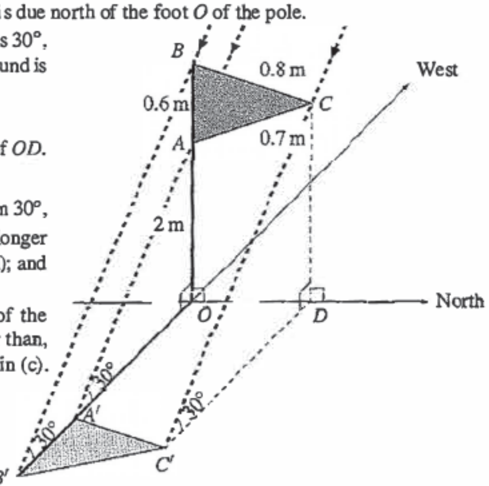
(c) Find the angle of elevation of T from Q, giving your answer correct to the nearest degree.

(d) (i) If he walks a further distance of 400 metres from Q in a direction Nθ°E to a point R (not shown in the figure) on the same horizontal plane, he finds that the angle of elevation of T is 20°. Find $\angle OQR$ and hence write down the value of θ to the nearest integer.

(ii) If he starts from Q again and walks the same distance of 400 metres in another direction to a point S on the same horizontal plane, he finds that the angle of elevation of T is again 20°. Find the bearing of S from Q, giving your answer correct to the nearest degree.

14B.14 HKCEE MA 1995 – I – 15

The figure shows a triangular road sign ABC attached to a vertical pole OAB standing on the horizontal ground. The plane ABC is vertical with $OA = 2$ m, $AB = 0.6$ m, $AC = 0.7$ m and $BC = 0.8$ m. D is a point on the horizontal ground vertically below C and is due north of the foot O of the pole.



The sun is due west. When its angle of elevation is 30°, the shadow of the road sign on the horizontal ground is $A'B'C'$.

(a) Find the lengths of OA' and $A'B'$.

(b) Calculate $\angle BAC$ and hence find the length of OD.

(c) Find the area of the shadow $A'B'C'$.

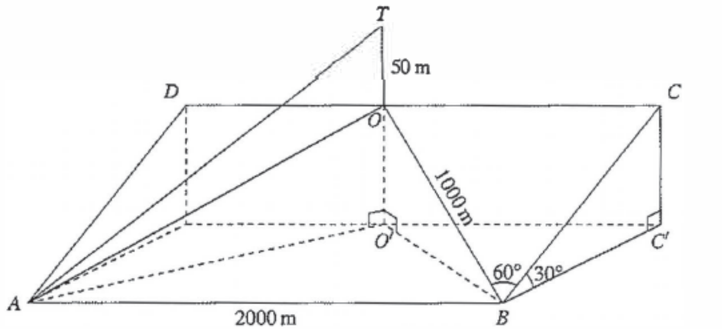
(d) If the angle of elevation of the sun is less than 30°,

(i) state whether the shadow of AB is longer than, shorter than, or equal to $A'B'$ in (a); and hence

(ii) state with reasons whether the area of the shadow of the road sign ABC is larger than, smaller than, or equal to that of $A'B'C'$ in (c).

14B.15 HKCEE MA 1996 – I – 15

In the figure, the rectangular plane $ABCD$ is a hillside with inclination 30°. C' and O' are vertically below C and O respectively so that A, B, C', O' are on the same horizontal plane. BO is a straight path on the hillside which makes an angle 60° with BC, and OT is a vertical tower. $AB = 2000$ m, $BO = 1000$ m and $OT = 50$ m.



(a) Find BC and CC'.

(b) Find the inclination of BO with the horizontal.

(c) Find AT.

(d) There are cable cars going directly from A to T. A man wants to go to T from B and he can do this by taking either one of the following two routes:

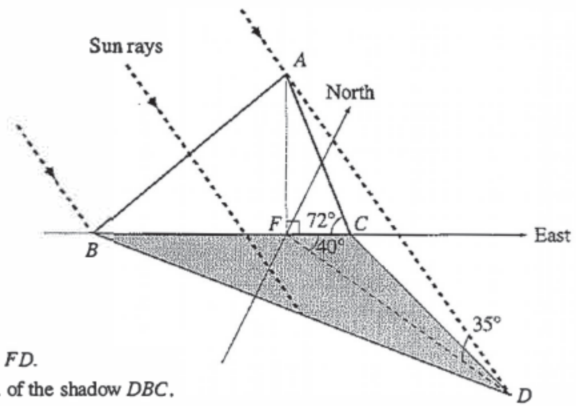
Route I: Walking uphill along BO at an average speed of 0.3 m/s and taking a lift in the tower for 1 minute from O to T.

Route II: Walking along BA at an average speed of 0.8 m/s and taking a cable car from A to T at an average speed of 3.2 m/s.

Determine which route takes a shorter time.

14B.16 HKCEE MA 1998 I – 17

In the figure, triangular sign post ABC stands vertically on the horizontal ground along the east west direction. $AC = 4$ m, $BC = 6$ m, $\angle ACB = 72°$ and F is the foot of the perpendicular from A to BC. When the sun shines from N50°W with an angle of elevation 35°, the shadow of the sign post on the horizontal ground is DBC.



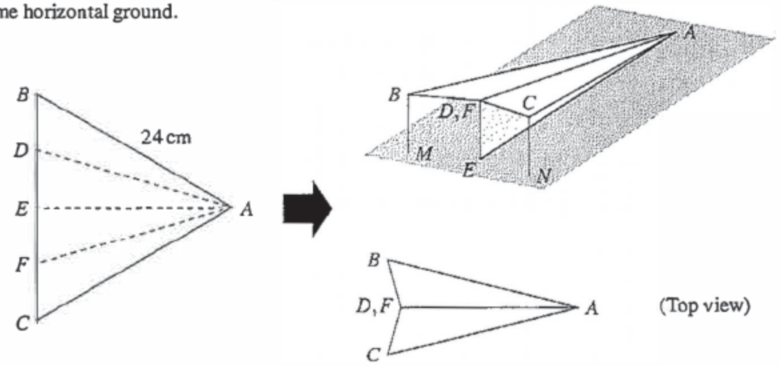
(a) Find AF and FD.

(b) Find the area of the shadow DBC.

(c) Suppose the sun shines from Nx°W, where $50 < x < 90$, but its angle of elevation is still 35°. State with reasons whether the area of the shadow of the sign post on the horizontal ground is greater than, smaller than or equal to the area obtained in (b).

14B.17 HKCEE MA 1999 – I – 18

In the figure, a paper card ABC in the shape of an equilateral triangle of side 24 cm is folded to form a paper aeroplane. D, E and F are points on edge BC so that $BD = DE = EF = FC$. The aeroplane is formed by folding the paper card along the lines AD, AE and AF so that AD and AF coincide. It is supported by two vertical sticks BM and CN of equal length so that A, B, D, F, C lie on the same plane and A, E, M, N lie on the same horizontal ground.



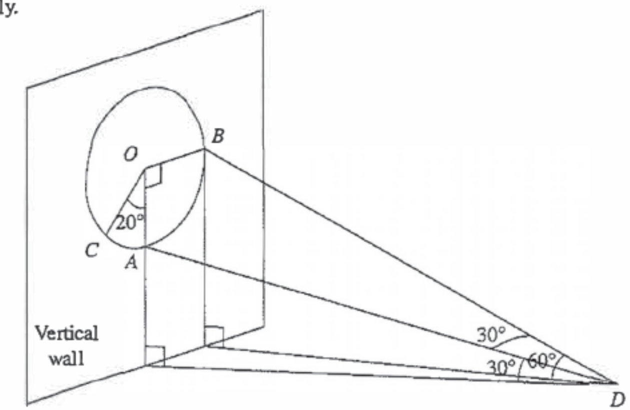
(a) Find the distance between the tips, B and C, of the wings of the aeroplane.

(b) Find the inclination of the wings of the aeroplane to the horizontal ground.

(c) Find the length of the stick CN.

14B.18 HKCEE MA 2000 I – 17

The figure shows a circle with centre O and radius 10 m on a vertical wall which stands on the horizontal ground. A, B and C are three points on the circumference of the circle such that A is vertically below O, $\angle AOB = 90°$ and $\angle AOC = 20°$. A laser emitter D on the ground shoots a laser beam at B. The laser beam then sweeps through an angle of 30° to shoot at A. The angles of elevation of B and A from D are 60° and 30° respectively.



(a) Let A be h m above the ground.

(i) Express AD and BD in terms of h.

(ii) Find h.

(b) Another laser emitter E on the ground shoots a laser beam at A with angle of elevation 25°. The laser beam then sweeps through an angle of 5° to shoot at C. Find $\angle ACE$.

14B.19 HKCEE MA 2001 – I – 16

Figure (1) shows a piece of pentagonal cardboard $ABCDE$. It is formed by cutting off two equilateral triangular parts, each of side x cm, from an equilateral triangular cardboard AFG. AB is 6 cm long and the area of $BCDE$ is $5\sqrt{3}\,\text{cm}^2$.

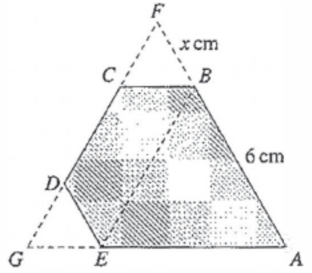


Figure (1)

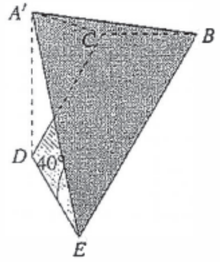


Figure (2)

(a) Show that $x^2 - 12x + 20 = 0$. Hence find x.

(b) The triangular part ABE is folded up along the line BE until the vertex A comes to the position A' (as shown in Figure (2)) such that $\angle A'ED = 40°$.

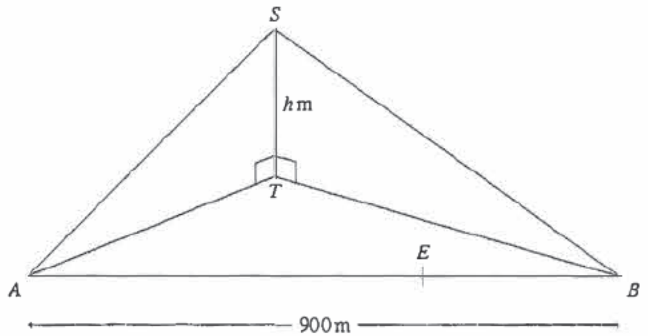
(i) Find the length of $A'D$.

(ii) Find the angle between the planes $BCDE$ and $A'BE$.

(iii) If A', B, C, D, E are the vertices of a pyramid with base $BCDE$, find the volume of the pyramid.

14B.20 HKCEE MA 2002 – I – 14

In the figure, AB is a straight track 900 m long on the horizontal ground. E is a small object moving along AB. ST is a vertical tower of height h m standing on the horizontal ground. The angles of elevation of S from A and B are 20° and 15° respectively. $\angle TAB = 30°$.



(a) Express AT and BT in terms of h. Hence find h.

(b) (i) Find the shortest distance between E and S.

(ii) Let θ be the angle of elevation of S from E. Find the range of values of θ as E moves along AB.

14B.21 HKCEE MA 2003 – I 14

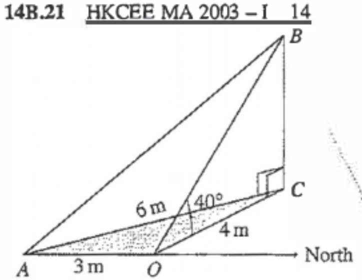


Figure (1)

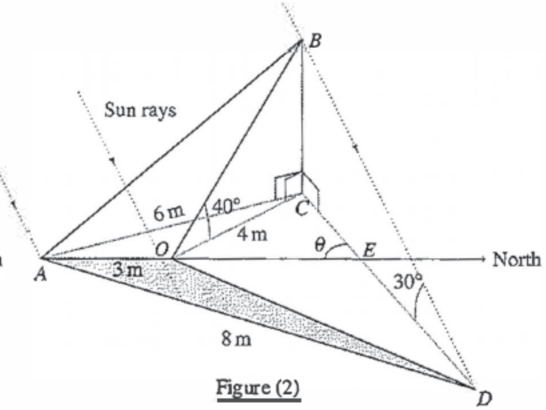


Figure (2)

Figure (1) shows a triangular metal plate OAB standing on the horizontal ground. The side OA lies along the north south direction on the ground. OB is inclined at an angle of 40° to the horizontal. The overhead sun casts a shadow of the plate, OAC, on the ground. $OA = 3$ m, $OC = 4$ m and $AC = 6$ m.

(a) Find $\angle OAC$.

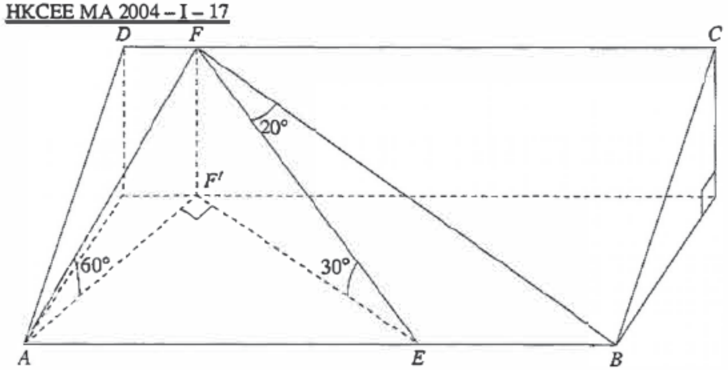
(b) In Figure (2), OAD is the shadow of the plate cast on the horizontal ground when the sun shines from SθW with an angle of elevation 30°. AO is produced to cut CD at E. $AD = 8$ m.

(i) Find CD.

(ii) Find $\angle CAD$.

(iii) Using $CE + ED = CD$, or otherwise, find θ.

14B.22 HKCEE MA 2004 – I – 17



In the figure, $ABCD$ is a rectangular inclined plane. E and F are points on the straight lines AB and CD respectively. F' is vertically below F. A, E, B and F' are on the same horizontal ground. $\angle AF'E = 90°$, $\angle FAF' = 60°$, $\angle FEF' = 30°$, $\angle EFB = 20°$ and $EF = 20$ m.

(a) Find

(i) FF' and AE,

(ii) $\angle AEF$.

(b) A small red toy car goes straight from E to B at an average speed of 2 m/s while a small yellow toy car goes straight from F to B at an average speed of 3 m/s. The two toy cars start going at the same time. Will the yellow toy car reach B before the red one? Explain your answer.

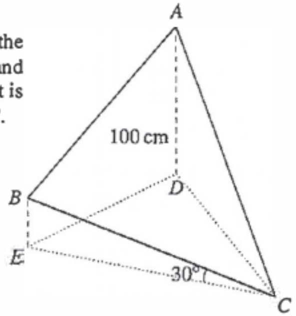
14B.23 HKCEE MA 2005 – I – 14

In the figure, a thin triangular board ABC is held with the vertex C on the horizontal ground. D and E are points on the ground vertically below A and B respectively. BC is inclined at an angle of 30° with the horizontal. It is known that $AD = 100\text{ cm}$, $BC = 120\text{ cm}$, $\angle CAB = 60^\circ$ and $\angle ABC = 80^\circ$.

(a) Find BE and CE.

(b) Find AB and AC.

(c) Find $\angle CDE$ and the shortest distance from C to DE.



14B.24 HKCEE MA 2006 – I – 17

In Figure (1), ABC is a triangular paper card. D is a point lying on AC such that BD is perpendicular to AC. It is known that $AB = 40\text{ cm}$, $BC = 60\text{ cm}$ and $AC = 90\text{ cm}$.

(a) Find AD.

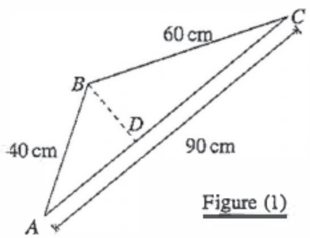


Figure (1)

(b) The triangular paper card in Figure (1) is folded along BD such that AB and BC lie on a horizontal plane as shown in Figure (2).

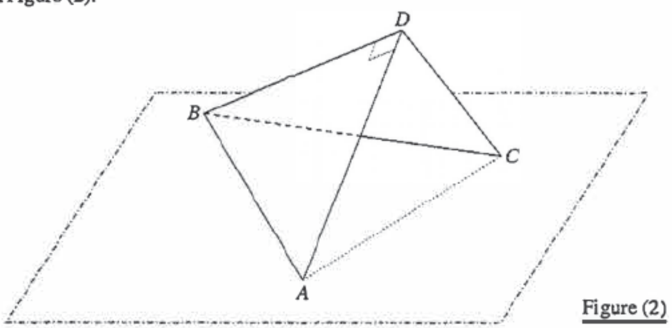


Figure (2)

(i) Suppose $\angle DAC = 62^\circ$.

(1) Find the distance between A and C on the horizontal plane.

(2) Using Heron's formula, or otherwise, find the area of $\triangle ABC$ on the horizontal plane.

(3) Find the height of the tetrahedron $ABCD$ from the vertex D to the base $\triangle ABC$.

(ii) Describe how the volume of the tetrahedron $ABCD$ varies when $\angle ADC$ increases from 30° to 150°. Explain your answer.

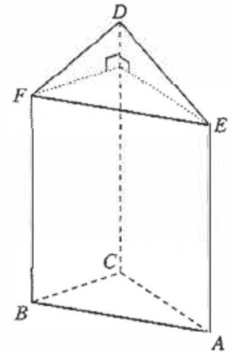
14B.25 HKCEE MA 2007 – I – 16

The figure shows a solid wooden souvenir $ABCDEF$ with the triangular base ABC lying on the horizontal ground. A, B and C are vertically below E, F and D respectively. DEF is an inclined triangular plane. It is given that $AB = 9\text{ cm}$, $BC = 5\text{ cm}$, $AC = 6\text{ cm}$, $AE = BF = 20\text{ cm}$ and $CD = 23\text{ cm}$.

(a) Find the area of the triangular base ABC and the volume of the souvenir $ABCDEF$.

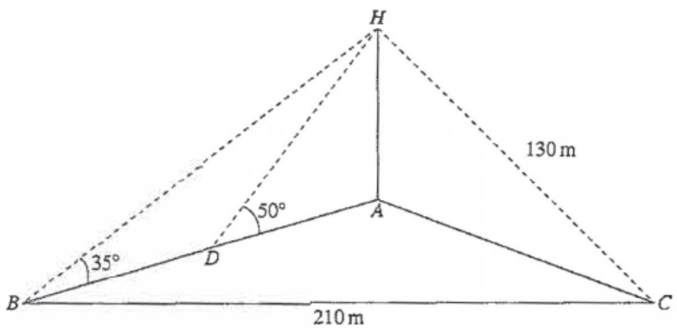
(b) Find $\angle DFE$ and the shortest distance from D to EF.

(c) Can a piece of thin rectangular metal plate of dimensions $5\text{ cm} \times 4\text{ cm}$ be fixed onto the triangular surface DEF so that the thin metal plate completely lies in the triangle DEF? Explain your answer.



14B.26 HKCEE MA 2008 – I – 15

In the figure, H is the top of a tower and A is vertically below H. AB, BC and CA are straight paths on the horizontal ground and D is a point on AB. Christine walks from A to D along AD and finds that the angle of elevation of H from D is 50°. She then walks 50 m to B along DB and finds that the angle of elevation of H from B is 35°.



(a) Find the distance between B and H.

(b) Christine walks 210 m from B to C along BC. It is given that the distance between C and H is 130 m.

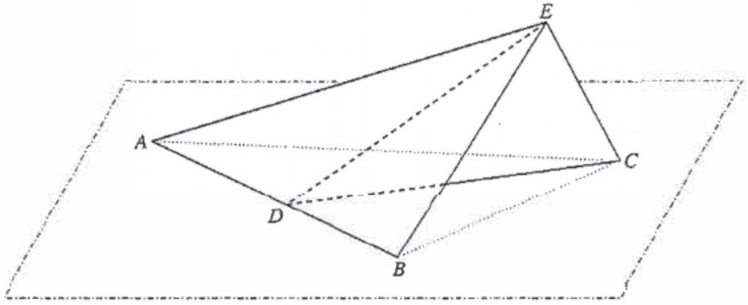
(i) Find $\angle CBH$.

(ii) Find the angle between the plane BCH and the horizontal ground.

(iii) When Christine walks from B to C along BC, is it possible for her to find a point K on BC such that the angle of elevation of H from K is 75°? Explain your answer.

14B.27 HKCEE MA 2009 – I – 17

The figure shows a geometric model fixed on the horizontal ground. The model consists of two thin triangular metal plates ABE and CDE, where D lies on AB and CE is perpendicular to the thin metal plate ABE. It is given that A, B, C and D lie on the horizontal ground. It is found that $AC = 28$ cm, $BC = 25$ cm, $BD = 6$ cm, $BE = 24$ cm and $\angle ABC = 57°$.



(a) Find
  (i) the length of CD,
  (ii) $\angle BAC$,
  (iii) the area of $\triangle ABC$,
  (iv) the shortest distance from E to the horizontal ground.
(b) A student claims that the angle between DE and the horizontal ground is $\angle CDE$. Do you agree? Explain your answer.

14B.28 HKCEE MA 2010 – I – 15

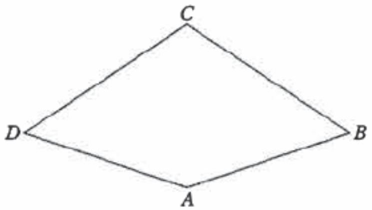


Figure (1)

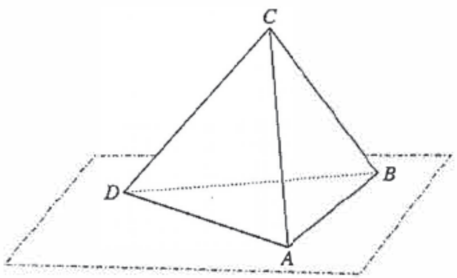


Figure (2)

(a) Figure (1) shows a piece of paper card $ABCD$ in the form of a quadrilateral with $AB = AD$ and $BC = CD$. It is given that $BC = 24$ cm, $\angle BAD = 146°$ and $\angle ABC = 59°$. Find the length of AB.
(b) The paper card described in (a) is folded along AC such that AB and AD lie on the horizontal ground as shown in Figure (2). It is given that $\angle BAD = 92°$.
  (i) Find the distance between B and D on the horizontal ground.
  (ii) Find the angle between the plane ABC and the plane ACD.
  (iii) Let P be a movable point on the slant edge AC. Describe how $\angle BPD$ varies as P moves from A to C. Explain your answer.

