13 Basic Trigonometry

13A Trigonometric functions

13A.1 HKCEE MA 1980(1/1*/3) – I – 4

If $0° < \theta < 360°$ and $\sin\theta = \cos 120°$, find θ.

13A.2 HKCEE MA 1981(1/2/3) – I – 4

Solve $\cos(200° + \theta) = \sin 120°$ where $0° \leq \theta \leq 180°$.

13A.3 HKCEE MA 1982(1/2/3) I 5

Solve $2\sin^2\theta + 5\sin\theta - 3 = 0$ for θ, where $0° \leq \theta < 360°$.

13A.4 HKCEE MA 1983(A/B) – I – 7

Find all the values of θ, where $0° \leq \theta \leq 360°$, such that $2\cos^2\theta + 5\sin\theta + 1 \quad 0$.

13A.5 HKCEE MA 1984(A/B) I 7

Given $\tan\theta = \dfrac{1+\cos\theta}{\sin\theta}$ $(0° < \theta < 90°)$,

(a) rewrite the above equation in the form $a\cos^2\theta + b\cos\theta + c = 0$ where a, b and c are integers;

(b) hence, solve the given equation.

13A.6 HKCEE MA 1985(A/B) I 6

Solve $2\tan^2\theta = 1 - \tan\theta$, where $0° \leq \theta < 360°$. (Give your answers correct to the nearest degree.)

13A.7 HKCEE MA 1986(A/B) I – 4

Solve $\sin^2\theta + 7\sin\theta = 5\cos^2\theta$ for $0° \leq \theta < 360°$.

13A.8 HKCEE MA 1987(A/B) – I – 4

Solve the equation $\sin^2\theta = \dfrac{3}{2}\cos\theta$, where $0° \leq \theta < 360°$.

13A.9 (HKCEE MA 1988 – I – 2)

Simplify

(a) $\dfrac{\sin(180° - \theta)}{\sin(90° + \theta)}$,

(b) $\sin^2(180° - \phi) + \sin^2(270° + \phi)$.

13A.10 HKCEE MA 1989 – I – 7

Rewrite the equation $3\tan\theta = 2\cos\theta$ in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers. Hence solve the equation for $0° \leq \theta < 360°$.

13A.11 HKCEE MA 1990 – I – 3

Rewrite $\sin^2\theta : \cos\theta = -3 : 2$ in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are integers. Hence solve for θ, where $0° \leq \theta < 360°$.

13A.12 HKCEE MA 1991 – I – 5

Solve $\sin^2\theta - 3\cos\theta - 1 = 0$ for $0° \leq \theta < 360°$.

13A.13 HKCEE MA 1992 I 1(b)

Find x if $\sin x = \dfrac{1}{2}$ and $90° < x < 180°$.

13A.14 HKCEE MA 1992 I 1(c)

Simplify $\dfrac{1-\sin^2 A}{\cos A}$.

13A.15 HKCEE MA 1993 – I – 3

Solve $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \dfrac{3}{2}$ for $0° \leq \theta < 360°$.

13A.16 HKCEE MA 1994 – I – 2(b)

If $\sin x° = \sin 36°$ and $90 < x < 270$, find the value of x.

13A.17 HKCEE MA 1994 I 2(c)

If $\cos y° = -\cos 36°$ and $180 < y < 360$, find the value of y.

13A.18 HKCEE MA 1995 – I – 6

Solve the trigonometric equation $2\sin^2\theta + 5\sin\theta - 3 = 0$ for $0° \leq \theta < 360°$.

13A.19 HKCEE MA 2010 – I – 4

For each positive integer n, the nth term of a sequence is $\tan\dfrac{180°}{n+2}$.

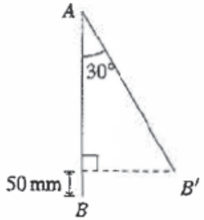
(a) Find the 2nd term of the sequence.

(b) Write down, in surd form, two different terms of the sequence such that the product of these two terms is equal to the 2nd term of the sequence.

13B Trigonometric ratios in right-angled triangles

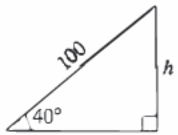
13B.1 HKCEE MA 1980(1/1*/3) I – 5

In the figure, AB is a vertical thin rod. It is rotated about A to position AB' such that $\angle BAB' = 30°$. If B' is 50mm higher than B, find the length of the rod.



13B.2 HKCEE MA 1993 I 1(b)

In the figure, find h.

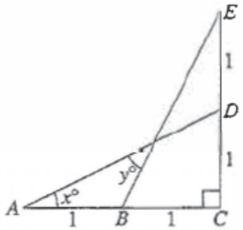


13B.3 HKCEE MA 1994 – I – 5

In the figure, calculate

(a) the length of BE,

(b) the values of x and y.



13B.4 HKCEE MA 1995 I 1(e)

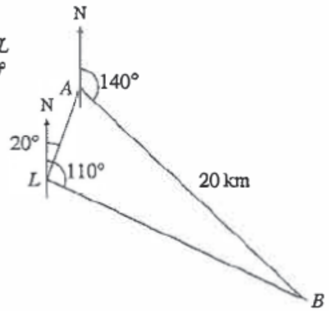
In the figure, ABC is a right-angled triangle. If $\cos A = \frac{1}{3}$, find AC.

13B.5 HKCEE MA 1997 I – 6

In the figure, the bearings of two ships A and B from a lighthouse L are 020° and 110° respectively. B is 20km and at a bearing of 140° from A. Find

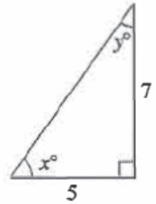
(a) the distance of L from B,

(b) the bearing of L from B.



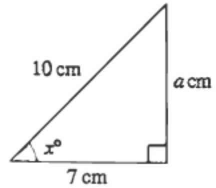
13B.6 HKCEE MA 1998 – I – 3

In the figure, find x and y.



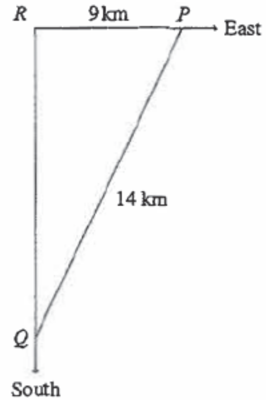
13B.7 HKCEE MA 2000 – I – 4

In the figure, find a and x.



13B.8 HKCEE MA 2008 I 4

In the figure, P, Q and R are three posting boxes on