14B.29 HKCEE MA 2011 – I – 17

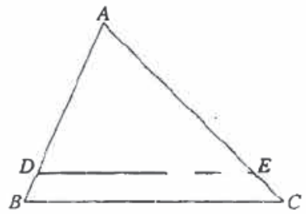


Figure (1)

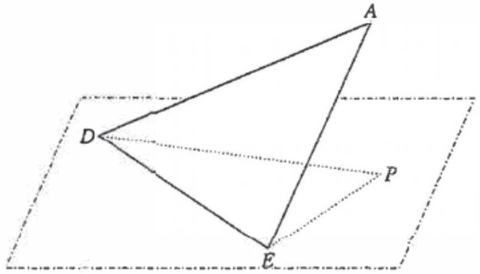
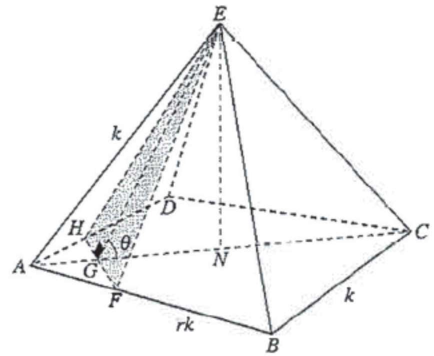


Figure (2)

In Figure (1), ABC is a thin triangular metal sheet. D and E are points lying on AB and AC respectively such that DE is parallel to BC and the distance between DE and BC is 4 cm. It is found that $AB = 20$ cm, $AC = 30$ cm and $\angle BAC = 56°$.
(a) Find
  (i) the length of BC,
  (ii) $\angle ACB$,
  (iii) the perpendicular distance from A to DE,
  (iv) the length of DE.
(b) The thin triangular metal sheet in Figure (1) is cut along DE. The metal sheet ADE is held with DE lying on the horizontal ground as shown in Figure (2). It is given that P is the projection of A on the horizontal ground and the area of $\triangle PDE$ is 120 cm^2. Find
  (i) the angle between the metal sheet ADE and the horizontal ground,
  (ii) the shortest distance from A to the horizontal ground.

14B.30 HKCEE AM 1981 – II – 10

In the figure, $ABCDE$ is a right pyramid with a square base $ABCD$. Each of the eight edges of the pyramid is of length k. F, G and H are points on AB, AC and AD, respectively, such that FGH is a straight line and $BF = DH = rk$, where $0 \le r \le 1$. $EG \perp HF$, $\angle EGC = \theta$ and N is the foot of the perpendicular from E to the base.



(a) Express FE^2 and FG^2 in terms of k and r.
(b) Express EG and EN in terms of k and r.
  Hence, or otherwise, show that $\sin\theta = \dfrac{1}{\sqrt{1+r^2}}$.
(c) Using the results of (b), find the range of the inclination of the plane EFH to the base as r varies from 0 to 1.

14B.31 HKCEE AM 1983 II – 8

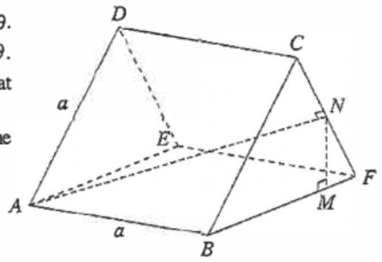
The figure shows a tent consisting of two inclined square planes $ABCD$ and $EFCD$ standing on the horizontal ground $ABFE$. The length of each side of the inclined planes is a. N is a point on CF such that $AN \perp CF$. Let $NF = x (\neq 0)$, $\angle CFB = \theta$ and M be a point on BF such that $NM \perp BF$.

(a) By considering $\triangle ABM$, express AM in terms of a, x and θ.

(b) By considering $\triangle ANF$, express AN in terms of a, x and θ.

(c) Using the results of (a) and (b), or otherwise, show that $x = 2a\cos^2\theta$.

(d) Given that $x = \frac{a}{2}$, find (correct to the nearest degree) the inclination of AN to the horizontal.

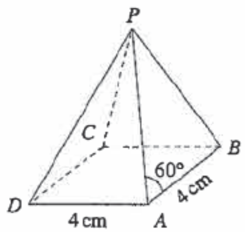


14B.32 HKCEE AM 1991 – II – 6

In the figure, $PABCD$ is a right pyramid with a square base of sides of length 4 cm. $\angle PAB = 60°$. Find, correct to the nearest 0.1 degree,

(a) the angle between the plane PAB and the base $ABCD$,

(b) the angle between the planes PAB and PAD.

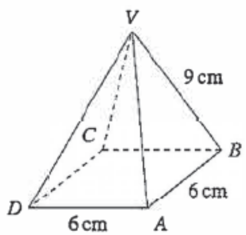


14B.33 HKCEE AM 1992 – II – 7

In the figure, $VABCD$ is a right pyramid with a square base of side 6 cm. $VB = 9$ cm. Find, correct to the nearest 0.1 degree,

(a) the angle between edge VB and the base $ABCD$,

(b) the angle between the planes VAB and VAD.

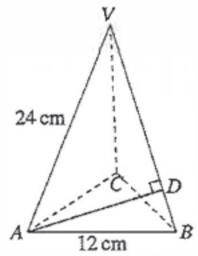


14B.34 HKCEE AM 1993 II – 7

In the figure, $VABC$ is a right pyramid whose base ABC is an equilateral triangle. $AB = 12$ cm and $VA = 24$ cm. D is a point on VB such that AD is perpendicular to VB. Find, correct to 3 significant figures,

(a) $\angle VBA$ and AD,

(b) the angle between the faces VAB and VBC.



14B.35 HKCEE AM 1994 – II – 12

A, B and C are three points on the horizontal ground and $AB = 100$ km. P is a point vertically above C (see Figure (1)). Let $\angle CAB = \alpha$, $\angle CBA = \beta$, $\angle PAC = \theta$.

(a) Show that

(i) $AC = \dfrac{100\sin\beta}{\sin(\alpha+\beta)}$ km.

(ii) $PC = \dfrac{100\sin\beta\tan\theta}{\sin(\alpha+\beta)}$ km.

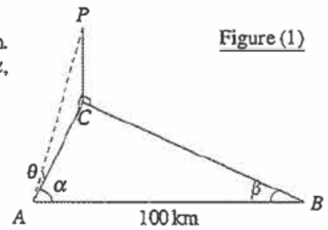


Figure (1)

(b) Suppose at P, $\alpha = 45°$, $\beta = 30°$ and $\theta = 20°$. An aeroplane climbs from P to a point P' along a straight path. The projection of P' on the ground is the point C' (see Figure (2)). Given that $\angle C'AB = 37°$, $\angle C'BA = 43°$ and $\angle P'AC'$ 17°, find, correct to 2 decimal places,

(i) AC and AC',

(ii) the distance between C and C',

(iii) the increase in height of the aeroplane as it climbs from P to P',

(iv) the angle of inclination PP'.

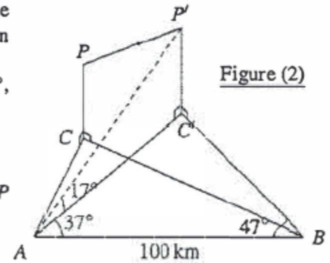


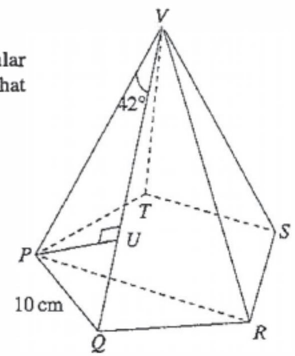
Figure (2)

14B.36 HKCEE AM 1995 – II – 7

In the figure, $VPQRST$ is a right pyramid whose base $PQRST$ is a regular pentagon. $PQ = 10$ cm and $\angle PVQ = 42°$. U is a point on VQ such that PU is perpendicular to VQ. Find, correct to 3 significant figures,

(a) PU and PR,

(b) the angle between the faces VPQ and VQR.



14B.37 HKCEE AM 1996 – II – 12

In Figure (1), ABC is a triangular piece of paper such that $\angle B = 45°$, $\angle C = 30°$ and $AC = 2$. D is the foot of perpendicular from A to BC.

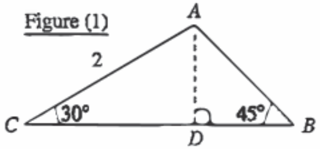


Figure (1)

(a) Find AB, BD and DC.

(b) The paper is folded along AD. It is then placed on a horizontal table such that the edges AB and AC lie on the table and the plane DAB is vertical. (See Figure (2).) E is the foot of perpendicular from D to AB.

(i) If θ is the angle between DC and the horizontal, show that $\sin\theta = \frac{\sqrt{6}}{6}$.

(ii) Find CE.
Hence show that $\angle EAC = 45°$.

(iii) Find the angle between the two planes DAB and DAC to the nearest degree.

[Hint: You may tear off Figure (3) to help you answer part (b).]

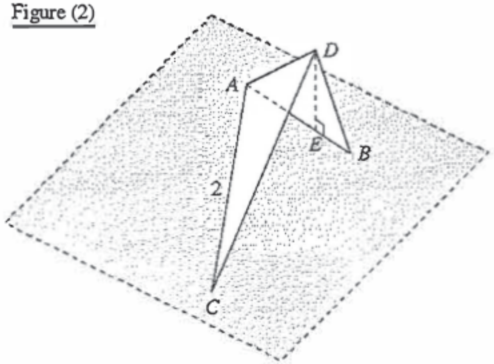


Figure (2)

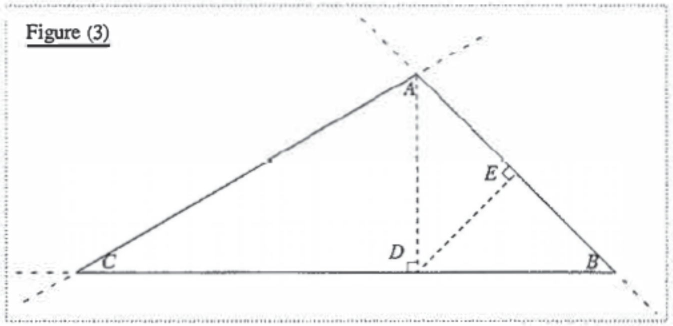


Figure (3)

14B.38 HKCEE AM 1997 – II – 12

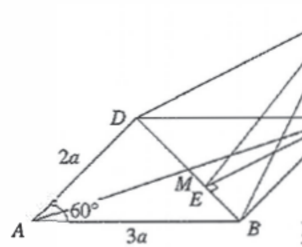


Figure (1)

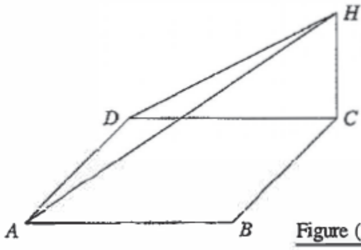


Figure (2)

In Figure (1), $ABCD$ is a parallelogram on a horizontal plane with $AB = 3a$, $AD = 2a$ and $\angle BAD = 60°$. H is a point vertically above C and $HC = \alpha$.

(a) (i) Find AC in terms of a.

(ii) If M is the mid-point of AC, find the angle of elevation of H from M to the nearest degree.

(b) E is a point on BD such that CE is perpendicular to BD.

(i) Find BD and CE in terms of a.

(ii) Using Pythagoras' theorem and its converse, show that HE is perpendicular to BD.
Hence find the angle between the planes HBD and $ABCD$ to the nearest degree.

(c) Figure (2) shows the planes HAD and $ABCD$. X is a point lying on both planes such that the angle between the two planes is $\angle HXC$. Find AX in terms of a.

14B.39 HKCEE AM 1998 – II – 13

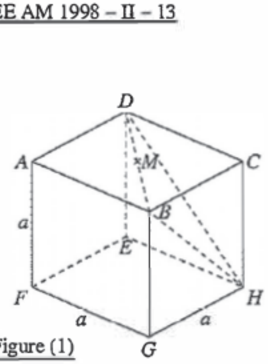


Figure (1)

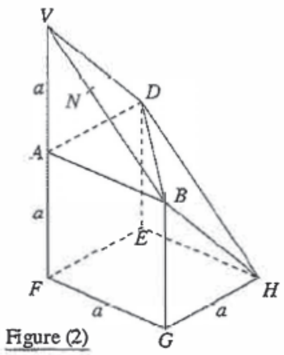


Figure (2)

(a) Figure (1) shows a solid cube $ABCDEFGH$ of side a. Let M be the mid point of BD.

(i) Find CM.

(ii) Find the angle between the lines CM and HM to the nearest degree.

(b) The tetrahedron $BCDH$ is cut off from the cube in (a) and is then placed on top of the solid $ABDEFGH$ as shown in Figure (2). The face BCD of the tetrahedron coincides with the face BAD of the solid $ABDEFGH$ such that vertex H of the tetrahedron moves to position V and vertex C coincides with A. The two faces BHD and BVD of the new solid lie on the same plane.

(i) Show that $\sin\angle FVH = \frac{\sqrt{3}}{3}$ and find the perpendicular distance from F to the face $BVDH$.

(ii) Let N be the point on VB such that DN and AN are both perpendicular to VB.

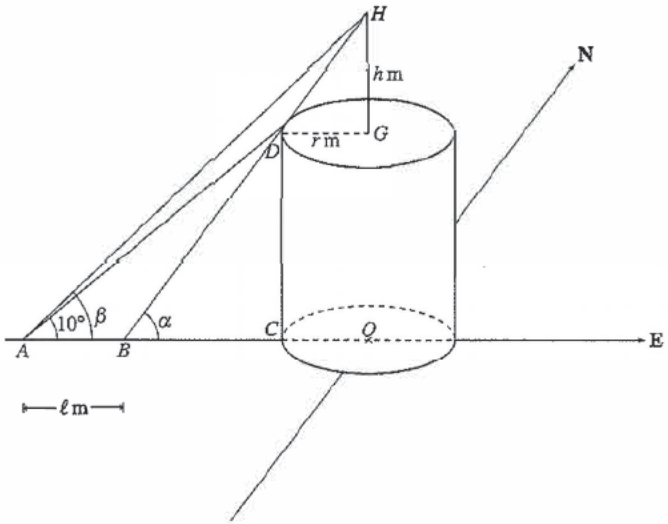
(1) Find DN.

(2) Find the angle between the faces BVD and BVA to the nearest degree.

(iii) A student says that the angle between the faces BHD and $ABGF$ is $\angle AND$. Explain briefly whether the student is correct.

14B.40 <u>HKCEE AM 1999 – II – 11</u>

The figure shows a right cylindrical tower with a radius of r m standing on horizontal ground. A vertical pole HG, h m in height, stands at the centre G of the roof of the tower. Let O be the centre of the base of the tower. C is a point on the circumference of the base of the tower due west of O and D is a point on the roof verticcally above C. A man stands at a point A due west of O. The angles of elevation of D and H from A are $10°$ and β respectively. The man walks towards the east to a point B where he can just see the top of the pole H as shown in the figure. (Note: If he moves forward, he can no longer see the pole.) The angle of elevation of H from B is α. Let $AB = \ell$ m.



(a) Show that $AD = \dfrac{\ell \sin\alpha}{\sin(\alpha - 10°)}$ m. Hence

  (i) express CD in terms of ℓ and α,

  (ii) show that $h = \dfrac{\ell \sin^2\alpha \sin(\beta - 10°)}{\sin(\alpha - 10°)\sin(\alpha - \beta)}$. (Hint: You may consider $\triangle ADH$.)

(b) **In this part, numerical answers should be given correct to two significant figures.**

  Suppose $\alpha = 15°$, $\beta = 10.2°$ and $\ell = 97$.

  (i) Find

    (1) the height of the pole HG,

    (2) the height and radius of the tower.

  (ii) P is a point south-west of O. Another man standing at P can just see the top of the pole H. Find

    (1) the distance of P from O,

    (2) the bearing of B from P.

14B.41 <u>HKCEE AM 2001 – 15</u>

Figure (1)

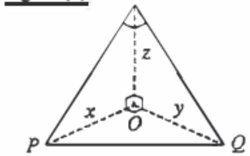
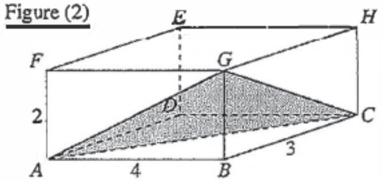


Figure (2)



(a) Figure (1) shows a pyramid $OPQR$. The sides OP, OQ and OR are of lengths x, y and z respectively, and they are mutually perpendicular to each other.

  (i) Express $\cos\angle PRQ$ in terms of x, y and z.

  (ii) Let S_1, S_2, S_3 and S_4 denote the areas of $\triangle OPR$, $\triangle OPQ$, $\triangle OQR$ and $\triangle PQR$ respectively. Show that $S_4{}^2 = S_1{}^2 + S_2{}^2 + S_3{}^2$.

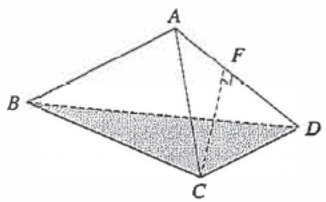
(b) Figure (2) shows a rectangular block $ABCDEFGH$. The lengths of sides AB, BC and AF are 4, 3 and 2 respectively. A pyramid $ABCG$ is cut from the block along the plane GAC.

  (i) Find the volume of the pyramid $ABCG$.

  (ii) Find the angle between the side AB and the plane GAC, giving your answer correct to the nearest degree.

14B.42 <u>HKCEE AM 2002 17</u>

The figure shows a tetrahedron $ABCD$ such that $AB = 28$, $CD = 30$, $AC = AD = 25$ and $BC = BD = 40$. F is the foot of perpendicular from C to AD.



(a) Find $\angle BFC$, giving your answer correct to the nearest degree.

(b) A student says that $\angle BFC$ represents the angle between the planes ACD and ABD.
  Explain whether the student is correct or not.

14B.43 <u>HKCEE AM 2003 18</u>

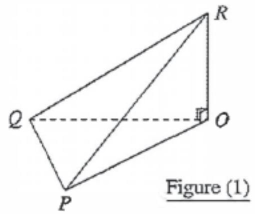


Figure (1)

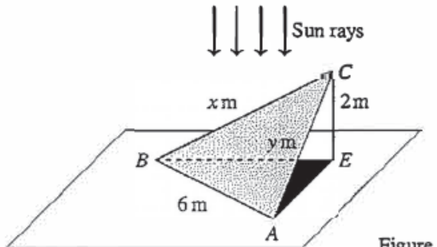


Figure (2)

(a) Figure (1) shows a tetrahedron $OPQR$ with RO perpendicular to the plane OPQ. Let θ be the angle between the planes RPQ and OPQ. Show that $\dfrac{\text{Area of } \triangle OPQ}{\text{Area of } \triangle RPQ} = \cos\theta$.

(b) In Figure (2), a pole of length 2 m is erected vertically at a point E on the horizontal ground. A triangular board ABC of area 12 m^2 is supported by the pole such that side AB touches the ground and vertex C is fastened to the top of the pole. $AB = 6$ m, $BC = x$ m and $CA = y$ m, where $6 > x > y$. The sun rays are vertical and cast a shadow of the board on the ground.

  (i) Find the area of the shadow.

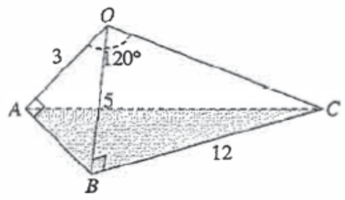
  (ii) Two other ways of supporting the board with the pole are to fasten vertex A or B to the top of the pole with the opposite side touching the ground. Among these three ways determine which one will give the largest shadow.

14B.44 HKCEE AM 2004 – 11

In the figure, $OABC$ is a pyramid such that $OA = 3$, $OB = 5$, $BC = 12$, $\angle AOC = 120°$ and $\angle OAB = \angle OBC = 90°$.

(a) Find AC.

(b) A student says that angle between the planes OBC and ABC can be represented by $\angle OBA$. Determine whether the student is correct or not.



14B.45 HKCEE AM 2006 – 17

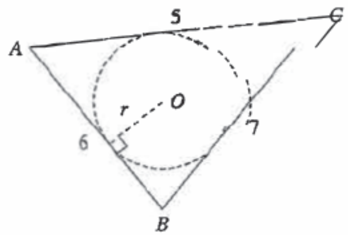


Figure (1)

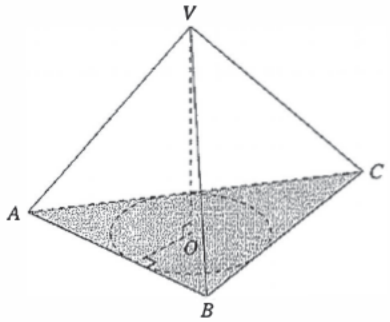


Figure (2)

(a) ABC is a triangle with $AB = 6$, $BC = 7$ and $CA = 5$. A circle is inscribed in the triangle (see Figure (1)). Let O be the centre of the circle and r be its radius.

(i) Find the area of $\triangle ABC$.

(ii) By considering the areas of $\triangle AOB$, $\triangle BOC$ and $\triangle COA$, show that $r = \frac{2\sqrt{6}}{3}$.

(b) $VABC$ is a tetrahedron with the $\triangle ABC$ described in (a) as the base (see Figure (2)). Furthermore, point O is the foot of perpendicular from V to the plane ABC. It is given that the angle between the planes VAB and ABC is 60°.

(i) Find the volume of the tetrahedron $VABC$.

(ii) Find the area of $\triangle VBC$.

(iii) Find the angle between the side AB and the plane VBC, giving your answer correct to the nearest degree.

14B.46 HKCEE AM 2008 – 16

The figure shows a triangular pyramid $VABC$. The base of the pyramid is a right-angled triangle with $AB = 2$ cm and $\angle BAC = 90°$. $\triangle VAB$ and $\triangle VAC$ are equilateral triangles.

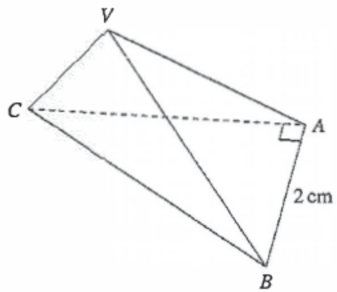
(a) Explain why the angle between the planes VAB and ABC **cannot** be represented by $\angle VAC$.

(b) Let D and E be the mid-points of AB and BC respectively.

(i) Show that the angle between the planes VAB and ABC can be represented by $\angle VDE$.

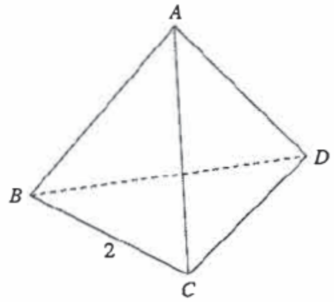
(ii) Show that $\angle VED = 90°$.

(c) Find the distance between the point C and the plane VAB.



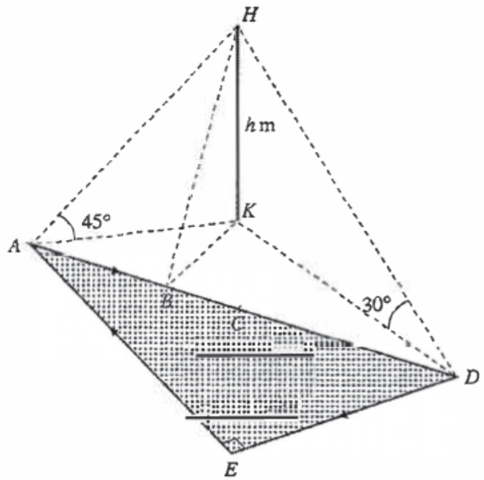
14B.47 HKCEE AM 2009 – 12

In the figure, $ABCD$ is a regular tetrahedron with length of each side 2. Find the angle between the planes ABC and BCD correct to the nearest degree.



14B.48 HKCEE AM 2009 – 18

The figure shows a park AED on a horizontal ground. The park is in the form of a right-angled triangle surrounded by a walking path with negligible width. Henry walks along the path at a constant speed. He starts from point A at 7:00 am. He reaches points B, C and D at 7:10 am, 7:15 am and 7:30 am respectively and returns to A via point E. The angles of elevation of H, the top of a tower outside the park, from A and D are 45° and 30° respectively. At point B, Henry is closest to the point K which is the projection of H on the ground. Let $HK = h$ m.



(a) Express DK in terms of h.

(b) Show that $AB = \sqrt{\frac{2}{3}}h$ m.

(c) Find the angle of elevation of H from C correct to the nearest degree.

(d) Henry returns to A at 8:10 am. It is known that the area of the park is 9450 m^2.

(i) Find h.

(ii) A vertical pole of length 3 m is located such that it is equidistant from A, D and E. Find the angle of elevation of H from the top of the pole correct to the nearest degree.

14B.49 HKCEE AM 2010 – 17

[Note: In this question, numerical answers may be given correct to 3 significant figures. You may use a ruler to tear off Figure (5) to help you if you attempt this question.]

Three faces of a tetrahedron (see Figure (4)) are formed by folding a triangular piece of paper ABC, where $AB = AC = 11$ cm, $\angle BAC = 120°$ and AD is an altitude (see Figure (1)), with the following steps.

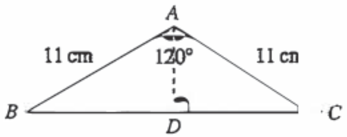


Figure (1)

Step 1: Fold AB over so that AB coincides with AD, then crease line AE (see Figure (2)).

(a) Calculate the length of AE and the area of $\triangle ABE$.

Step 2: Fold AC over so that AC coincides with AE, then crease line AF (see Figure (3)).

(b) Calculate the length of AF.

Step 3: Unfold the paper. Then fold the paper along AE and AF such that AB coincides with AC completely (see Figure (4)).

(c) It is known that the volume of the tetrahedron is 22.582 cm^3 (correct to 5 significant figures).

(i) Find the angle between the line AF and the plane $\triangle ABE$ in the tetrahedron.

(ii) Find the angle between the planes $\triangle ABE$ and $\triangle ABF$ in the tetrahedron.

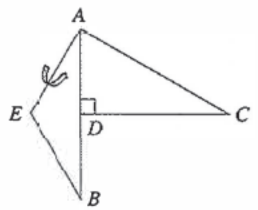


Figure (2)

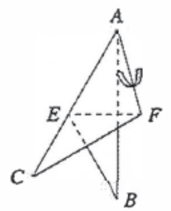


Figure (3)

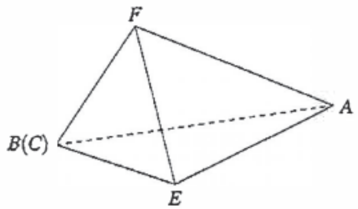


Figure (4)

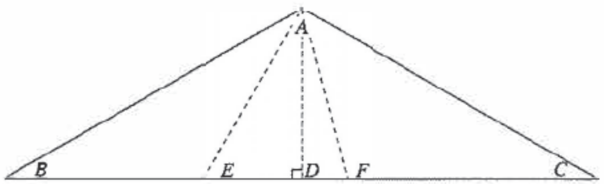


Figure (5)

14B.50 HKCEE AM 2011 – 13

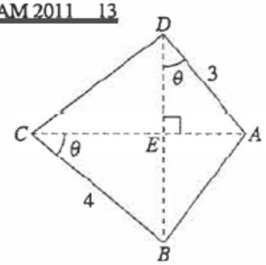


Figure (1)

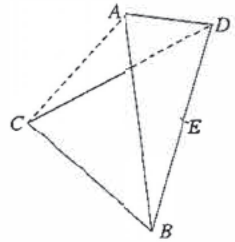


Figure (2)

In Figure (1), $ABCD$ is a quadrilateral with diagonals AC and BD perpendicular to each other and intersecting at E. It is given that $AD = 3$, $BC = 4$ and $\angle ADE = \angle BCE = \theta$, where $0° < \theta < 90°$.

(a) (i) Show that $AB = 5\sin\theta$.

(ii) Express CD in terms of θ.

(b) The quadrilateral is folded along BD as shown in Figure (2). Let the planes ABD and BCD be Π_1 and Π_2 respectively. Let $\angle ABC = \alpha$. It is given that

the angle between the lines AB and BC the angle between the planes Π_1 and Π_2.

(i) By considering the length of AC, show that $\cos\alpha = \dfrac{4\sin\theta}{5 - 3\cos\theta}$.

(ii) Prove that α is acute.

(iii) Furthermore, it is given that

the angle between the line AB and Π_2 = the angle between the line AD and Π_2.

State with reason whether the angle between the line AC and Π_2 is greater than, less than or equal to the angle between the line AB and Π_2.

14B.51 HKDSE MA SP – I – 18

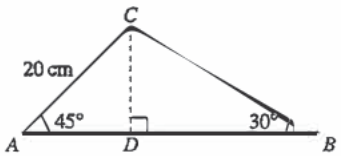


Figure (1)

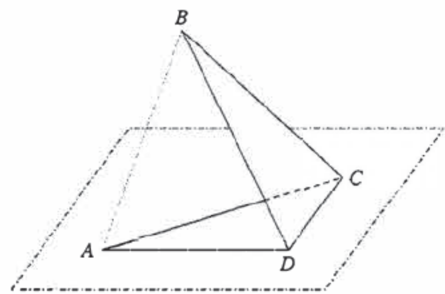


Figure (2)

In Figure (1), ABC is a triangular paper card. D is a point lying on AB such that CD is perpendicular to AB. It is given that $AC = 20$ cm, $\angle CAD = 45°$ and $\angle CBD = 30°$.

(a) Find, in surd form, BC and BD.

(b) The triangular paper card in Figure (1) is folded along CD such that $\triangle ACD$ lies on the horizontal plane as shown in Figure (2).

(i) If the distance between A and B is 18 cm, find the angle between the plane BCD and the horizontal plane.

(ii) Describe how the volume of the tetrahedron $ABCD$ varies when $\angle ADB$ increases from 40° to 140°. Explain your answer.

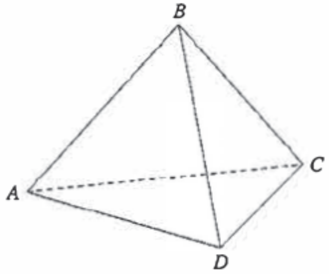
14B.52 HKDSE MA PP I 18

The figure shows a geometric model $ABCD$ in the form of a tetrahedron. It is found that $\angle ACB = 60°$, $AC = AD = 20$ cm, $BC = BD = 12$ cm and $CD = 14$ cm.

(a) Find the length of AB.

(b) Find the angle between the plane ABC and the plane ABD.

(c) Let P be a movable point on the slant edge AB. Describe how $\angle CPD$ varies as P moves from A to B. Explain your answer.



14B.53 HKDSE MA 2012 I – 18

Figure (1) shows a right pyramid $VABCD$ with a square base, where $\angle VAB = 72°$. The length of a side of the base is 20 cm. Let P and Q be the points lying on VA and VD respectively such that PQ is parallel to BC and $\angle PBA = 60°$. A geometric model is made by cutting off the pyramid $VPBCQ$ from $VABCD$ as shown in Figure (2).

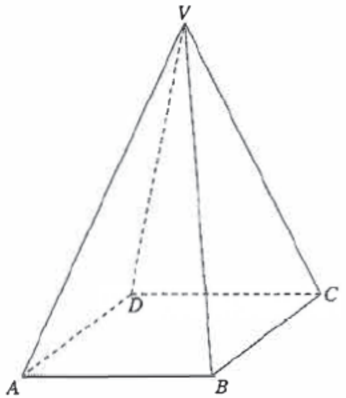


Figure (1)

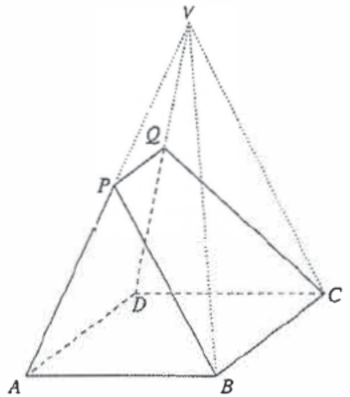


Figure (2)

(a) Find the length of AP.

(b) Let α be the angle between the plane $PBCQ$ and the base $ABCD$.

  (i) Find α.

  (ii) Let β be the angle between PB and the base $ABCD$. Which one of α and β is greater? Explain your answer.

14B.54 HKDSE MA 2013 – I – 18

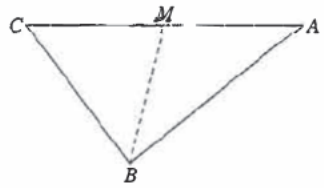


Figure (1)

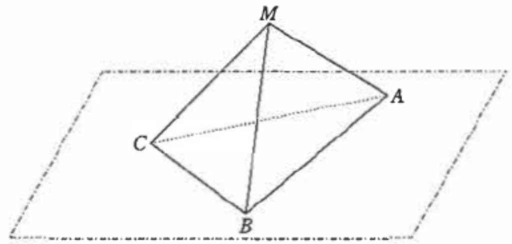


Figure (2)

(a) Figure (1) shows a piece of triangular paper card ABC with $AB = 28$ cm, $BC = 21$ cm and $AC = 35$ cm. Let M be a point lying on AC such that $\angle BMC = 75°$. Find

  (i) $\angle BCM$,

  (ii) CM.

(b) Peter folds the triangular paper card described in (a) along BM such that AB and BC lie on the horizontal ground as shown in Figure (2). It is given that $\angle AMC = 107°$.

  (i) Find the distance between A and C on the horizontal ground.

  (ii) Let N be a point lying on BC such that MN is perpendicular to BC. Peter claims that the angle between the face BCM and the horizontal ground is $\angle ANM$. Do you agree? Explain your answer.

14B.55 HKDSE MA 2014 I – 17

Figure (1) shows a solid pyramid $VABCD$ with a rectangular base, where $AB = 18$ cm, $BC = 10$ cm, $VB = VC = 30$ cm and $\angle VAB = \angle VDC = 110°$.

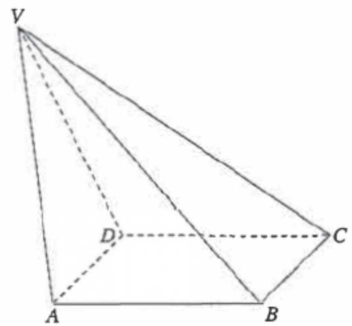


Figure (1)

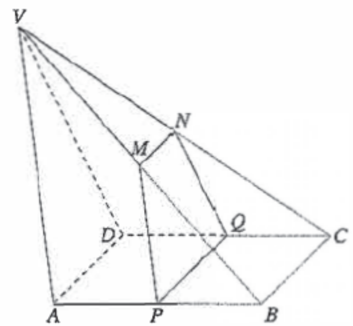


Figure (2)

(a) Find $\angle VBA$.

(b) P, Q, M and N are the mid points of AB, CD, VB and VC respectively. A geometric model is made by cutting off $PBCQNM$ from $VABCD$ as shown in Figure (2). A craftsman claims that the area of the trapezium $PQNM$ is less than 70 cm^2. Do you agree? Explain your answer.

14B.56 HKDSE MA 2015 – I 19

In Figure (1), $ABCDB'$ is a pentagonal paper card. It is given that $AB = AB' = 40\text{ cm}$, $BC = B'D = 24\text{ cm}$ and $\angle ABC = \angle AB'D = 80°$.

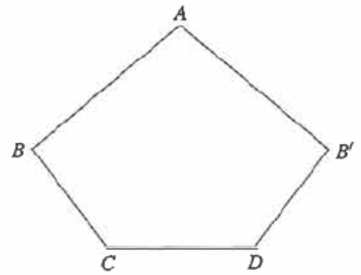


Figure (1)

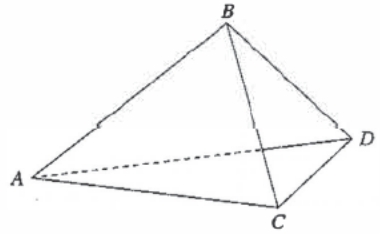
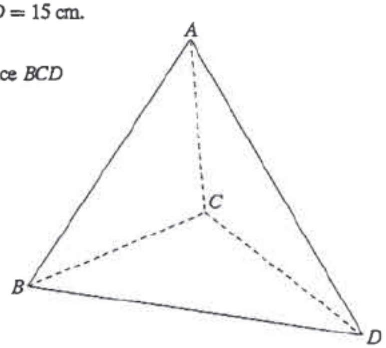


Figure (2)

(a) Suppose that $105° \le \angle BCD \le 145°$.
  (i) Find the distance between A and C.
  (ii) Find $\angle ACB$.
  (iii) Describe how the area of the paper card varies when $\angle BCD$ increases from 105° to 145°. Explain your answer.
(b) Suppose that $\angle BCD = 132°$. The paper card in Figure (1) is folded along AC and AD such that AB and AB' join together to form a pyramid $ABCD$ as shown in Figure (2). Find the volume of the pyramid $ABCD$.

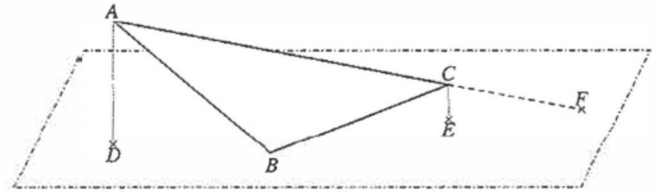
14B.57 HKDSE MA 2016 – I – 19

The figure shows a geometric model $ABCD$ in the form of a tetrahedron. It is given that $\angle BAD = 86°$, $\angle CBD = 43°$, $AB = 10\text{ cm}$, $AC = 6\text{ cm}$, $BC = 8\text{ cm}$ and $BD = 15\text{ cm}$.
(a) Find $\angle ABD$ and CD.
(b) A craftsman claims that the angle between AB and the face BCD is $\angle ABC$. Do you agree? Explain your answer.



14B.58 HKDSE MA 2017 – I 19

ABC is a thin triangular metal sheet, where $BC = 24\text{ cm}$, $\angle BAC = 30°$ and $\angle ACB = 42°$.
(a) Find the length of AC.
(b) In the figure, the thin metal sheet ABC is held such that only the vertex B lies on the horizontal ground. D and E are points lying on the horizontal ground vertically below the vertices A and C respectively. AC produced meets the horizontal ground at the point F. A craftsman finds that $AD = 10\text{ cm}$ and $CE = 2\text{ cm}$.
  (i) Find the distance between C and F.
  (ii) Find the area of $\triangle ABF$.
  (iii) Find the inclination of the thin metal sheet ABC to the horizontal ground.
  (iv) The craftsman claims that the area of $\triangle BDF$ is greater than 460 cm^2. Do you agree? Explain your answer.



14B.59 HKDSE MA 2018 I – 17

(a) In Figure (1), $ABCD$ is a paper card in the shape of a parallelogram. It is given that $AB = 60\text{ cm}$, $\angle ABD = 20°$ and $\angle BAD = 120°$. Find the length of AD.
(b) The paper card in Figure (1) is folded along BD such that the distance between A and C is 40 cm (see Figure (2)).
  (i) Find $\angle ABC$.
  (ii) Find the angle between the plane ABD and the plane BCD.

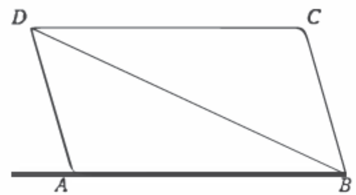


Figure (1)

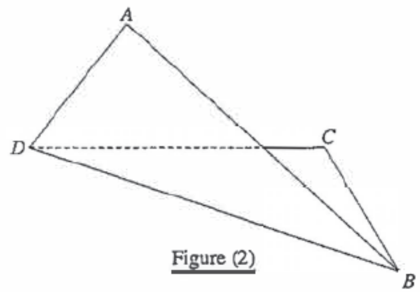


Figure (2)

14B.60 HKDSE MA 2019 I – 18

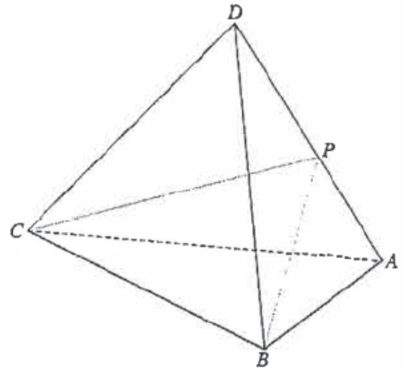
The figure shows a tetrahedron $ABCD$. Let P be a point lying on AD such that BP is perpendicular to AD. A craftsman finds that $AC = AD = CD = 13$ cm, $BC = 8$ cm, $BD = 12$ cm and $\angle ABD = 72°$.

(a) Find

(i) $\angle BAD$,

(ii) CP.

(b) The craftsman claims that $\angle BPC$ is the angle between the face ABD and the face ACD. Is the claim correct? Explain your answer.



14B.61 HKDSE MA 2020 – I – 19

$PQRS$ is a quadrilateral paper card, where $PQ = 60$ cm , $PS = 40$ cm , $\angle PQR = 30°$, $\angle PRQ = 55°$ and $\angle QPS = 120°$. The paper card is held with QR lying on the horizontal ground as shown in Figure 3.

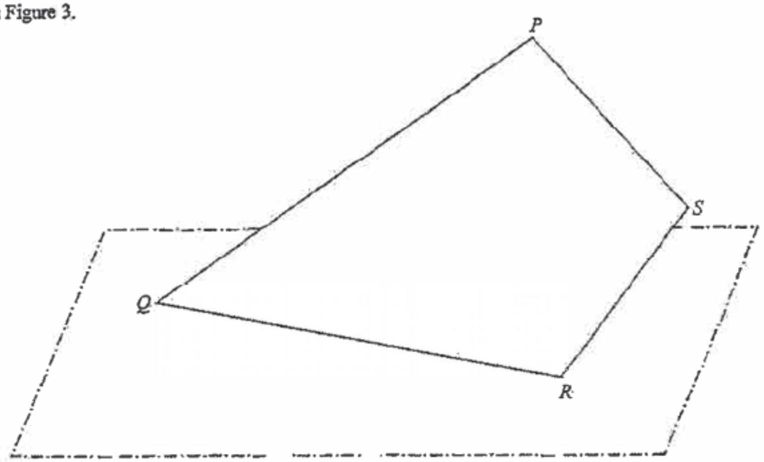


Figure 3

(a) Find the length of RS . (3 marks)

(b) Find the area of the paper card (2 marks)

(c) It is given that the angle between the paper card and the horizontal ground is $32°$.

(i) Find the shortest distance from P to the horizontal ground.

(ii) A student claims that the angle between RS and the horizontal ground is at most $20°$. Is the claim correct? Explain your answer.

(7 marks)

14 Applications of Trigonometry

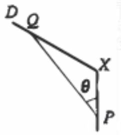
14A Two-dimensional applications

14A.1 HKCEE MA 1981(2/3) – I – 11

(a) Distance at noon $= \sqrt{24^2+9^2-2\cdot 24\cdot 9\cos 60^\circ}$
$= 21$ (km)

(b) At 4 p.m.,
Distance travelled by $P = 4.5\times 4 = 18$ (km)
$\Rightarrow\ PX = 24-18 = 6$ (km)
Distance travelled by $Q = 6\times 4 = 24$ (km)
$\Rightarrow\ QX = 24-9 = 15$ (km) [Q has gone past X.]
$\therefore$ Distance at 4 p.m. $\sqrt{6^2+15^2-2\cdot 6\cdot 15\cos 60^\circ}$
$= \sqrt{171} = 13.1$ (km, 3 s.f.)

(c) $\theta = \cos^{-1}\dfrac{(\sqrt{171})^2+6^2-15^2}{2(\sqrt{171})(6)} = 96.59^\circ$
$\therefore$ Bearing $= 360^\circ - 96.59^\circ$
$= 263^\circ$
or N97°W (nearest deg)



14A.2 HKCEE MA 1982(3) – I – 2

$\angle A = \cos^{-1}\dfrac{4^2+5^2-7^2}{2\cdot 4\cdot 5} = 102^\circ$ (nearest deg)

14A.3 HKCEE MA 1985(A/B) – I – 13

(a) $DE^2 = BD^2+BE^2-2\cdot BD\cdot BE\cos\angle B$
$= (2-x)^2+x^2-\cancel{2}(2-x)(x)\cancel{\cos 60^\circ}$
$= 3x^2-6x+4$

(b) Area of $\triangle DEF = \frac{1}{2}DE\cdot DE\sin 60^\circ$
$= \frac{1}{2}(3x^2-6x+4)\cdot\frac{\sqrt{3}}{2}$
$= \frac{\sqrt{3}}{4}(3x^2-6x+4)$
$= \frac{3\sqrt{3}}{4}\left(x^2-2x+\frac{4}{3}\right)$
$= \frac{3\sqrt{3}}{4}\left(x^2-2x+1+\frac{1}{3}\right)$
$= \frac{3\sqrt{3}}{4}(x-1)^2+\frac{\sqrt{3}}{4}$
$\therefore$ Minimum area is attained when $x=1$.

(c) $\frac{3\sqrt{3}}{4}(x-1)^2+\frac{\sqrt{3}}{4} < \frac{\sqrt{3}}{3}$
$(x-1)^2 \le \frac{1}{9}$
$\frac{-1}{3} \le x-1 \le \frac{1}{3} \ \Rightarrow\ \frac{2}{3}\le x\le\frac{4}{3}$

14A.4 HKCEE MA 1989 – I – 6

(a) $\angle ABD = \angle ACD = 60^\circ$ ($\angle$s in the same segment)
$\angle BAD = 180^\circ-(60^\circ+40^\circ)$ (opp. $\angle$s, cyclic quad.)
$= 80^\circ$

(b) $\dfrac{BD}{\sin\angle BAD} = \dfrac{AD}{\sin\angle ABD}$
$BD = \dfrac{10\sin 80^\circ}{\sin 60^\circ} = 11.37$ (cm, 2 d.p.)

14A.5 HKCEE MA 1997 – I – 5

(a) $AC = \sqrt{3^2+4^2} = 5$
(b) $AD = \sqrt{5^2+6^2-2\cdot 5\cdot 6\cos 60^\circ} = \sqrt{31}$ (= 5.57, 3.s.f)
(c) Area $= \frac{1}{2}(5)(6)\sin 60^\circ = \frac{15\sqrt{3}}{2}$ (= 13.0, 3 s.f.)

14A.6 HKCEE MA 2000 – I – 13

(a) $\angle A = \angle ABC = \angle BCD$ (given)
$= (5-2)180^\circ \div 5$ ($\angle$ sum of polygon)
$= 108^\circ$
$\angle GCD = 90^\circ$ (property of square)
$\Rightarrow\ \angle BCG = 108^\circ - 90^\circ = 18^\circ$
$BC = CD = CG$ (given)
$\angle GBC = \angle BGC$ (base $\angle$s, isos. $\triangle$)
In $\triangle BCG$, $\angle GBC = (180^\circ - \angle BCG)\div 2$ ($\angle$ sum of $\triangle$)
$= 81^\circ$
$\angle ABP = 108^\circ - 81^\circ = 27^\circ$
$\angle APB = 180^\circ - \angle A - \angle ABP = 45^\circ$ ($\angle$ sum of $\triangle$)

(b) $AP - \dfrac{\sin\angle ABP}{\sin\angle APB}AB - \dfrac{\sin 27^\circ}{\sin 45^\circ}AB - 0.642AB$
$PE = AB \quad AP = (1-0.642)AB = 0.358AB < AP$
i.e. AP is longer.

14A.7 HKCEE MA 2001 – I – 9

$\dfrac{AB}{\sin 50^\circ} = \dfrac{8}{\sin(180^\circ-50^\circ-70^\circ)}$
$\Rightarrow\ AB = 7.0764 = 7.08$ (cm, 3 s.f.)
$\therefore$ Area $= \frac{1}{2}(8)(7.0764)\sin 70^\circ = 26.6$ (cm², 3 s.f.)

14B Three-dimensional applications

14B.1 HKCEE MA 1980(1/1*/3) – I – 9

(a) (i) In $\triangle PAC$, $x = \dfrac{h}{\tan\alpha}$
(ii) In $\triangle PBC$, $y = \dfrac{h}{\tan\beta}$

(b) In $\triangle ABC$,
$x^2+400^2 = y^2$
$\left(\dfrac{h}{\tan 60^\circ}\right)^2+160000 = \left(\dfrac{h}{\tan 30^\circ}\right)^2$
$\dfrac{h^2}{3}+160000 = 3h^2$
$h^2 = 60000 \ \Rightarrow\ h = 245$ (3 s.f.)

14B.2 HKCEE MA 1982(1/2/3) – I – 8

(a) $8x+4y+9 = 69 \ \Rightarrow\ y = 15-2x$
$AC^2 = 9^2-y^2 \ \Rightarrow\ 2x^2 = 81-y^2$
$2x^2 = 81-(15-2x)^2$
$x^2-10x+24 = 0 \ \Rightarrow\ x = 4$ or 6
When $x=4$, $y = 15-2(4) = 7$
When $x=6$, $y = 15 \quad 2(6) = 3$

(b) $\angle ABC = \cos\ ^{1}\frac{y}{9} = \cos^{-1}\frac{7}{9} = 39^\circ$ (nrst deg)

14B.3 HKCEE MA 1983(A/B) – I – 13

(a) In $\triangle ACH$, $AC = \dfrac{50}{\tan 45^\circ} = 50$ (m)
In $\triangle BCH$, $BC = \dfrac{50}{\tan 30^\circ} = 50\sqrt{3}$ (m)
In $\triangle ABC$, $AB = \sqrt{(50)^2+(50\sqrt{3})^2} = 100$ (m)

(b) (i) $\dfrac{AC\cdot BC}{2} = \dfrac{CP\cdot AB}{2}$ (= Area of $\triangle ABC$)
$\Rightarrow\ CP = \dfrac{(50)(50\sqrt{3})}{100} = 25\sqrt{3} = 43.3$ (m, 3 s.f.)
(ii) Required $\angle = \angle HPC = \tan^{-1}\dfrac{HC}{CP} = 49^\circ$ (nrst deg)

14B.4 HKCEE MA 1984(A/B) – I – 13

(a) (i) In $\triangle ACH$,
$HA = 20\tan 15^\circ = 5.25898 = 5.36$ (m, 2 d.p.)
(ii) In $\triangle ABH$,
$AB = \dfrac{HA}{\tan 30^\circ} = 9.28203 = 9.28$ (m, 2 d.p.)

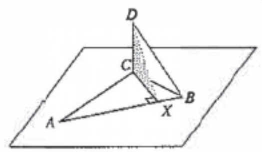
(b) Given: $\angle ABC = 90^\circ$ ($\angle$ in semi-circle)
(i) $BC = \sqrt{AC^2-AB^2} = 17.71564 = 17.72$ (m, 2 d.p.)
(ii) Area $= \frac{1}{2}AB\cdot BC = 82.22$ m² (2 d.p.)

14B.5 HKCEE MA 1985(A/B) – I – 8

(a) In $\triangle ABC$, $\dfrac{BC}{\sin 30^\circ} = \dfrac{100}{\sin(180^\circ-30^\circ-45^\circ)} = \dfrac{AC}{\sin 45^\circ}$
$\Rightarrow\ BC = 51.76381 = 51.8$ (m, 1 d.p.)
$AC = 73.20508 = 73.2$ (m, 1 d.p.)

(b) (i) In $\triangle BCD$, $CD = BC\tan 25^\circ = 24.13789$
$= 24.1$ (m, 1 d.p.)

(ii)



(1) In $\triangle CXB$, $CX = BC\sin 45^\circ = 36.60254$
$= 36.6$ (m, 1 d.p.)
(2) Required $\angle = \angle DXC$
$= \tan^{-1}\dfrac{CD}{CX} = 33^\circ$ (nrst deg)

14B.6 HKCEE MA 1986(A/B) – I – 10

(a) In $\triangle QRS$, $\dfrac{QS}{\sin 35^\circ} = \dfrac{500}{\sin(180^\circ-50^\circ-35^\circ)}$
$\Rightarrow\ QS = 287.88370$ (m)
$\therefore$ In $\triangle PQS$,
Required distance $= PS = QS\tan 15^\circ$
$= 77.13821 = 77.1$ (m, 3 s.f.)

(b) In $\triangle QRS$, $\dfrac{RS}{\sin 50^\circ} = \dfrac{500}{\sin 95^\circ} \ \Rightarrow\ RS = 384.48530$ (m)
$\therefore$ In $\triangle PRS$, Required $\angle = \angle PRS = = \tan^{-1}\dfrac{PS}{RS}$
$= 11^\circ$ (nrst deg)

14B.7 HKCEE MA 1987(A/B) – I – 11

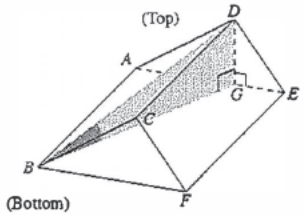
(a) In $\triangle ADE$, $AE = \sqrt{3^2+2^2-2\cdot 3\cdot 2\cos 80^\circ}$
$= 3.30397 = 3.304$ (cm, 3 d.p.)

(b) In $\triangle ADE$, $\angle DAE = \cos^{-1}\dfrac{AE^2+3^2-2^2}{2\cdot AE\cdot 3}$
$= 36.59365^\circ = 36.594^\circ$ (3 d.p.)

(c) In $\triangle ADG$, $DG = 3\sin\angle DAE$
$= 1.7884077 = 1.788$ (cm, 3 d.p.)

(d) In $\triangle ABD$, $BD = \sqrt{3^2+3^2}$
$= \sqrt{18} = 4.243$ (cm, 3 d.p.)

(e)



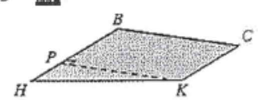
Required $\angle = \angle DBG = \sin\ ^{1}\dfrac{DG}{BD} = 24\ 931^\circ$ (3 d.p.)

14B.8 HKCEE MA 1988 – I – 13

(a) In $\triangle ABH$, $HB = \dfrac{3}{\tan\theta}$
In $\triangle DCK$, $KC = \dfrac{2}{\tan\theta}$

(b) (i) $S_1 = \dfrac{(2+3)(6)}{2} = 15$ (m²)
(ii) $S_2 = \dfrac{\left(\frac{3}{\tan\theta}+\frac{2}{\tan\theta}\right)(6)}{2} = \dfrac{15}{\tan\theta}$ (m²)
$\therefore\ \dfrac{S_1}{S_2} = \dfrac{15}{\frac{15}{\tan\theta}} = \tan\theta$

(c)



Let P be the foot of perpendicular from K to BH.
$PK = 6$ m, $PH = \dfrac{3}{\tan 30^\circ} - \dfrac{2}{\tan 30^\circ} = \sqrt{3}$ (m)
$\therefore\ HK = \sqrt{PK^2+PH^2} = \sqrt{39}$ (m)

14B.9 <u>HKCEE MA 1989 – I – 10</u>

(a) In $\triangle ABB'$, $AB' = 10\cos 45° = 5\sqrt{2}$ (m) (7.07 m, 3 s.f.)

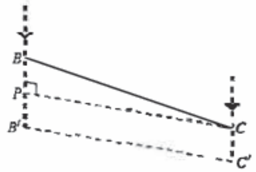
In $\triangle ACC'$, $AC' = 10\cos 30° = 5\sqrt{3}$ (m) (8.66 m, 3 s.f.)

(b) In $\triangle ABC$, $BC = \sqrt{10^2+10^2} = \sqrt{200}$ (m) (14.1 m, 3 s.f.)

In $\triangle ABB'$, $BB' = 10\sin 45° = 5\sqrt{2}$ (m) (7.07 m, 3 s.f.)

In $\triangle ACC'$, $CC' = 10\sin 30° = 5$ (m)

(c)



Let P be the foot of perpendicular from C to BB'.

$BP = BB' - CC' = 5(\sqrt{2}-1)$ m

$\therefore B'C' = PC$
$= \sqrt{BC^2 - BP^2}$
$= \sqrt{200 - 25(\sqrt{2}-1)^2}$
$= \sqrt{125+50\sqrt{2}}$ (m) (14.0 m, 3 s.f.)

(d) In $\triangle AB'C'$,

$\angle B'AC' = \cos^{-1}\dfrac{AB'^2+AC'^2-B'C'^2}{2\cdot AB'\cdot AC'}$
$= \cos^{-1}\dfrac{(5\sqrt{2})^2+(5\sqrt{3})^2-(125+50\sqrt{2})}{2(5\sqrt{2})(5\sqrt{3})}$
$= 125.2644° = 125°$ (3 s.f.)

Hence, Area $= \dfrac{1}{2}(AB')(AC')\sin\angle B'AC' = 25$ (m^2)

14B.10 <u>HKCEE MA 1990 – I – 10</u>

(a) $\angle TAO = 30°$, $\angle TBO = 60°$

In $\triangle TOA$, $OA = \dfrac{h}{\tan 30°} = \sqrt{3}h$ (m)

In $\triangle TOB$, $OB = \dfrac{h}{\tan 60°} = \dfrac{h}{\sqrt{3}}$ (m)

(b) In $\triangle OAB$,

$AB = \sqrt{OA^2+OB^2-2\cdot OA\cdot OB\cos(20°+40°)}$
$= \sqrt{\dfrac{10}{3}h^2 - h^2} = \sqrt{\dfrac{7}{3}}h$ (m)

$\therefore h = 500 \div \sqrt{\dfrac{7}{3}} = 327.3268 = 327$ (m, 3 s.f.)

(c) $\angle OAB = \cos^{-1}\dfrac{OA^2+500^2-OB^2}{2\cdot OA\cdot 500}$
$= 19.1066° = 19°$ (nearest deg)

(i) N$(20°+19°)$E = N39°E

(ii) S39°W

14B.11 <u>HKCEE MA 1992 – I – 15</u>

(a) In $\triangle ABD$, $BD = \sqrt{3^2+3^2} = \sqrt{18}$ (m)

In $\triangle BDE$, $ED = \sqrt{BD^2 - BE^2} = \sqrt{14}$ (m)

In $\triangle ABE$, $AE = \sqrt{AB^2 - BE^2} = \sqrt{5}$ (m)

(b) In $\triangle ADE$, $\angle ADE = \cos^{-1}\dfrac{3^2+14-5}{2\cdot 3\cdot\sqrt{14}}$
$= 36.69923° = 36.7°$ (3 s.f.)

(c) Required $\angle = \angle BDE = \sin^{-1}\dfrac{BE}{BD}$
$= 28.12551° = 28.1°$ (3 s.f.)

(d) In $\triangle ADC$, $\angle ADC = 2\angle ADE = 73.39845°$

$AC = \sqrt{3^2+3^2-2\cdot 3\cdot 3\cos 73.39845°}$
$= 3.58569$ (m)

Denote the intersection of the diagonals of the square $ABCD$ by P. Since $BD \perp AC$ at P, the required angle is $\angle APC$ (in Figure (2)).

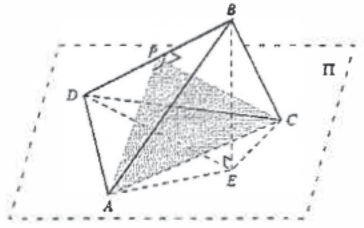


Figure (2)

$AP = PC = \dfrac{1}{2}BD = \dfrac{\sqrt{18}}{2}$

In $\triangle APC$, $\angle APC = \cos^{-1}\dfrac{(\frac{\sqrt{18}}{2})^2+(\frac{\sqrt{18}}{2})^2-3.58569^2}{2(\frac{\sqrt{18}}{2})(\frac{\sqrt{18}}{2})}$
$= 115°$ (3 s.f.)

14B.12 <u>HKCEE MA 1993 – I – 12</u>

(a) (i) In $\triangle APQ$, $AQ = \dfrac{h}{\tan 45°} = h$ (m)

In $\triangle BPQ$, $BQ = \dfrac{h}{\tan 60°} = \dfrac{h}{\sqrt{3}}$ (m)

(ii) In $\triangle ABQ$,

$100^2 = h^2 + \left(\dfrac{h}{\sqrt{3}}\right)^2 - 2(h)\left(\dfrac{h}{\sqrt{3}}\right)\cos 80°$

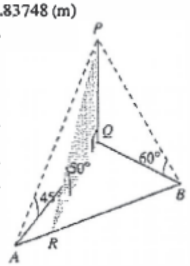
$10000 = \left(\dfrac{4}{3} - \dfrac{2\cos 80°}{\sqrt{3}}\right)h^2$

$h = 93.954854 = 94.0$ (3 s.f.)

$\angle QAB = \cos^{-1}\dfrac{AQ^2+100^2-BQ^2}{2\cdot AQ\cdot 100}$
$= 32.29019° = 32.3°$ (3 s.f.)

(b) In $\triangle PQR$, $QR = \dfrac{h}{\tan 50°} = 78.83748$ (m)

(From A to B, the angle of elevation increases from 45° until it reaches the maximum. Supposing the max is reached at point M, R must lie between A and R as the angle of elevation between M and B must be larger than 60°. Since $\angle AMQ = 90°$, $\angle ARQ$ must be obtuse.)



<u>Method 1</u>

In $\triangle AQR$, $AQ^2+AR^2-2\cdot AQ\cdot AR\cos\angle QAB = QR^2$

$AR^2 - 158.8501AR + 2612.1658 = 0$

$AR = 140.22$ (rej.) or 18.6 (m, 3 s.f.)

<u>Method 2</u>

In $\triangle AQR$, $\dfrac{\sin\angle ARQ}{AQ} = \dfrac{\sin\angle QAR}{QR}$

$\sin\angle ARQ = \dfrac{h\sin 32.29019°}{\frac{h}{\tan 50°}}$

$\angle ARQ = 39.54201°$ (rej.) or $140.45799°$

$\Rightarrow \angle AQR = 180° - 32.29019° - 140.45799°$
$= 7.25182°$

$\therefore AR = \dfrac{QR\sin\angle AQR}{\sin\angle QAR} = 18.6$ (m, 3 s.f.)

14B.13 <u>HKCEE MA 1994 – I – 14</u>

(a) In $\triangle OPQ$, $\dfrac{OQ}{\sin 50°} = \dfrac{500}{\sin 70°} = \dfrac{OP}{\sin(180°-50°-70°)}$

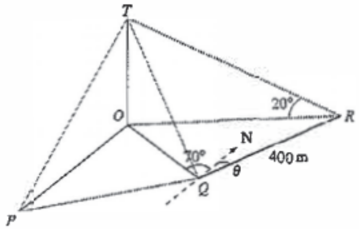
$\Rightarrow OQ = 407.60373 = 408$ (m, 3 s.f.)

$OP = 460.80249 = 461$ (m, 3 s.f.)

(b) In $\triangle OPT$, $h = OP\tan 30° = 266.04444 = 266$ (m, 3 s.f.)

(c) In $\triangle OQT$. Required $\angle = \angle OQT$
$= \tan^{-1}\dfrac{h}{OQ} = 33°$ (nrst deg)

(d) (i)



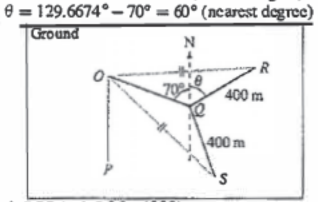
In $\triangle ORT$, $OR = \dfrac{h}{\tan 20°} = 730.9511$ (m)

In $\triangle OQR$,

$\angle OQR = \cos^{-1}\dfrac{OQ^2+QR^2-OR^2}{2\cdot OQ\cdot QR}$
$= \cos^{-1}\dfrac{407.60373^2+400^2-730.9511^2}{2\cdot 407.60373\cdot 400}$
$= 129.6674° = 130°$ (nearest degree)

$\therefore \theta = 129.6674° - 70° = 60°$ (nearest degree)

(ii)



$\because \triangle OQR \cong \triangle OQS$ (SSS)

$\therefore \angle OQS = \angle OQR = 130°$

$\Rightarrow$ Bearing $= 360° - 70° - 130° = 160°$ (S20°E)

14B.14 <u>HKCEE MA 1995 – I – 15</u>

(a) In $\triangle OAA'$, $OA' = \dfrac{2}{\tan 30°} = 2\sqrt{3}$ (m) (3.46 m, 3 s.f.)

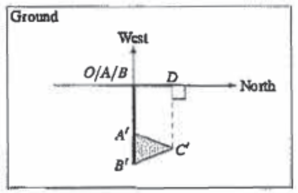
In $\triangle OBB'$, $OB' = \dfrac{2+0.6}{\tan 30°} = 2.6\sqrt{3}$

$\Rightarrow A'B' = 2.6\sqrt{3} - 2\sqrt{3} = 0.6\sqrt{3}$ (m) (1.04 m, 3 s.f.)

(b) In $\triangle ABC$, $\angle BAC = \cos^{-1}\dfrac{0.6^2+0.7^2-0.8^2}{2\cdot 0.6\cdot 0.7}$
$= 75.52249° = 75.5°$ (3 s.f.)

$\therefore OD = AC\sin\angle BAC = 0.67777 = 0.678$ (m, 3 s.f.)

(c)



As the height of $\triangle A'B'C'$ with $A'B'$ as base is also OD,

Area of shadow $= \dfrac{A'B'\cdot OD}{2} = 0.352$ m^2 (3 s.f.)

(d) (i) Let the angle of elevation be θ.

$\because A'B' = \dfrac{0.6}{\tan\theta}$

$\therefore \theta < 30° \Rightarrow \tan\theta < \tan 30° \Rightarrow \dfrac{0.6}{\tan\theta} > \dfrac{0.6}{\tan 30°}$

Thus, $A'B'$ will become longer.

(ii) Since the area of the shadow is $\dfrac{A'B'\cdot OD}{2}$, when the angle of elevation is smaller, $A'B'$ is longer while OD is <u>unchanged</u>, the area of the shadow is <u>larger</u>.

14B.15 <u>HKCEE MA 1996 – I – 15</u>

(a) In $\triangle OBC$, $BC = 1000\cos 60° = 500$ (m)

In $\triangle BCC'$, $CC' = 500\sin 30° = 250$ (m)

(b) $OO' = CC' = 250$ m

$\therefore$ In $\triangle OO'B$, Required $\angle = \angle OBO'$
$= \sin^{-1}\dfrac{250}{1000}$
$= 14.4775° = 14.5°$ (3 s.f.)

(c) *<u>Method 1 to find O'A</u>*

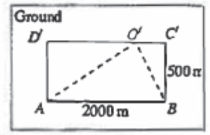
Denote the foot of perpendicular from D to the horizontal ground by D'.

In $\triangle OO'B$,

$O'B = \sqrt{1000^2 - 250^2}$
$= \sqrt{937500}$ (m)

In $\triangle BCC'$,

$BC' = 500\cos 30°$
$= 250\sqrt{3}$ (m)



$\therefore$ In $\triangle O'BC'$, $O'C' = \sqrt{O'B^2 - BC'^2} = \sqrt{750000}$ (m)

In $\triangle AO'D'$, $AD' = BC' = 250\sqrt{3}$ (m)

$D'O' = AB - O'C' = (2000 - \sqrt{750000})$ m

$AO' = \sqrt{AD'^2 + D'O'^2}$
$= \sqrt{4937500 - 4000\sqrt{750000}}$ m

<u>Method 2 to find O'A</u>

In $\triangle OO'B$, $O'B = \sqrt{1000^2 - 250^2} = \sqrt{937500}$ (m)

In $\triangle OBC$, $OC = 1000\sin 60° = 500\sqrt{3}$ (m)

$\Rightarrow O'C' = OC = 500\sqrt{3}$ m

$\therefore \cos\angle O'BA = \sin\angle O'BC' = \dfrac{O'C'}{O'B} = \dfrac{500\sqrt{3}}{\sqrt{937500}} \left(= \sqrt{\dfrac{4}{5}}\right)$

In $\triangle O'AB$,

$O'A = \sqrt{2000^2 + 937500 - 2\cdot 2000\cdot\sqrt{937500}\cos\angle O'BA}$
$= \sqrt{4937500 - 4000\sqrt{937500}\sqrt{\dfrac{4}{5}}}$
$= \sqrt{4937500 - 4000\sqrt{750000}}$

<u>Hence</u>, In $\triangle AO'T$,

$AT = \sqrt{AO'^2 + O'T^2}$
$= \sqrt{4937500 - 4000\sqrt{750000} + (250+50)^2}$
$= 1250.3593 = 1250$ (m, 3 s.f.)

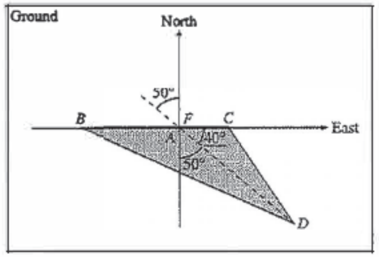
(d) Time for Rt I $= \dfrac{1000}{0.3} + 60 = 3393$ (s)

Time for Rt II $= \dfrac{2000}{0.8} + \dfrac{1250.3593}{3.2} = 2891$ (s) < 3393 (s)

$\therefore$ Route II takes a shorter time.

14B.16 HKCEE MA 1998 – I – 17

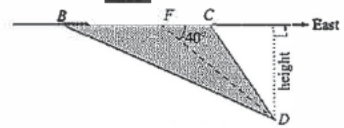
(The sun shining from N50°W is indicated in the diagram by $\angle CFD = 40°$.)



(a) In $\triangle ACF$, $AF = 4\sin 72° = 3.80423 = 3.80$ (m, 3 s.f.)

In $\triangle ADF$, $FD = \dfrac{AF}{\tan 35°} = 5.43300 = 5.43$ (m, 3 s.f.)

(b)



Height of $\triangle DBC$ with BC as base $= FD\sin 40°$
$= 3.49226$ m

$\therefore$ Area of shadow $= \dfrac{BC\cdot(FD\sin 40°)}{2} = 10.5$ (m², 3 s.f.)

(c) Area of shadow $= \dfrac{BC\cdot FD\sin(90° - x°)}{2} = \dfrac{BC\cdot FD}{2}\cos x°$

Since FD only depends on the $\angle$ of elevation (recall that $FD = \dfrac{AF}{\tan(\angle \text{ of elvn})}$),

$50 < x < 90 \Rightarrow \cos 50° > \cos x° > \cos 90°$

Hence the area becomes smaller.

14B.17 HKCEE MA 1999 – I – 18

$BD = DE = EF = FC = 6$ cm

(a) Method 1 to find AD

In $\triangle ABD$, $AD = \sqrt{24^2+6^2-2\cdot 24\cdot 6\cos 60°}$
$= \sqrt{468} = 21.6$ (cm, 3 s.f.)

Method 2 to find AD

In $\triangle ABE$ (before folding), $AE = \sqrt{24^2-12^2} = \sqrt{432}$ (cm)

In $\triangle ADE$, $AD = \sqrt{432+6^2} = \sqrt{468} = 21.6$ (cm, 3 s.f.)

Method 1

$\angle BDA = \cos^{-1}\dfrac{BD^2+AD^2-AB^2}{2\cdot BD\cdot AD} = 106.10211°$

$\therefore$ In $\triangle BCD$ (after folding),

$\angle BDC = 360° - 2(106.10211°) = 147.79577°$

$BC = \sqrt{6^2+6^2-2\cdot 6\cdot 6\cos 147.79577°}$
$= 11.52923 = 11.5$ (cm, 3 s.f.)

Method 2

Area of $\triangle ABD = \frac{1}{2}(6)(24)\sin 60° = 36\sqrt{3}$

Height of $\triangle ABD$ with base $AD = \dfrac{36\sqrt{3}\times 2}{AD} = \dfrac{72}{\sqrt{156}}$ (cm)

$\therefore BC = 2\times\dfrac{72}{\sqrt{156}} = 11.52923 = 11.5$ (cm, 3 s.f.)

Method 3

In $\triangle ABD$, $\dfrac{\sin\angle BAD}{6} = \dfrac{\sin 60°}{AD} \Rightarrow \angle BAD = 13.89789°$

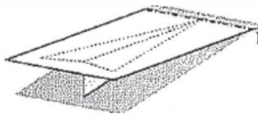
$\Rightarrow \angle BAC$ (after folding) $= 2\angle BAD = 27.79577°$

$\therefore$ In $\triangle ABC$ (after folding),

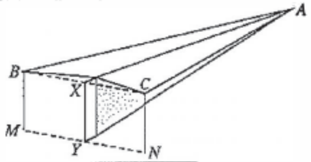
$BC = \sqrt{24^2+24^2-2\cdot 24\cdot 24\cos 27.79577°}$
$= 11.52923 = 11.5$ (cm, 3 s.f.)

(b) Required $\angle = \angle DAE = \tan^{-1}\dfrac{DE}{AE}$
$= \tan^{-1}\dfrac{6}{\sqrt{24^2-12^2}}$
$= 16.10211° = 16.1°$ (3 s.f.)

Note: Normally we need to look for the line of intersection of the 2 planes to locate the dihedral angle. In this problem, however, the planes intersect at only a point, and we could only assume that the aeroplane is positioned symmetrically, and that AE is perpendicular to the line of intersection.



(c) $BCNM$ is a rectangle. Suppose AD produced meets BC at X and AE produced meets MN at Y as shown.
Then $BM = XY = CN$.



In $\triangle ABX$, $AX = \sqrt{AB^2-\left(\dfrac{BC}{2}\right)^2} = 23.2974$ cm

$\therefore$ In $\triangle AXY$,
$CN = XY = AX\sin\angle DAE = 6.46$ cm (3 s.f.)

14B.18 HKCEE MA 2000 – I – 17

(a) (i) $AD = \dfrac{h}{\sin 30°} = 2h$ (m)

$BD = \dfrac{h+OA}{\sin 60°} = \dfrac{10+h}{\frac{\sqrt{3}}{2}} = \dfrac{2}{\sqrt{3}}(10+h)$ (m)

(ii) In $\triangle OAB$, $AB = \sqrt{10^2+10^2} = \sqrt{200}$ (m)

In $\triangle ABD$,

$AB^2 = AD^2 + BD^2 - 2AD\cdot BD\cos 30°$

$200 = 4h^2 + \frac{4}{3}(10+h)^2 - 2\cdot 2h\cdot\frac{2}{\sqrt{3}}(10+h)\cdot\frac{\sqrt{3}}{2}$

$200 = 4h^2 + \frac{4}{3}(100+20h+h^2) - 40h - 4h^2$

$0 = h^2 - 10h - 50$

$h = \dfrac{10\pm\sqrt{100+200}}{2}$

$= 5+5\sqrt{3}$ or $5-5\sqrt{3}$ (rejected)

$= 13.66025 = 13.7$ (3 s.f.)

(b) (Similar approach as (a))

$AE = \dfrac{h}{\sin 25°} = 32.32291$ (m)

$AC = \sqrt{10^2+10^2-2\cdot 10\cdot 10\cos 20°} = 3.47296$ (m)

$\therefore \dfrac{\sin\angle ACE}{AE} = \dfrac{\sin 5°}{AC}$

$\angle ACE = 54.2°$ or $126°$ (3 s.f.)

14B.19 HKCEE MA 2001 – I – 16

(a) Area of $BCDE = \triangle AFG - 2\triangle BCF - \triangle ABE$

$5\sqrt{3} = \dfrac{(6+x)^2\sin 60°}{2} - x^2\sin 60° - \dfrac{(6)^2\sin 60°}{2}$

$= \dfrac{\sqrt{3}}{4}[(6+x)^2 - 2x^2 - 6^2]$

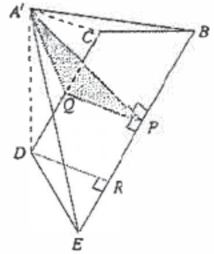
$20 = 12x - x^2$

$0 = x^2 - 12x + 20$

$x = 10$ (rejected) or 2

(b) (i) In $\triangle A'DE$, $A'D = \sqrt{6^2+2^2-2\cdot 6\cdot 2\cos 40°}$
$= 4.64919 = 4.65$ (3 s.f.)

(ii) Let P and Q be the mid-points of BE and CD respectively as shown. By symmetry, $A'P \perp BE$ and $QP \perp BE$. Hence, the required angle is $\angle A'PQ$.



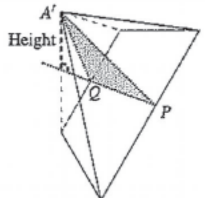
Let R be the foot of perpendicular from DE to BE.
Then $PQ = RD = x\sin 60° = \sqrt{3}$ (cm).
In $\triangle A'EP$, $A'P = A'E\sin 60° = 3\sqrt{3}$ (cm)
In $\triangle A'DQ$, $DQ = CD \div 2 = 2$ cm
$\Rightarrow A'Q = \sqrt{A'D^2 - DQ^2} = 4.19701$ cm

$\therefore$ In $\triangle A'PQ$, $\angle A'PQ = \cos^{-1}\dfrac{PQ^2+A'P^2-A'Q^2}{2\cdot PQ\cdot A'P}$

$= \cos^{-1}\dfrac{3+27-4.19701^2}{2\cdot\sqrt{3}\cdot 3\sqrt{3}}$

$= 46.52332° = 46.5°$ (3 s.f.)

(iii)



Height of pyramid $= A'P\cos\angle A'PQ = 3.77061$ cm

$\therefore$ Area $= \frac{1}{3}\times 5\sqrt{3}\times 3.77061 = 10.9$ (cm³)

14B.20 HKCEE MA 2002 – I – 14

(a) In $\triangle AST$, $AT = \dfrac{h}{\tan 20°}$; In $\triangle BST$, $BT = \dfrac{h}{\tan 15°}$.

In $\triangle ABT$, $\cos 30° = \dfrac{AT^2+AB^2-BT^2}{2AT\cdot BT}$

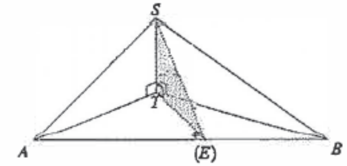
$\dfrac{900\sqrt{3}h}{\tan 20°} = \left(\dfrac{h}{\tan 20°}\right)^2 + 900^2 - \left(\dfrac{h}{\tan 15°}\right)^2$

$0 = 6.37957h^2 + 4282.8934h - 810000$

$h = 153.86177$ or -825 (rej)

$= 154$ (3 s.f.)

(b) (i) The shortest distance occurs when $TE \perp AB$ and thus $SE \perp AB$.



Method 1

In $\triangle AET$, $ET = AT\sin 30° = 211.3659$ (m)

In $\triangle EST$, $SE = \sqrt{ST^2+ET^2} = 261.436$
$= 261$ (m, 3 s.f.)

Method 2

In $\triangle AST$, $SA = \dfrac{h}{\sin 20°} = 449.86172$ m

In $\triangle BST$, $SB = \dfrac{h}{\sin 15°} = 594.47623$ m

In $\triangle ABS$, $\angle SAB = \cos^{-1}\dfrac{SA^2+AB^2-SB^2}{2SA\cdot AB}$
$= 35.5313°$

$\therefore$ In $\triangle SAE$, $SE = SA\sin\angle SAB = 261$ m (3 s.f.)

Method 3

In $\triangle AST$, $SA = \dfrac{h}{\sin 20°} = 449.86172$m

In $\triangle BST$, $SB = \dfrac{h}{\sin 15°} = 594.47623$ m

In $\triangle ABS$, let $s = \dfrac{SA+SB+900}{2} = 972.1690$ m.

$\Rightarrow$ Area $= \sqrt{s(s-SA)(s-SB)(s-900)}$
$= 117646.36$ (m²)

$\therefore SE = \dfrac{\text{Area}\times 2}{AB} = 261$ m (3 s.f.)

(ii) At E as in (b)(i), $\angle SET = \tan^{-1}\dfrac{ST}{ET} = 36.1°$.

$\therefore$ From A to B, θ increases from 20° at A to 36.1° at E as in (b)(i), and then decreases to 15° at B (since SE is the 'line of greatest slope').

14B.21 HKCEE MA 2003 – I – 14

(a) In $\triangle OAC$, $\angle OAC = \cos^{-1}\dfrac{3^2+6^2-4^2}{2\cdot 3\cdot 6}$
$= 36.33606° = 36.3°$ (3 s.f.)

(b) (i) In $\triangle OBC$, $BC = 4\tan 40° = 3.35640$ (m)

In $\triangle BCD$, $CD = \dfrac{BC}{\tan 30°} = 5.81345 = 5.81$ (m, 3 s.f.)

(ii) In $\triangle ACD$, $\angle CAD = \cos^{-1}\dfrac{6^2+8^2-CD^2}{2\cdot 6\cdot 8}$
$= 46.39976° = 46.4°$ (3 s.f.)

(iii) In $\triangle ACE$, $\dfrac{CE}{\sin\angle OAC} = \dfrac{6}{\sin\theta} \Rightarrow CE = \dfrac{3.55512}{\sin\theta}$

In $\triangle ADE$, $\dfrac{DE}{\sin(\angle CAD - \angle OAC)} = \dfrac{8}{\sin(180° - \theta)}$

$\Rightarrow DE = \dfrac{1.39794}{\sin\theta}$

$\therefore CE + ED = CD$

$\Rightarrow \dfrac{3.55512}{\sin\theta} + \dfrac{1.39794}{\sin\theta} = 5.81345$

$\dfrac{4.95306}{\sin\theta} = 5.81345$

$\theta = 58.4$ or $121.6°$ (rejected)

14B.22 HKCEE MA 2004 – I – 17

(a) (i) In $\Delta EFF'$, $FF' = 20\sin 30° = 10$ (m)

$$EF' = \frac{10}{\tan 30°} = 10\sqrt{3} \text{ (m)}$$

In $\Delta AFF'$, $AF' = \frac{10}{\tan 60°} = \frac{10}{\sqrt{3}}$ (m)

In $\Delta AEF'$, $AE = \sqrt{AF'^2 + EF'^2}$

$$= \sqrt{\frac{1000}{3}} = 18.3 \text{ (m, 3 s.f.)}$$

(ii) In $\Delta AFF'$, $AF = \frac{FF'}{\sin 60°} = \frac{20}{\sqrt{3}}$ m

In ΔAEF, $\angle AEF = \cos^{-1}\frac{AE^2 + EF^2 - AF^2}{2AE \cdot EF}$

$$= \cos^{-1}\frac{\frac{1000}{3} + 400 - \frac{400}{3}}{2 \cdot \sqrt{\frac{1000}{3}} \cdot 20}$$

$$= 34.75634° = 34.8° \text{ (3 s.f.)}$$

(b) In ΔBEF, $\angle BEF = 180° - 34.75634° = 145.24366°$

$\angle FBE = 34.75634° - 20° = 14.75634°$

$$\frac{20}{\sin 14.75634°} = \frac{BE}{\sin 20°} = \frac{BF}{\sin 145.24366°}$$

$\Rightarrow$ $BE = 26.85576$ m, $BF = 44.76385$ m

$\therefore$ Time red car takes $= BE \div 2 = 13.4$ s

Time yellow car takes $= BF \div 3 = 14.9$ s > 13.4 s

$\therefore$ NO.

14B.23 HKCEE MA 2005 – I – 14

(a) In ΔBCE, $BE = 120\sin 30° = 60$ (cm)

$CE = 120\cos 30° = 60\sqrt{3} = 104$ (cm, 3 s.f.)

(b) In ΔABC, $\angle C = 180° - 80° - 60° = 40°$

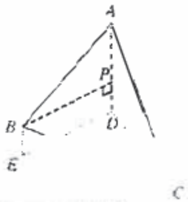
$$\therefore \frac{120}{\sin 60°} = \frac{AB}{\sin 40°} = \frac{AC}{\sin 80°}$$

$\Rightarrow$ $AB = 89.0673 = 89.1$ (cm, 3 s.f.)

$AC = 136.4590 = 136$ (cm, 3 s.f.)

(c) In ΔACD, $CD = \sqrt{AC^2 - AD^2} = 92.8496$ cm

In $ABED$, let P be on AD such that $BP \perp AD$.



$DE = PB = \sqrt{AB^2 - (AD - BE)^2} = 79.5800$ cm

$\therefore$ In ΔCDE, $\angle CDE = \cos^{-1}\frac{CD^2 + DE^2 - CE^2}{2CD \cdot DE}$

$= 73.674°$

Shortest distance from C to DE

$= CQ$ in the figure

$= CD\sin\angle CDE = 89.1$ cm (3 s.f.)

14B.24 HKCEE MA 2006 – I – 17

(a) In ΔABC, $\cos\angle BAC = \frac{40^2 + 90^2 - 60^2}{2 \cdot 40 \cdot 90} = \frac{61}{72}$

In ΔABD, $AD = 40\cos\angle BAD = \frac{305}{9}$ (cm)

(b) (i) (1) $DC = 90 - \frac{305}{9} = \frac{505}{9}$ (cm)

In ΔACD,

$$\left(\frac{505}{9}\right)^2 = \left(\frac{305}{9}\right)^2 + AC^2 - 2\left(\frac{305}{9}\right)(AC)\cos 62°$$

$0 = AC^2 - 31.81974AC - 2000$

$AC = 63.37695$ or -31.6 (rejected)

63.4 (cm, 3 s.f.)

(2) Let $s = \frac{40 + 60 + 63.37695}{2} = 81.6885$ (cm)

Area of $\Delta ABC = \sqrt{s(s-40)(s-60)(s-63.37695)}$

$= 1162.961 = 1160$ (cm², 3 s.f.)

(3) For tetrahedron $ABCD$, note that BD is its height when ΔACD is its base.

Area of $\Delta ACD = \frac{AD \cdot AC\sin 62°}{2} = 948.186$ cm²

$\therefore$ Required height $= \frac{3 \times \text{Volume of } ABCD}{\text{Area of } \Delta ABC}$

$$= \frac{\text{Area of } \Delta ACD \times BD}{\text{Area of } \Delta ABC}$$

$$= \frac{948.186 \times \sqrt{40^2 - \left(\frac{305}{9}\right)^2}}{1162.961}$$

$= 17.3$ (cm, 3 s.f.)

(ii) Volume of $ABCD = \frac{1}{3}$(Area of ΔACD)(BD)

$= \frac{1}{3}AD \cdot DC \cdot BD\sin\angle ADC$

$\therefore$ Volume of $ABCD \propto \sin\angle ADC$

Thus, when $\angle ADC$ increases from 30° to 150°, the volume increases from $\frac{1}{3}AD \cdot DC \cdot BD \cdot \frac{1}{2} = 6734$ cm³ to $\frac{1}{3}AD \cdot DC \cdot BD \cdot 1 = 13469$ cm³ when $\angle ADC = 90°$, and then decreases back to 6734 cm³.

14B.25 HKCEE MA 2007 – I – 16

(a) Let $s = \frac{5 + 6 + 9}{2} = 10$ (cm)

Area of $\Delta ABC = \sqrt{s(s-5)(s-6)(s-9)}$

$= \sqrt{200} = 14.1$ (cm², 3 s.f.)

$\therefore$ Volume of souvenir

$=$ Volume of prism $+$ Volume of pyramid

$= \sqrt{200} \times 20 + \frac{1}{3} \times \sqrt{200} \times (23 - 20)$

$= 21\sqrt{200} = 297$ (cm³, 3 s.f.)

(b) Let P be the point on CD such that plane PEF is parallel to plane ABC as shown. $DP = 3$ cm, $EF = AB = 9$ cm, $FP = BC = 5$ cm, $EP = AC = 6$ cm

In ΔDFP, $DF = \sqrt{3^2 + 5^2} = \sqrt{34}$ (cm)

In ΔDEP, $DE = \sqrt{3^2 + 6^2} = \sqrt{45}$ (cm)

$\therefore$ In ΔDEF, $\angle DFE = \cos^{-1}\frac{DF^2 + EF^2 - DE^2}{2DF \cdot EF}$

$= 48.16875° = 48.2°$ (3 s.f.)

$\therefore$ Required distance $= DF\sin\angle DFE = 4.3447$

$= 4.34$ cm (3 s.f.)

(c) Area of metal plane $= 4 \times 5 = 20$ (cm²)

Area of $\Delta DEF = \frac{4.3447 \times 9}{2} = 19.6 < 20$ (cm²)

$\therefore$ NO.

14B.26 HKCEE MA 2008 – I – 15

(a) In ΔBDH, $\angle BHD = 50° - 35° = 15°$

$\angle BDH = 180° - 50° = 130°$

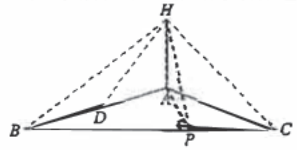
$\frac{BH}{\sin 130°} = \frac{50}{\sin 15°}$ $\Rightarrow$ $BH = 147.98842 = 148$ (m, 3 s.f.)

(b) (i) In ΔBCH, $\angle CBH = \cos^{-1}\frac{BC^2 + BH^2 - CH^2}{2BC \cdot BH}$

$= 37.81747° = 37.8°$ (3 s.f.)

(ii) In ΔABH, $AH = BH\sin 35° = 84.88267$ m

Let P be on BC such that $HP \perp BC$. Then $AP \perp BC$.



In ΔBHP, $HP = BH\sin\angle CBH = 90.73880$ m

In ΔAHP, Required $\angle = \angle HPA = \sin^{-1}\frac{AH}{HP}$

$= 69.3°$ (3 s.f.)

(iii) As the largest possible $\angle$ of elevation is $69.3° < 75°$, it is impossible.

14B.27 HKCEE MA 2009 – I – 17

(a) (i) In ΔBCD, $CD = \sqrt{6^2 + 25^2 - 2 \cdot 6 \cdot 25\cos 57°}$

$= 22.30714 = 22.3$ (cm, 3 s.f.)

(ii) In ΔABC, $\frac{\sin\angle BAC}{25} = \frac{\sin 57°}{28}$

$\angle BAC = 48.48766°$ or $131.5°$ (rej.)

$= 48.5°$ (3 s.f.)

(iii) In ΔABC, $\angle ACB = 180° - 48.48766° - 57°$

$= 74.51234°$

Area of $\Delta ABC = \frac{1}{2}AC \cdot BC\sin 74.51234°$

$= 337.29079 = 337$ (cm², 3 s.f.)

(iv) Since $\Delta CDE \perp \Delta ABE$, we have $CE \perp \Delta ABE$.

In ΔBCE, $CE = \sqrt{BC^2 - BE^2} = 7$ cm

In ΔACE, $AE = \sqrt{AC^2 - CE^2} = \sqrt{735}$ cm

In ΔABC, $\frac{AB}{\sin 74.51234°} = \frac{28}{57°}$

$AB = 32.17385$ (cm)

Let $s = \frac{AB + AE + BE}{2} = 41.64237$ cm

Area of $\Delta ABE = \sqrt{s(s-AB)(s-AE)(s-BE)}$

$= 317.9377$ (cm²)

$\therefore$ Required dist $= \frac{3 \times \text{Volume of } ABCE}{\text{Area of } \Delta ABC}$

$$= \frac{\text{Area of } \Delta ABE \times CE}{\text{Area of } \Delta ABC}$$

$$= \frac{337.29079 \times 7}{317.9377}$$

$= 6.59835 = 6.60$ (cm, 3 s.f.)

(b) *Method 1* – Finding the angles explicitly

In ΔCDE, $\angle CDE = \sin^{-1}\frac{CE}{CD} = 18.29°$

Denoting the distance from E to the ground (i.e. that found in (a)(iv)) by h cm and the angle between CE and the ground be θ,

$\theta = \sin^{-1}\frac{h}{DE} = 18.15° \neq 18.29°$

$\therefore$ NO.

Method 2 – Considering the projection of E

(If the student is correct, the projection of E on the ground would lie on CD.)

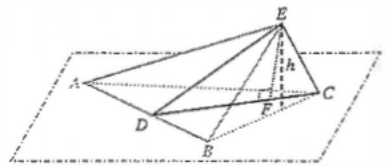
Let F be the projection of E onto CD.

$$EF = \frac{2 \times \text{Area of } \Delta CDE}{CD}$$

$$\frac{CE \times DE}{CD}$$

$$= \frac{7 \times \sqrt{22.30714^2 - 7^2}}{22.30714} = 6.65 \neq 6.60 \text{ (cm)}$$

Hence, the ~~projection~~ of E onto the ground is *not* on CD, and thus the angle between DE and the ground is *not* the angle between DE and DC, i.e. $\angle CDE$. The student is disagreed.



(Remark: This diagram is for illustration only. In the real situation, the "h" is *behind* ΔCDE, and would be too hard to visualise in the given diagram. But the key point is the same, that the dashed "h" is different from EF – in fact, h is shorter than EF since it is the *shortest* distance from E to the ground.)

14B.28 HKCEE MA 2010 – I – 15

(a) In ΔABC, $\angle CAB = 146° = 2 = 73°$,

$\angle ACB = 180° - 73° - 59° = 48°$

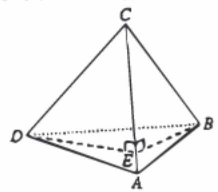
$\frac{AB}{\sin 48°} = \frac{24}{\sin 73°}$ $\Rightarrow$ $AB = 18.65041 = 18.7$ (cm, 3 s.f.)

(b) (i) In ΔABD, $BD = \sqrt{AB^2 + AB^2 - 2 \cdot AB \cdot AB\cos 92°}$

$= 26.83196 = 26.8$ (cm, 3 s.f.)

(ii) Let the diagonals of the kite intersect at E.

Then $DE \perp AC$ and $BE \perp AC$.



In ΔBCE, $BE = BC\sin\angle BCE = 17.83548$ (cm)

$DE = BE = 17.83548$ cm

In ΔBDE, Required $\angle = \angle BED$

$$= \cos^{-1}\frac{BE^2 + DE^2 - BD^2}{2BE \cdot DE}$$

$= 97.6°$ (3 s.f.)

(iii) In ΔBCD, $\angle BCD = \cos^{-1}\frac{BC^2 + CD^2 - BD^2}{2BC \cdot CD}$

$= 68.0°$

As P moves from A to E, $\angle BPD$ increases from 92° to 97.6°. As P moves from E to C, $\angle BPD$ decreases from 97.6° to 68.0°.

14B.29 HKCEE MA 2011 – I – 17

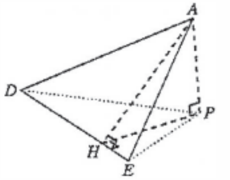
(a) (i) In ΔABC, $BC=\sqrt{20^2+30^2-2\cdot 20\cdot 30\cos 56^\circ}$
$25.07924=25.1$ (cm, 3 s.f.)

(ii) $\angle ACB=\cos^{-1}\dfrac{25.07924^2+30^2-20^2}{2\cdot 25.07924\cdot 30}$
$=41.38645^\circ=41.4^\circ$ (3 s.f.)

(iii) Required distance $=AC\sin\angle ACB-4$
$=15.83403=15.8$ (cm, 3.s.f.)

(iv) $\dfrac{DE}{BC}=\dfrac{\perp \text{ dist from } A \text{ to } DE}{\perp \text{ dist from } A \text{ to } BC}$
$DE=\dfrac{15.83403}{15.83403+4}\cdot 25.07924$
$=20.02142=20.0$ (cm, 3 s.f.)

(b) (i) Let H be the point on DE such that $AB\perp DE$ and $PH\perp DE$.



$AH=15.83403$ cm
$PH=\dfrac{2\times \text{Area of } \Delta PDE}{DE}=11.98716$ cm
$\therefore$ Required $\angle=\angle AHP$
$=\cos^{-1}\dfrac{PH}{AH}=40.8^\circ$ (3 s.f.)

(ii) Required distance $=AP$
$=\sqrt{AH^2-PH^2}=10.3$ cm (3 s.f.)

14B.30 HKCEE AM 1981 – II – 10

(a) In ΔBEF, $\angle EBF=60^\circ$
$FE^2=k^2+(rk)^2-2\cdot k\cdot rk\cos 60^\circ$
$=k^2+r^2k^2-rk^2=(1-r+r^2)k^2$
$FG^2=\left(\dfrac{1}{2}FH\right)^2=\dfrac{1}{4}(HA^2+FA^2)$
$=\dfrac{1}{4}[2\times(k-rk)^2]=\dfrac{(1-r)^2k^2}{2}$

(b) In ΔEFG, $EG=\sqrt{FE^2-FG^2}$
$=\sqrt{(1-r+r^2)k^2-\dfrac{1-2r+r^2}{2}k^2}$
$=\sqrt{\dfrac{1+r^2}{2}}k$
In ΔACD, $AC^2=AD^2+DC^2=2k^2$
$AN^2=\dfrac{1}{2^2}(2k^2)=\dfrac{1}{2}k^2$
In ΔAEN, $EN=\sqrt{AE^2-AN^2}=\sqrt{k^2-\dfrac{1}{2}k^2}=\dfrac{1}{\sqrt{2}}k$
$\therefore \sin\theta=\dfrac{EN}{EG}=\dfrac{\frac{1}{\sqrt{2}}k}{\frac{\sqrt{1+r^2}}{\sqrt{2}}k}=\dfrac{1}{\sqrt{1+r^2}}$

(c) The inclination is θ.
$0<r<1 \Rightarrow 1<1+r^2<2$
$\Rightarrow 1>\sin\theta>\dfrac{1}{\sqrt{2}} \Rightarrow 90^\circ>\theta>45^\circ$
Hence, when r varies from 0 to 1, the inclination decreases from 90° to 45°.

14B.31 HKCEE AM 1983 – II – 8

$\angle CBF=\angle CFB=\theta$

(a) In ΔBCF, $BF=2\times BC\cos\theta=2a\cos\theta$
In ΔFMN, $MF=x\cos\theta$
$\therefore$ In ΔABM, $AM=\sqrt{AB^2+BM^2}$
$=\sqrt{a^2(2a\cos\theta-x\cos\theta)^2}$
$=\sqrt{a^2+(2a-x)^2\cos^2\theta}$

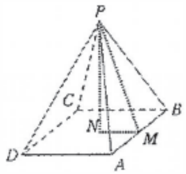
(b) In ΔABF, $AF=\sqrt{AB^2+BF^2}$
$=\sqrt{a^2+(2a\cos\theta)^2}=\sqrt{(1+4\cos^2\theta)a^2}$
$\therefore$ In ΔANF, $AN=\sqrt{AF^2-NF^2}$
$=\sqrt{(1+4\cos^2\theta)a^2-x^2}$

(c) In ΔFMN, $NM=x\sin\theta$
In ΔAMN, $AN^2=AM^2+NM^2$
$(1+4\cos^2\theta)a^2-x^2=a^2+(2a-x)^2\cos^2\theta+x^2\sin^2\theta$
$\cancel{(1+4\cos^2\theta)a^2}-x^2=\cancel{a^2+4a^2\cos^2\theta}-4ax\cos^2\theta$
$+x^2\cos^2\theta+x^2\sin^2\theta$
$4ax\cos^2\theta=x^2(\cos^2\theta+\sin^2\theta)+x^2$
$4ax\cos^2\theta=2x^2 \Rightarrow x=2a\cos^2\theta$

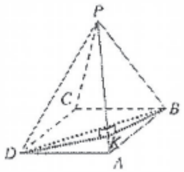
(d) $\dfrac{a}{2}=2a\cos^2\theta \Rightarrow \cos\theta=\dfrac{1}{2} \Rightarrow \theta=60^\circ$
$NM=x\sin\theta=\dfrac{\sqrt{3}}{4}a$
$AM=\sqrt{a^2+(2a-x)^2\cos^2\theta}=\sqrt{a^2+\dfrac{9a^2}{4}\cdot\dfrac{1}{4}}=\dfrac{5}{4}a$
$\therefore$ Inclination $=\angle NAM=\tan^{-1}\dfrac{NM}{AM}=19^\circ$ (nrst deg)

14B.32 HKCEE AM 1991 – II – 6

(a) Let M and N be the mid-points of AB and CD respectively.
Then $PM\perp AB$ and $PN\perp CD$.
In ΔAPM, $PM=AM\tan 60^\circ=2\sqrt{3}$ cm
In ΔMNP,
Required $\angle=\angle PMN$
$=\cos^{-1}\dfrac{MN}{PM}$
$=\cos^{-1}\dfrac{2}{2\sqrt{3}}$
$=54.7^\circ$
(nearest 0.1°)

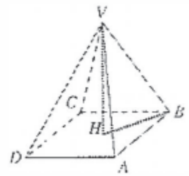


(b) Let K be on PA such that $DK\perp PA$. Then $BK\perp PA$.
In ΔABD,
$BD=\sqrt{4^2+4^2}$
$=\sqrt{32}$ (cm)
In ΔADK,
$DK=4\sin 60^\circ$
$=2\sqrt{3}$ (cm)
Similarly, $BK=2\sqrt{3}$ cm
In ΔBDK,
Required $\angle=\angle BKD$
$=\cos^{-1}\dfrac{(2\sqrt{3})^2+(2\sqrt{3})^2-32}{2\cdot 2\sqrt{3}\cdot 2\sqrt{3}}$
$=109.5^\circ$ (nearest 0.1°)

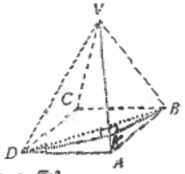


14B.33 HKCEE AM 1992 – II – 7

(a) Let H be the projection of V onto $ABCD$.
$BH=\dfrac{1}{2}BD$
$=\dfrac{1}{2}\sqrt{6^2+6^2}$
$=3\sqrt{2}$ (cm)
Required $\angle=\angle VBH$
$=\cos^{-1}\dfrac{3\sqrt{2}}{9}$
$=61.9^\circ$
(nearest 0.1°)



(b) Let K be on VA such that $BK\perp VA$. Then $DK\perp VA$.
$\angle VAB=\cos^{-1}\dfrac{\frac{1}{2}AB}{VA}$
$=70.5288^\circ$
$DK=BK=AB\sin\angle VAB$
$=5.6569$ cm
Required $\angle$
$=\angle BKD$
$=\cos^{-1}\dfrac{5.6569^2+5.6569^2-(2\cdot 3\sqrt{2})^2}{2\cdot 5.6569\cdot 5.6569}$
$=97.2^\circ$ (nearest 0.1°)



14B.34 HKCEE AM 1993 – II – 7

(a) $\angle VBA=\cos^{-1}\dfrac{\frac{1}{2}AB}{VB}=75.52249^\circ=75.5^\circ$ (3 s.f.)
$AD=AB\sin\angle VBA=11.61895=11.6$ (cm, 3 s.f.)

(b) $DC=AD=11.61895$ cm
Required $\angle=\angle ADC$
$=\cos^{-1}\dfrac{AD^2+DC^2-AC^2}{2AD\cdot DC}=62.2^\circ$ (3 s.f.)

14B.35 HKCEE AM 1994 – II – 12

(a) (i) In ΔABC, $\dfrac{AC}{\sin\beta}=\dfrac{100}{\sin(180^\circ-\alpha-\beta)}$
$AC=\dfrac{100\sin\beta}{\sin(\alpha+\beta)}$ (km)

(ii) In ΔACP, $PC=AC\tan\theta=\dfrac{100\sin\beta\tan\theta}{\sin(\alpha+\beta)}$ km

(b) (i) $AC=\dfrac{100\sin 30^\circ}{\sin(45^\circ+30^\circ)}$
$=51.76381=51.76$ (km, 2 d.p.)
$AC'=\dfrac{100\sin 43^\circ}{\sin(37^\circ+43^\circ)}$
$=69.25193=69.25$ (km, 2 d.p.)

(ii) $\angle CAC'=45^\circ-37^\circ=8^\circ$
In $\Delta ACC'$, $CC'=\sqrt{AC^2+AC'^2-2AC\cdot AC'\cos 8^\circ}$
$=19.38059=19.38$ (km, 2 d.p.)

(iii) $PC=\dfrac{100\sin 30^\circ\tan 20^\circ}{\sin(45^\circ+30^\circ)}=18.84049$ (km)
$P'C'=\dfrac{100\sin 43^\circ\tan 17^\circ}{\sin(37^\circ+43^\circ)}=21.17244$ (km)
$\therefore$ Increase in height $=P'C'-PC$
$=2.33195=2.33$ (km, 2 d.p.)

(iv) Required $\angle=\tan^{-1}\dfrac{2.33195}{19.38059}=6.86^\circ$ (2 d.p.)

14B.36 HKCEE AM 1995 – II – 7

(a) $\angle PQU=(180^\circ-42^\circ)\div 2=69^\circ$
$PU=10\sin 69^\circ=9.33580=9.34$ (cm, 3 s.f.)
$\angle PQR=180^\circ(5-3)\div 5=108^\circ$
$PR=\sqrt{10^2+10^2-2\cdot 10\cdot 10\cos 108^\circ}$
$=16.18034=16.2$ (cm, 3 s.f.)

(b) Required $\angle=\angle PUR$
$=\cos^{-1}\dfrac{PU^2+RU^2-PR^2}{2PU\cdot RU}=120^\circ$ (3 s.f.)

14B.37 HKCEE AM 1996 – II – 12

(a) $AD=AC\sin 30^\circ=1$, $DC=2\cos 30^\circ=\sqrt{3}$
$AB=\dfrac{AD}{\sin 45^\circ}=\sqrt{2}$, $BD=\dfrac{AD}{\tan 45^\circ}=1$

(b) (i) E is the mid-pt of AB (since ΔABD is right-angled isosceles).
$\Rightarrow AE=DE=BE=\dfrac{\sqrt{2}}{2}$
$\because \theta=\angle DCE$
$\Rightarrow \sin\theta=\dfrac{DE}{DC}=\dfrac{\frac{\sqrt{2}}{2}}{\sqrt{3}}=\dfrac{\sqrt{6}}{6}$

(ii) $CE=\sqrt{CD^2-DE^2}=\sqrt{\dfrac{5}{2}}$
Hence, in ΔACE,
$\angle EAC=\cos^{-1}\dfrac{AE^2+AC^2-CE^2}{2AE\cdot AC}=45^\circ$

(ii) In ΔABC, $BC=\sqrt{2^2+2-2\cdot 2\cdot\sqrt{2}\cos 45^\circ}=\sqrt{2}$
In ΔBCD, since $\angle ADC=\angle ADB=90^\circ$,
Required $\angle=\angle CDB=\cos^{-1}\dfrac{3+1-2}{2(\sqrt{3})(1)}$
$=55^\circ$ (nearest degree)

14B.38 HKCEE AM 1997 – II – 12

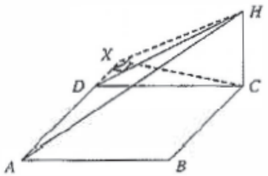
(a) (i) In ΔABC, $AC=\sqrt{AB^2+BC^2-2AB\cdot BC\cos\angle ABC}$
$=\sqrt{(3a)^2+(2a)^2-2(3a)(2a)\cos 120^\circ}$
$=\sqrt{9a^2+4a^2+6a^2}=\sqrt{19}a$

(ii) Required $\angle=\angle HMC$
$=\tan^{-1}\dfrac{HC}{MC}=25^\circ$ (nearest deg)

(b) (i) In ΔABD, $BD=\sqrt{(3a)^2+(2a)^2-2(3a)(2a)\cos 60^\circ}$
$=\sqrt{7}a$
Area of $\Delta BCD=\dfrac{1}{2}(3a)(2a)\sin 60^\circ=\dfrac{3\sqrt{3}}{2}a^2$
$\therefore CE=\dfrac{2\cdot\text{Area of }\Delta BCD}{BD}=\dfrac{3\sqrt{3}a^2}{\sqrt{7}a}=\dfrac{3\sqrt{21}}{7}a$

(ii) In ΔBCE, $BE^2=BC^2-CE^2$
In ΔBCH, $BH^2=BC^2+HC^2$
In ΔCEH, $HE^2=HC^2+CE^2$
$\therefore HE^2+BE^2=(HC^2+CE^2)+(BC^2-CE^2)$
$=HC^2+BC^2=BH^2$
$\therefore HE\perp BD$
Hence, required $\angle=\angle HEC=\tan^{-1}\dfrac{HC}{CE}$
$=27^\circ$ (nearest deg)

(c) X is on AD extended such that $CX \perp AX$.



In $\triangle ACD$,

$\cos\angle DAC = \dfrac{(2a)^2+(\sqrt{19}a)^2-(3a)^2}{2(2a)(\sqrt{19}a)} = \dfrac{7}{2\sqrt{19}}$

$\therefore\ AX = AC\cos\angle XAC = \sqrt{19}a\cdot\dfrac{7}{2\sqrt{19}} = \dfrac{7}{2}a$

14B.39 <u>HKCEE AM 1998 – II – 13</u>

(a) (i) $CM = \dfrac{1}{2}AC = \dfrac{\sqrt{2}}{2}a$

(ii) Required $\angle = \angle CMH = \tan^{-1}\dfrac{HC}{CM} = 55^\circ$ (nrst deg)

(b) (i) $FH = \sqrt{2}a,\quad FV = 2a$

In $\triangle FVH$, $HV = \sqrt{(\sqrt{2}a)^2+(2a)^2} = \sqrt{6}a$

$\therefore\ \sin\angle FVH = \dfrac{FH}{HV} = \dfrac{\sqrt{2}a}{\sqrt{6}a} = \dfrac{\sqrt{3}}{3}$

Let the projection of F on $BVDH$ be P.

By symmetry, F lies on HV as shown.

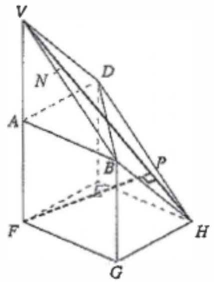
In $\triangle FVP$,

Required distance

$= FP$

$= FV\sin\angle FVP$

$= \dfrac{2\sqrt{3}}{3}a$



(ii) (1) Since $VB = BD = DV = \sqrt{2}a$, $\angle DVB = 60^\circ$.

$\Rightarrow\ DN = \sqrt{2}a\sin 60^\circ = \dfrac{\sqrt{6}}{2}a$

(2) <u>*Method 1*</u>

$AN = AB\sin 45^\circ = \dfrac{\sqrt{2}}{2}a$

$\therefore$ Required $\angle = \angle AND$

$= \cos^{-1}\dfrac{AN^2+DN^2-AD^2}{2AN\cdot DN}$

$= \cos^{-1}\dfrac{\frac{1}{2}a^2+\frac{3}{2}a^2-a^2}{2\cdot\frac{\sqrt{2}}{2}a\cdot\frac{\sqrt{6}}{2}a}$

$= 55^\circ$ (nearest degree)

<u>*Method 2*</u>

In fact, since AD is perpendicular to plane BVA, it is perpendicular to any line on plane BVA.

$\therefore$ Required $\angle = \angle AND$

$= \sin^{-1}\dfrac{AD}{DN} = 55^\circ$ (nearest deg)

(iii) BHD and BVD is the same plane, and $ABGF$ and BVA is also the same plane. Hence the required angle is the same one as in (b)(ii)(2). $\therefore$ YES.

14B.40 <u>HKCEE AM 1999 – II – 11</u>

(a) In $\triangle ABD$, $\dfrac{AD}{\sin(180^\circ-\alpha)} = \dfrac{\ell}{\sin(\alpha-10^\circ)}$

$AD = \dfrac{\ell\sin\alpha}{\sin(\alpha-10^\circ)}$ (m)

(i) In $\triangle ACD$, $CD = AD\sin 10^\circ = \dfrac{\ell\sin\alpha\sin 10^\circ}{\sin(\alpha-10^\circ)}$ m

(ii) In $\triangle ADH$, $\dfrac{DH}{\sin(\beta-10^\circ)} = \dfrac{AD}{\sin(\alpha-\beta)}$

$DH = \dfrac{\ell\sin\alpha\sin(\beta-10^\circ)}{\sin(\alpha-10^\circ)\sin(\alpha-\beta)}$ (m)

$\therefore$ In $\triangle DGH$, $h = DH\sin\alpha$

$= \dfrac{\ell\sin^2\alpha\sin(\beta-10^\circ)}{\sin(\alpha-10^\circ)\sin(\alpha-\beta)}$

(b) (i) (1) $HG = \dfrac{97\sin^2 15^\circ\sin 0.2^\circ}{\sin 5^\circ\sin 4.8^\circ}$

$= 3.11003 = 3.1$ (m, 2 s.f.)

(2) Height of tower $= \dfrac{97\sin 15^\circ}{\sin 5^\circ}$

$= 288.0527 = 290$ (m, 2 s.f.)

Radius of tower $= DH\cos\alpha$

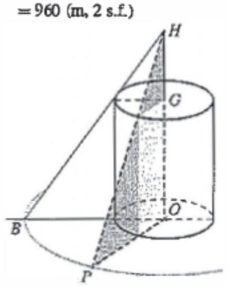
$= \dfrac{97\sin 15^\circ\sin 0.2^\circ\cos 15^\circ}{\sin 5^\circ\sin 4.8^\circ}$

$= 11.60678 = 12$ (m, 2 s.f.)

(ii) (1) $PO = BO = \dfrac{h+CD}{\tan\alpha}$

$= 963.476$

$= 960$ (m, 2 s.f.)



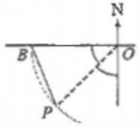
(2) $\angle OBP = (180^\circ-45^\circ)\div 2$

$= 67.5$

$\therefore$ Bearing of B from P

N$(90^\circ-67.5^\circ)$W

$=$ N22.5°W



14B.41 <u>HKCEE AM 2001 – 15</u>

(a) (i) $PR^2 = x^2+z^2,\ PQ^2 = x^2+y^2,\ QR^2 = y^2+z^2$

$\cos\angle PRQ = \dfrac{PR^2+QR^2-PQ^2}{2PR\cdot PQ}$

$= \dfrac{(x^2+z^2)+(y^2+z^2)-(x^2+y^2)}{2\sqrt{x^2+z^2}\sqrt{y^2+z^2}}$

$= \dfrac{z^2}{\sqrt{(x^2+z^2)(y^2+z^2)}}$

(ii) $S_1 = \dfrac{xz}{2},\ S_2 = \dfrac{xy}{2},\ S_3 = \dfrac{yz}{2}$

$\sin\angle PRQ = \sqrt{1-\cos^2\angle PRQ}$

$= \sqrt{1-\dfrac{z^4}{(x^2+z^2)(y^2+z^2)}}$

$= \sqrt{\dfrac{x^2y^2+x^2z^2+y^2z^2+z^4-z^4}{(x^2+z^2)(y^2+z^2)}}$

$= \sqrt{\dfrac{x^2y^2+x^2z^2+y^2z^2}{(x^2+z^2)(y^2+z^2)}}$

$\therefore\ S_4 = \dfrac{1}{2}PR\cdot QR\sin\angle PRQ$

$= \dfrac{1}{2}\sqrt{x^2+z^2}\sqrt{y^2+z^2}\sqrt{\dfrac{x^2y^2+x^2z^2+y^2z^2}{(x^2+z^2)(y^2+z^2)}}$

$= \dfrac{1}{2}\sqrt{x^2y^2+x^2z^2+y^2z^2}$

$\Rightarrow\ {S_4}^2 = \dfrac{x^2y^2}{4}+\dfrac{x^2z^2}{4}+\dfrac{y^2z^2}{4} = {S_2}^2+{S_1}^2+{S_3}^2$

(b) (i) Volume $= \dfrac{1}{3}\times\left(\dfrac{4\times 3}{2}\right)\times 2 = 4$

(ii) Height of pyramid with $\triangle GAC$ as base

$= \dfrac{3\times\text{Volume}}{\text{Area of }\triangle GAC}$

$= \dfrac{3\times 4}{\sqrt{(\frac{4\times 2}{2})^2+(\frac{4\times 3}{2})^2+(\frac{2\times 3}{2})^2}} = \dfrac{12}{\sqrt{61}}$

$\therefore$ Required $\angle = \sin^{-1}\dfrac{\frac{12}{\sqrt{61}}}{AB} = 23^\circ$ (nearest degree)

14B.42 <u>HKCEE AM 2002 – 17</u>

(a) <u>*Method 1 to find CF*</u>

In $\triangle ACD$, $\cos\angle ADC = \dfrac{\frac{1}{2}CD}{AD} = \dfrac{3}{5}$ (since $\triangle ACD$ is isos.)

$\Rightarrow\ \sin\angle ADC = \dfrac{4}{5}$

$\therefore\ CF = CD\sin ADC = 24$

<u>*Method 2 to find CF*</u>

Area of $\triangle ACD = \dfrac{1}{2}\times 30\sqrt{25^2-(30\div 2)^2} = 300$

$\therefore\ CF = \dfrac{2\times\text{Area of }\triangle ACD}{AD} = \dfrac{2\times 300}{25} = 24$

<u>*Then...*</u>

In $\triangle ACF$, $AF = \sqrt{AC^2-CF^2} = 7$

In $\triangle ABD$, $\cos\angle BAD = \dfrac{28^2+25^2-40^2}{2\cdot 28\cdot 25} = \dfrac{-191}{1400}$

$\therefore$ In $\triangle ABF$, $BF = \sqrt{28^2+7^2-2\cdot 28\cdot 7\cos\angle BAD}$

$= 886.48 = 29.77381$

$\therefore$ In $\triangle BCF$, $\angle BFC = \cos^{-1}\dfrac{886.48+24^2-40^2}{2\cdot\sqrt{886.48}\cdot 24}$

$= 96^\circ$ (nearest degree)

(b) <u>*Method 1*</u>

$AB^2 = 784$

$AF^2+BF^2 = 935.48 \neq AB^2$

$\therefore\ \angle AFB \neq 90^\circ$

<u>*Method 2*</u>

$\angle AFB = \cos^{-1}\dfrac{AF^2+BF^2-AB^2}{2AF\cdot BF} = 69^\circ \neq 90^\circ$

<u>*Method 3*</u>

$\cos\angle BAD = \dfrac{-191}{4000} < 0$

$\Rightarrow\ \angle BAF > 90^\circ\ \Rightarrow\ \angle AFB < 90^\circ$

<u>*Hence*</u>

BF is not perpendicular to AD.

Thus, $\angle BFC$ is not the dihedral angle.

14B.43 <u>HKCEE AM 2003 – 18</u>

(a) Let S be on PQ such that $RS \perp PQ$ and $OS \perp PQ$.

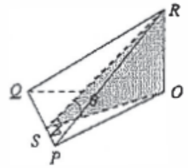
Then $\cos\theta = \dfrac{OS}{RS}$.

$\therefore\ \dfrac{\text{Area of }\triangle OPQ}{\text{Area of }\triangle RPQ}$

$= \dfrac{\frac{1}{2}\cdot OS\cdot PQ}{\frac{1}{2}\cdot RS\cdot PQ}$

$= \dfrac{OS}{RS}$

$= \cos\theta$



(b) (i) Let D be on AB such that $CD \perp AB$ and $ED \perp AB$.

$CD = \dfrac{2\times 12}{6} = 4$ (m)

$\angle$ between board and shadow $= \sin^{-1}\dfrac{2}{4} = 30^\circ$

By (a)(i), Area of shadow $=$ (Area of board) $\cos 30^\circ$

$= 12\cos 30^\circ = 6\sqrt{3}$ (m²)

(ii) $\because$ AC is the longest side

$\therefore$ Height of $\triangle ABC$ from B to AC is the shortest.

Area of shadow $= 12\cos\phi$, where ϕ is the angle of inclination of the board.

Since $\sin\phi = \dfrac{2}{\text{Height of }\triangle ABC}$, ϕ is the smallest (i.e. $\cos\phi$ largest) when B is fastened to the pole.

$\therefore$ B fastened will give the largest shadow.

14B.44 <u>HKCEE AM 2004 – 11</u>

(a) In $\triangle OBC$, $OC = \sqrt{5^2+12^2} = 13$

In $\triangle OAC$, $AC = \sqrt{3^2+13^2-2\cdot 3\cdot 13\cos 12^\circ}$

$= \sqrt{217}$ ($= 14.7$, 3 s.f.)

(b) In $\triangle OAB$, $AB = \sqrt{5^2-3^2} = 4$

In $\triangle ABC$,

<u>*Method 1*</u> $AC^2 = 217$

$AB^2+BC^2 = 4^2+12^2 = 160 \neq AC^2$

$\therefore\ \angle ABC \neq 90^\circ$

<u>*Method 2*</u> $\angle ABC = \cos^{-1}\dfrac{4^2+12^2-217}{2\cdot 4\cdot 12} = 126^\circ \neq 90^\circ$

<u>*Hence*</u>, the student is not correct.

14B.45 HKCEE AM 2006 – 17

(a) (i) Let $s=\frac{5+6+7}{2}=9$

Area $=\sqrt{s(s-5)(s-6)(s-7)}$
$=\sqrt{216}$ $(=14.7,\ 3\text{ s.f.})$

(ii) Area of $\Delta ABC=\Delta AOB+\Delta BOC+\Delta COA$

$$\sqrt{216}=\frac{6r}{2}+\frac{7r}{2}+\frac{5r}{2}$$
$$r=\frac{\sqrt{216}}{9}\left(=\frac{2\sqrt{6}}{3}\right)$$

(b) (i) $VO=r\tan 60°=2\sqrt{2}$

$\therefore$ Volume of $VABC=\frac{1}{3}\times\sqrt{216}\times 2\sqrt{2}$
$=8\sqrt{3}$ $(=13.9,\ 3\text{ s.f.})$

(ii) Height of ΔVBC from V to $BC=\sqrt{VO^2+r^2}=\sqrt{\frac{32}{3}}$

$\therefore$ Area of $\Delta VBC=\frac{1}{2}\times\sqrt{\frac{32}{3}}\times 7=\frac{14\sqrt{6}}{3}$
$(=11.4,\ 3\text{ s.f.})$

(iii) Height of pyramid from A to ΔVBC

$$=\frac{3\times\text{Volume of pyramid}}{\text{Area of }\Delta VBC}=\frac{3\times 8\sqrt{3}}{\frac{14\sqrt{6}}{3}}=\frac{18\sqrt{2}}{7}$$

$\therefore$ Required $\angle=\sin^{-1}\frac{\frac{18\sqrt{2}}{7}}{6}=37°$ (nearest degree)

14B.46 HKCEE AM 2008 – 16

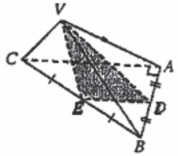
(a) Since VA is not perpendicular to AB, $\angle VAC$ is not the $\angle$ between the planes.

(b) (i) *Criterion 1: $VD\perp AB$*

$\because$ ΔVAB is equilateral and $BD=DA$
$\therefore$ $VD\perp AB$ (property of isos. Δ)

Criterion 2: $ED\perp AB$

$\because$ $BD=DA$ and $BE=EC$
$\therefore$ $DE//AC$ (mid-pt thm)
$\Rightarrow$ $\angle EDB=\angle CAB=90°$
(corr. $\angle$s, $DE//AC$)



Hence, the $\angle$ between VAB and ABC is $\angle VDE$.

(ii) $VA=VB=VC=AC=2$ cm

$ED=\frac{1}{2}AC=1$ cm
$BC=\sqrt{2^2+2^2}=\sqrt{8}$ (cm)
$VE=\sqrt{VB^2-(BC\div 2)^2}=\sqrt{2}$ cm
$VD=\sqrt{VA^2-(AB\div 2)^2}=\sqrt{3}$ cm
$\because$ $VD^2=3$
$VE^2+ED^2=2+1=3=VD^2$
$\therefore$ $\angle VED=90°$

(c) Area of $\Delta ABC=\frac{1}{2}\times 2\times 2=2$ (cm^2)

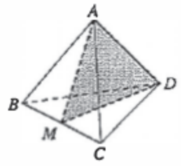
Volume of pyramid $=\frac{1}{3}\times$ Area of $\Delta ABC\times VE$
$=\frac{2\sqrt{2}}{3}$ cm^3

Area of $\Delta VAB=\frac{1}{2}\times 2\times 2\sin 60°=\sqrt{3}$ (cm^2)

$\therefore$ Required dist. $=\frac{3\times\text{Volume of pyramid}}{\text{Area of }\Delta VAB}$

$$=\frac{3\times\frac{2\sqrt{2}}{3}}{\sqrt{3}}=\frac{2\sqrt{6}}{3}\ (=1.63\text{ cm},\ 3\text{ s.f.})$$

14B.47 HKCEE AM 2009 – 12



Let M be the mid-point of BC.
$AM=AC\sin\angle ACB=\sqrt{3}$
$DM=\sqrt{3}$
$\therefore$ Required $\angle=\angle AMD$

$$=\cos^{-1}\frac{3+3-2^2}{2\cdot\sqrt{3}\cdot\sqrt{3}}$$
$=71°$ (nearest deg)

14B.48 HKCEE AM 2009 – 18

(a) In ΔDHK, $DK=\frac{h}{\tan 30°}=\sqrt{3}h$ (m)

(b) In ΔAHK, $AK=\frac{h}{\tan 45°}=h$ (m)

From the time taken, $BD=2AB$.
Since B is the closest point on AD to K, $KB\perp AD$.
In ΔABK, $BK^2=AK^2-AB^2=h^2-AB^2$
In ΔBDK, $BK^2=DK^2-BD^2=3h^2-4AB^2$

$\therefore$ $h^2-AB^2=3h^2-4AB^2$
$3AB^2=2h^2$
$AB=\sqrt{\frac{2}{3}}h$ (m)

(c) $BC=\frac{1}{2}AB=\frac{1}{\sqrt{6}}h$ m

$BK=\sqrt{h^2-AB^2}=\frac{1}{\sqrt{3}}h$ m

In ΔBCK, $CK=\sqrt{BK^2+BC^2}=\frac{1}{\sqrt{2}}h$ m

$\therefore$ Required $\angle=\angle HCK=\tan^{-1}\frac{HK}{CK}=55°$ (nearest deg)

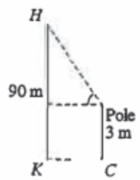
(d) (i) $AD=3AB=\sqrt{6}h$ m (30 mins)

$$AE+ED=4AB=\frac{4\sqrt{6}}{3}h\text{ m}\quad(40\text{ mins})$$
$$(AE+ED)^2=\frac{32}{3}h^2$$
$$AE^2+ED^2+2AE\cdot ED=\frac{32}{3}h^2$$
$$AD^2+2(9450\times 2)=\frac{32}{3}h^2$$
$$6h^2+37800=\frac{32}{3}h^2$$
$$h^2=8100\ \Rightarrow\ h=90$$

(ii) The pole is to be located at the circumcentre of ΔADE.

Since it is a right-angled triangle, the circumcentre is the mid-point of its hypotenuse.
i.e. The pole is located at C.

$\therefore$ Required $\angle$ of elevation
$=\tan^{-1}\frac{HK-3}{CK}$
$=\tan^{-1}\frac{87}{\frac{1}{\sqrt{2}}(90)}$
$=54°$ (nearest degree)



14B.49 HKCEE AM 2010 – 17

(a) In ΔABD, $AD=11\cos 60°=5.5$ (cm)

In ΔAED, $AE=\frac{AD}{\cos 30°}=\frac{11}{\sqrt{3}}=6.35$ (cm, 3 s.f.)

$\therefore$ Area of ΔABE
$=\frac{1}{2}\cdot 11\cdot\frac{11}{\sqrt{3}}\sin 30°$
$=\frac{121}{4\sqrt{3}}$
$=17.5$ (cm^2, 3 s.f.)

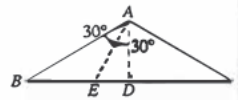


Figure (1)

(b) $\angle FAC=(120°-30°)\div 2)$
$=45°$
$\angle ACF=(180°-120°)\div 2$
$=30°$

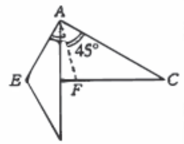
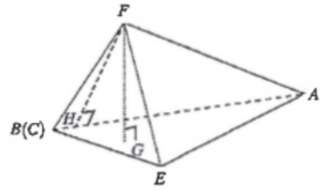


Figure (2)

In ΔACF, $\frac{AF}{\sin 30°}=\frac{11}{\sin(180°-45°-30°)}$
$AF=5.69402=5.69$ (cm, 3 s.f.)

(c)



(i) Let G be the projection of F onto ΔABE.

$FG=\frac{3\times\text{Volume of tetrahedron}}{\text{Area of }\Delta ABE}=3.87899$ cm

$\therefore$ Required $\angle=\angle FAG$
$=\sin^{-1}\frac{3.87899}{AF}$
$=42.94060°=42.9°$ (3 s.f.)

(ii) Let H be the projection of F onto AB. Then, since GH is the projection of FH onto ΔABE, the required angle is $\angle FHG$.

In ΔAFH, $FH=AF\sin\angle FAC=4.02628$ cm

$\therefore$ Required $\angle=\angle FHG=\sin^{-1}\frac{FG}{FH}=74.5°$ (3 s.f.)

14B.50 HKCEE AM 2011 – 13

(a) (i) In ΔADE, $AE=3\sin\theta$
In ΔBCE, $BE=4\sin\theta$
$\therefore$ In ΔABE, $AB=\sqrt{AE^2+BE^2}=5\sin\theta$

(ii) $CD=\sqrt{DE^2+CE^2}=\sqrt{(3\cos\theta)^2+(4\cos\theta)^2}$
$=5\cos\theta$

(b) (i) In ΔABC, $AC^2=AB^2+BC^2-2AB\cdot BC\cos\alpha$
$=25\sin^2\theta+16-40\sin\theta\cos\alpha$

In ΔAEC, $AC^2=AE^2+EC^2-2AE\cdot EC\cos\alpha$
$=9\sin^2\theta+16\cos^2\theta$
$-24\sin\theta\cos\theta\cos\alpha$

$\therefore$ $25\sin^2\theta+16-40\sin\theta\cos\alpha$
$=9\sin^2\theta+16\cos^2\theta-24\sin\theta\cos\theta\cos\alpha$
$16\sin^2\theta+16(1-\cos^2\theta)=8\sin\theta(5-3\cos\theta)\cos\alpha$
$32\sin^2\theta=8\sin\theta(5-3\cos\theta)\cos\alpha$
$\cos\alpha=\frac{4\sin\theta}{5-3\cos\theta}$

(ii) $\because$ $\sin\theta>0$ and $5-3\cos\theta\geq 2>0$
$\therefore$ $\cos\alpha=\frac{4\sin\theta}{5-3\cos\theta}>0$ $\Rightarrow$ α is acute.

(iii) From the given info, since the distance between A and Π_2 is the same,

$AB=AD$ $\Rightarrow$ $5\sin\theta=3$ $\Rightarrow$ $\sin\theta=\frac{3}{5}$
$\Rightarrow$ $\cos\theta=\frac{4}{5}$

$\Rightarrow$ $\cos\alpha=\frac{4(\frac{3}{5})}{5-3(\frac{4}{5})}=\frac{12}{13}$

$AC=\sqrt{25\sin^2\theta+16-40\sin\theta\cos\alpha}$
$=\sqrt{\frac{37}{13}}<3=AB$

Hence, the angle between AC and Π_2 is greater than the angle between AB and Π_2.

14B.51 HKDSE MA SP – I – 18

(a) In ΔACD, $CD=20\sin 45°=10\sqrt{2}$ (cm)
$AD=20\cos 45°=10\sqrt{2}$ (cm)

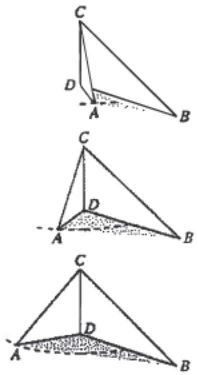
In ΔBCD, $BC=\frac{CD}{\sin 30°}=20\sqrt{2}$ cm
$BD=\frac{CD}{\tan 30°}=10\sqrt{6}$ cm

(b) (i) In ΔABD, Required $\angle=\cos^{-1}\frac{AD^2+BD^2-AB^2}{2AD\cdot BD}$

$=\cos^{-1}\frac{200+600-324}{2\cdot 10\sqrt{2}\cdot 10\sqrt{6}}$
$=46.60321°$
$=46.6°$ (3 s.f.)

(ii) $\because$ $CD\perp AD$ and $CD\perp BD$
$\therefore$ $CD\perp$ Plane ABD
$\Rightarrow$ Volume of $ABCD=\frac{1}{3}\times$ Area of $\Delta ABD\times CD$
$=\frac{1}{6}AD\cdot BD\cdot CD\sin\angle ADB$
$\Rightarrow$ Volume of $ABCD\propto\sin\angle ADB$

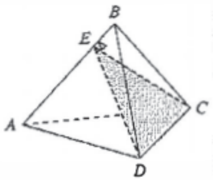
Hence, when $\angle ADB$ increases from 40° to 90°, the volume increases (from 525 cm^3 to 816 cm^3); when $\angle ADB$ increases from 90° to 140°, the volume decreases (from 816 cm^3 to 525 cm^3).



14B.52 HKDSE MA PP – I – 18

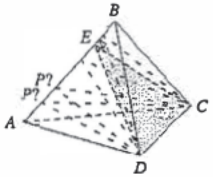
(a) In $\triangle ABC$, $AB=\sqrt{20^2+12^2-2\cdot 20\cdot 12\cos 60^\circ}$
$=\sqrt{304}=17.4$ (cm, 3 s.f.)

(b) Let E be on AB such that $CE\perp AB$. Since $\triangle ABC$ and $\triangle ABD$ are congruent, $DE\perp AB$ as well.



In $\triangle ABC$,
$CE=\dfrac{2\times \text{Area of }\triangle ABC}{AB}$
$=\dfrac{2\times\frac{1}{2}\cdot 12\cdot 20\sin 60^\circ}{\sqrt{304}}$
$=11.92079$ (cm)
$DE=CE=11.92079$ cm
$\therefore$ In $\triangle CDE$,
Required $\angle=\angle CED$
$=\cos^{-1}\dfrac{CE^2+DE^2-CD^2}{2CE\cdot DE}$
$=\cos^{-1}\dfrac{11.92079^2+11.92079^2-14^2}{2\cdot 11.92079\cdot 11.92079}$
$=71.9^\circ$ (3 s.f.)

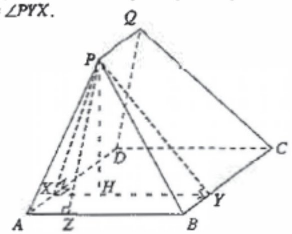
(c) $\angle CAD=\cos^{-1}\dfrac{20^2+20^2-14^2}{2\cdot 20\cdot 20}=41.0^\circ$
$\angle CBD=\cos^{-1}\dfrac{12^2+12^2-14^2}{2\cdot 12\cdot 12}=71.4^\circ$
As P moves from A to B, $\angle CPD$ increases from 41.0° to $\angle CED=71.9^\circ$ at E and then decreases to 71.4°.



14B.53 HKDSE MA 2012 – I – 18

(a) In $\triangle ABP$, $\angle APB=180^\circ-72^\circ-60^\circ=48^\circ$
$\dfrac{AP}{\sin 60^\circ}=\dfrac{20}{\sin 48^\circ}\Rightarrow AP=23.30704=23.3$ (cm, 3 s.f.)

(b) Since the pyramid is square-based and right, all lateral faces are congruent. Thus, all their base angles are 72°.
Let X, Y, Z and H be the projections of P on AD, BC, AB and $ABCD$ respectively. Then PXY is perpendicular to $ABCD$. (This is assumed by the symmetry without proof.)
$\therefore\ \alpha=\angle PYX$.



(i) Method 1 – Use $\triangle PXY$ to find α
In $\triangle ABP$, $\dfrac{BP}{\sin 72^\circ}=\dfrac{20}{\sin 48^\circ}$
$\Rightarrow BP=25.595456$ (cm)
By the symmetry of the pyramid, $PQCB$ and $PQDA$ are isosceles trapeziums.
In $\triangle APX$, $PX=AP\sin 72^\circ=22.166315$ cm
$AX=AP\cos 72^\circ=7.202272$ cm
$\Rightarrow PQ=AD-2AX=5.595456$ cm
In $\triangle BPY$, $BY=AX=7.202272$ cm
$PY=\sqrt{PB^2-BY^2}=24.561242$ cm
$\therefore$ In $\triangle PXY$, $XY=AB=20$ cm
$\Rightarrow \alpha=\cos^{-1}\dfrac{XY^2+PY^2-PX^2}{2XY\cdot PY}=58.6^\circ$ (3 s.f.)

Method 2 – Use $\triangle PHY$ to find α
In $\triangle APZ$, $AZ=AP\cos 72^\circ=7.202272$ cm
$PZ=AP\sin 72^\circ=22.166315$ cm
In $\triangle APX$, $AX=AP\cos 72^\circ=7.202272$ cm
$\Rightarrow$ In $\triangle PHZ$, $HZ=AX=7.202272$ cm
$PH=\sqrt{PZ^2-HZ^2}=20.963606$ cm
$\therefore$ In $\triangle PHY$, $HY=ZB=AB-AZ=12.797728$ cm
$\alpha=\tan^{-1}\dfrac{PH}{HY}=58.6^\circ$ (3 s.f.)

Method 3
In $\triangle ABP$, $\dfrac{AP}{\sin 60^\circ}=\dfrac{BP}{\sin 72^\circ}\Rightarrow AP=\dfrac{BP\sin 60^\circ}{\sin 72^\circ}$
In $\triangle ABX$, $AX=AP\cos 72^\circ$
$=\dfrac{BP\sin 60^\circ}{\sin 72^\circ}\cos 72^\circ$
$=\dfrac{BP\sin 60^\circ}{\tan 72^\circ}$
In $\triangle BPZ$, $BZ=BP\cos 60^\circ$
In $\triangle PHY$, $HY=BZ=BP\cos 60^\circ$
$PY=\dfrac{HY}{\cos\alpha}=\dfrac{BP\cos 60^\circ}{\cos\alpha}$
$\therefore$ In $\triangle BPY$, $BY=AX=\dfrac{BP\sin 60^\circ}{\tan 72^\circ}$
$BP^2=BY^2+PY^2$
$=\dfrac{BP^2\sin^2 60^\circ}{\tan^2 72^\circ}+\dfrac{BP^2\cos^2 60^\circ}{\cos^2\alpha}$
$\dfrac{\cos^2 60^\circ}{\cos^2\alpha}=1-\dfrac{\sin^2 60^\circ}{\tan^2 72^\circ}$
$\dfrac{\cos^2\alpha}{\cos^2 60^\circ}=\dfrac{\tan^2 72^\circ}{\tan^2 72^\circ-\sin^2 60^\circ}$
$\cos\alpha=\sqrt{\dfrac{\tan^2 72^\circ\cos^2 60^\circ}{\tan^2 72^\circ-\sin^2 60^\circ}}\Rightarrow\alpha=58.6^\circ$

Method 4
In $\triangle ABP$, $\dfrac{\tan 72^\circ}{\tan 60^\circ}=\dfrac{\frac{PL}{AL}}{\frac{PL}{BL}}=\dfrac{BL}{AL}$
Similarly, in $\triangle PXY$, $\dfrac{\tan\theta}{\tan\alpha}=\dfrac{YH}{XH}$
$=\dfrac{BL}{AL}=\dfrac{\tan 72^\circ}{\tan 60^\circ}$
$\Rightarrow \tan\alpha=\dfrac{\tan 60^\circ}{\tan 72^\circ}\tan\theta$
In $\triangle APZ$, $AZ=AP\cos 72^\circ$
In $\triangle APX$, $PX=AP\sin 72^\circ$
In $\triangle PHX$, $HX=AZ=AP\cos 72^\circ$
$\therefore\ \cos\theta=\dfrac{HX}{PX}=\dfrac{AP\cos 72^\circ}{AP\sin 72^\circ}=\dfrac{1}{\tan 72^\circ}$
$\Rightarrow \tan\theta=\sqrt{\tan^2 72^\circ-1}$
Hence, $\tan\alpha=\dfrac{\tan 60^\circ}{\tan 72^\circ}\tan\theta$
$=\dfrac{\tan 60^\circ}{\tan 72^\circ}\sqrt{\tan^2 72^\circ-1}\Rightarrow\alpha=58.6^\circ$

(ii) $\sin\alpha=\dfrac{PH}{PY}$, $\sin\beta=\dfrac{PH}{PB}$
$\because\ PY<PB$
$\therefore\ \dfrac{PH}{PY}>\dfrac{PH}{PB}\Rightarrow\sin\alpha>\sin\beta\Rightarrow\alpha>\beta$

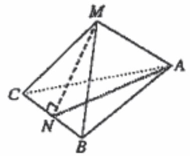
14B.54 HKDSE MA 2013 – I – 18

(a) (i) In $\triangle ABC$, $\angle BCM=\cos^{-1}\dfrac{21^2+35^2-28^2}{2\cdot 21\cdot 35}$
$=53.13010^\circ=53.1^\circ$ (3 s.f.)

(ii) In $\triangle BCM$, $\angle CBM=51.86990^\circ$
$\dfrac{CM}{\sin 51.86990^\circ}=\dfrac{21}{\sin 75^\circ}$
$CM=17.10155=17.1$ (cm, 3 s.f.)

(b) (i) $AM=35-17.101545=17.89845$ (cm)
In $\triangle ACM$,
$AC=\sqrt{AM^2+CM^2-2AM\cdot CM\cos\angle AMC}$
$=28.13898=28.1$ (cm, 3 s.f.)

(ii) In $\triangle CMN$, $CN=CM\cos\angle MCN$
$=17.10155\cos 53.13030^\circ$
$=10.26093$ (cm)
$\Rightarrow BN=21-10.26093=10.73907$ (cm)
In $\triangle ABC$, $\angle ABC=\cos^{-1}\dfrac{28^2+21^2-28.13898^2}{2\cdot 28\cdot 21}$
$=68.38516^\circ$



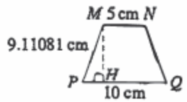
Method 1 – Check whether $AN\perp BC$
In $\triangle ABN$, $AN=\sqrt{AB^2+BN^2-2AB\cdot BN\cos\angle ABC}$
$=26.03454$ cm
$\because\ AB^2=784$
$AN^2+BN^2=681\neq AB^2$
$\therefore\ \angle ANB\neq 90^\circ$
i.e. $\angle ANM$ is not the described angle. Disagreed.

Method 2 – Check if N is the projection of A onto BC
Suppose the projection of A onto BC is P.
In $\triangle ABP$, $BP=AB\cos\angle ABC=10.31423\text{ cm}\neq BN$.
i.e. N is not the projection of A onto BC.
Hence, $\angle ANM$ is not the described angle. Disagree.

14B.55 HKDSE MA 2014 – I – 17

(a) In $\triangle VAB$, $\dfrac{\sin\angle AVB}{18}=\dfrac{\sin 110^\circ}{30}$
$\angle AVB=34.32008^\circ$ or 145.7° (rej.)
$\therefore\ \angle VBA=180^\circ-110^\circ-34.32008^\circ$
$=35.67992^\circ=35.7^\circ$ (3 s.f.)

(b) In $\triangle VAB$, $VA=\sqrt{18^2+30^2-2\cdot 18\cdot 30\cos 35.67992^\circ}$
$=18.22161$ cm
In $\triangle VBC$, $\because\ VM=MB$ and $VN=NC$
$\therefore\ MN=\frac{1}{2}BC=5$ cm (mid-pt theorem)
Similarly, $MP=\frac{1}{2}VA=9.11081$ cm
Let the projection of M onto PQ be H.
In $\triangle MPH$,
$PH=(PQ-MN)\div 2$
$=2.5$ cm
$MH=\sqrt{MP^2-PH^2}$
$=8.7611$ cm



$\therefore$ Area of $PQNM=\dfrac{(5+10)(8.7611)}{2}=65.7<70$ (cm^2)
$\therefore$ The craftsman is agreed.

14B.56 HKDSE MA 2015 – I – 19

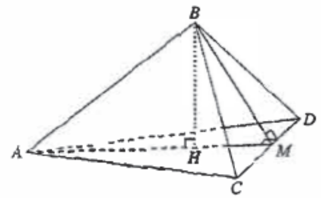
(a) (i) In $\triangle ABC$, $AC=\sqrt{40^2+24^2-2\cdot 40\cdot 24\cos 80^\circ}$
$=42.92546=42.9$ (cm, 3 s.f.)

(ii) $\dfrac{\sin\angle ACB}{40}=\dfrac{\sin 80^\circ}{42.92546}$
$\angle ACB=66.59082^\circ$ or 113° (rej.) $=66.6^\circ$ (3 s.f.)

(iii) Note how the given information had fixed the areas of $\triangle ABC$ and $\triangle AB'D$. Hence, the only varying part of the paper card is $\triangle ACD$.

Area of $\triangle ABC$ = Area of $\triangle AB'D$
$=\frac{1}{2}(40)(24)\sin 80^\circ=472.71$ (cm^2),
which is a constant.
Area of $\triangle ACD=\frac{1}{2}AC^2\sin\angle ACD$
$=921.30\sin(\angle BCD-66.6^\circ)$
$\because\ 105^\circ\leq\angle BCD\leq 145^\circ$
$\therefore\ 38.4^\circ\leq\angle ACD\leq 78.4^\circ$
Hence, as $\angle BCD$ increases from 105° to 145°, the area of the paper card increases.
(from $472.71\times 2+921.30\sin 38.4^\circ=1518$ (cm^2)
to $472.71\times 2+921.30\sin 78.4^\circ=1848$ (cm^2))

(b) Let the projection of B onto ACD be H and the mid-point of CD be M. By symmetry, we have $BM\perp CD$, $AM\perp CD$ and H lying on AM.



$\angle ACD=132^\circ-66.59082^\circ=65.40918^\circ$
In $\triangle ACM$, $AM=AC\sin(132^\circ-65.40918^\circ)$
$=39.39231$ (cm)
$CM=AC\cos(132^\circ-65.40918^\circ)$
$=17.86279$ (cm)
In $\triangle BCM$, $BM=\sqrt{BC^2-CM^2}=16.02875$ cm
In $\triangle ABM$, $\angle BAM=\cos^{-1}\dfrac{AB^2+AM^2-BM^2}{2AB\cdot AM}$
$=23.2791^\circ$
$\Rightarrow BH=AB\sin\angle BAH=15.8084$ cm
$\therefore$ Vol of pyramid $=\frac{1}{3}(921.30\sin 65.40918^\circ)(15.8084)$
$=4410$ (cm^3, 3 s.f.)

14B.57 HKDSE MA 2016 – I – 19

(a) In $\triangle ABD$, $\dfrac{\sin\angle ADB}{10}=\dfrac{\sin 86^\circ}{15}$

$\angle ADB = 41.68560^\circ$ or 138.3° (rej.)

$\Rightarrow \angle ABD = 180^\circ - 86^\circ - 41.68560^\circ$

$= 52.31440^\circ = 52.3^\circ$ (3 s.f.)

In $\triangle BCD$, $CD=\sqrt{8^2+15^2-2\cdot 8\cdot 15\cos 43^\circ}$

$= 10.65247 = 10.7$ (cm, 3 s.f.)

(b) We need to verify $AC\perp BC$ and $AC\perp CD$.

In $\triangle ABC$, $AC^2+BC^2=6^2+8^2$

$=100=AB^2$

$\therefore AC\perp BC$

In $\triangle ABD$, $AD^2=AB^2+BD^2-2AB\cdot BD\cos\angle ABD$

$=141.60$

$\because AC^2+CD^2=149.48\neq AD^2$

$\therefore AC$ is not perpendicular to CD.

Since C is not the projection of A onto BCD, $\angle ABC$ is not the described angle. The craftsman is disagreed.

14B.58 HKDSE MA 2017 – I – 19

(a) In $\triangle ABC$, $\angle B = 180^\circ - 30^\circ - 42^\circ = 108^\circ$

$\dfrac{AC}{\sin 108^\circ}=\dfrac{24}{\sin 30^\circ} \Rightarrow AC = 45.65071 = 45.7$ (cm, 3 s.f.)

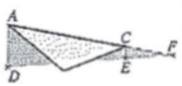
(b) (i) $\because \triangle ADF\sim\triangle CEF$

$\therefore \dfrac{10}{2}=\dfrac{45.65071+CF}{CF}$

$4CF = 45.65071$

$CF = 11.41268$

$= 11.4$ (cm, 3 s.f.)



(ii) *Method 1*

In $\triangle ABC$, $\dfrac{AB}{\sin 42^\circ}=\dfrac{24}{\sin 30^\circ}\Rightarrow AB = 32.11827$ (cm)

Area of $\triangle ABF = \dfrac{1}{2}AB\cdot AF\sin\angle FAB$

$= 458.1943 = 458$ (cm², 3 s.f.)

Method 2

Area of $\triangle ABF$

$=$ Area of $\triangle ABC$ + Area of $\triangle FBC$

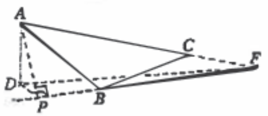
$=\dfrac{1}{2}AC\cdot BC\sin\angle ACB+\dfrac{1}{2}CF\cdot BC\sin(180^\circ-\angle ACB)$

$=\dfrac{1}{2}(AC+CF)BC\sin\angle ACB = 458$ cm² (3 s.f.)

(iii) In $\triangle FBC$, $BF=\sqrt{BC^2+CF^2-2BC\cdot CF\cos\angle BCF}$

$= 33.36690$ (cm)

(Or use $\triangle ABF$ to find BF.)



Let the projection of A onto BF be P.

$AP=\dfrac{2\times\text{Area of }\triangle ABF}{BF}=27.46400$ cm

$\therefore$ Inclination $=\sin^{-1}\dfrac{AD}{AP}=21.4^\circ$ (3 s.f.)

(iv) Since P is also the projection of D onto BF,

Area of $\triangle BDF=\dfrac{1}{2}BF\cdot DP$

$<\dfrac{1}{2}BF\cdot AP = 458 < 460$

$\therefore$ The craftsman is disagreed.

14B.59 HKDSE MA 2018 – I – 17

(a) In $\triangle ABD$, $\dfrac{AD}{\sin 20^\circ}=\dfrac{60}{\sin(180^\circ-120^\circ-20^\circ)}$

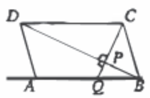
$AD = 31.92533 = 31.9$ (cm, 3 s.f.)

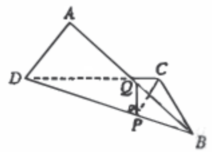
(b) (i) In $\triangle ABC$, $\angle ABC=\cos^{-1}\dfrac{AB^2+BC^2-AC^2}{2AB\cdot BC}$

$=\cos^{-1}\dfrac{60^2+31.92533^2-40^2}{2\cdot 60\cdot 31.92533}$

$=37.99208^\circ = 38.0^\circ$ (3 s.f.)

(ii) Let P be on BD such that $CP\perp BD$, and CP extended meet AB at Q (in Figure (1)). Then the angle between ABD and BCD in Figure (2) is $\angle CPQ$.





In $\triangle BCP$, $BP = BC\cos 40^\circ = 24.45622$ cm

In $\triangle BPQ$, $BQ=\dfrac{BP}{\cos 20^\circ}=26.02577$ cm

In $\triangle BCQ$, $CQ=\sqrt{BQ^2+BC^2-2BQ\cdot BC\cos\angle QBC}$

$= 19.67077$ cm

$\therefore$ In $\triangle CPQ$,

Required $\angle = \angle CPQ=\cos^{-1}\dfrac{PQ^2+CP^2-CQ^2}{2PQ\cdot CP}$

$= 71.9^\circ$ (3 s.f.)

14B.60 HKDSE MA 2019 – I – 18

(a) (i) In $\triangle ABD$, $\dfrac{\sin\angle BAD}{12}=\dfrac{\sin 72^\circ}{13}$

$\angle BAD = 61.38987^\circ$ or 118.6° (rej.)

$= 61.4^\circ$ (3 s.f.)

(ii) $\angle ADB = 180^\circ - 72^\circ - 61.38987^\circ = 46.61013^\circ$

$\Rightarrow DP = BD\cos\angle ADB = 8.24351$ cm

In $\triangle CDP$,

$CP=\sqrt{12^2+8.24351^2-2\cdot 12\cdot 8.24351\cos 60^\circ}$

$= 11.39253 = 11.4$ (cm, 3 s.f.)

(b) Since $BP\perp AD$, we need to check whether $CP\perp AD$.

In $\triangle CDP$, $CD^2=169$

$CP^2+DP^2=197.7\neq 169 = CD^2$

Hence, $\angle CPD\neq 90^\circ$, and thus $\angle BPC$ is not the angle between ABD and ACD. The claim is disagreed.

14B.61 HKDSE MA 2020 – I – 19

19a

$\dfrac{PR}{\sin\angle PQR}=\dfrac{PQ}{\sin\angle PRQ}$

$\dfrac{PR}{\sin 30^\circ}=\dfrac{60}{\sin 55^\circ}$

$PR\approx 36.62323766$ cm

$\angle PQR+\angle PRQ+\angle QPR=180^\circ$ ($\angle$ sum of Δ)

$30^\circ+55^\circ+\angle QPR=180^\circ$

$\angle QPR=95^\circ$

$\angle QPR+\angle RPS=\angle QPS$

$95^\circ+\angle RPS=120^\circ$

$\angle RPS=25^\circ$

$RS^2=PR^2+PS^2-2(PR)(PS)\cos\angle RPS$

$RS^2=36.62323766^2+40^2-2(36.62323766)(40)\cos 25^\circ$

$RS\approx 16.90879944$ cm

$RS\approx 16.9$ cm (corr. to 3 sig fig.)

b

The area of the paper card

$\dfrac{1}{2}(PQ)(PR)\sin\angle QPR+\dfrac{1}{2}(PR)(PS)\sin\angle RPS$

$=\dfrac{1}{2}(60)(36.62323766)\sin 95^\circ+\dfrac{1}{2}(36.62323766)(40)\sin 25^\circ$

≈ 1404.069236

≈ 1400 cm² (corr. to 3 sig. fig.)

ci

Let A be the perpendicular foot of P on the line passing through Q and R and O be the projection of P on the horizontal ground.

Then, $\angle OAP=32^\circ$.

$\sin\angle PQA=\dfrac{PA}{PQ}$

$\sin 30^\circ=\dfrac{PA}{60}$

$PA=30$ cm

$\sin\angle OAP=\dfrac{OP}{PA}$

$\sin 32^\circ=\dfrac{OP}{30}$

$OP=30\sin 32^\circ$ cm

$OP=15.9$ cm (corr. to 3 sig. fig.)

ii

Produce PS and QR to intersect at the point B.

$\angle PQB+\angle PBQ+\angle QPB=180^\circ$ ($\angle$ sum of Δ)

$30^\circ+\angle PBQ+120^\circ=180^\circ$

$\angle PBQ=30^\circ$

$\angle PBQ=\angle PQB$

$PQ=PB$ (sides opp. eq. $\angle$s)

$PS+SB=PQ$

40 cm $+SB=60$ cm

$SB=20$ cm

Let C be the perpendicular foot of S on AB.

$\angle PBA=\angle SBC$ (common $\angle$)

$\angle PAB=\angle SCB$ (by construction)

$\angle APB=\angle CSB$ (3rd $\angle$ of Δ)

$\Delta PAB\sim\Delta SCB$ (A.A.A.)

$\dfrac{PA}{SC}=\dfrac{PB}{SB}$ (corr. sides, ~Δs)

$\dfrac{30\sin 32^\circ}{SC}=\dfrac{60}{20}$

$SC=10\sin 32^\circ$ cm

$\sin\angle SRC=\dfrac{SC}{RS}$

$\sin\angle SRC=\dfrac{10\sin 32^\circ}{16.90879944}$

$\angle SRC\approx 18.26416068^\circ$

The angle between RS and the horizontal ground $\approx 18.26416068^\circ$

The angle between RS and the horizontal ground $< 20^\circ$

Hence, the student's claim is agreed with.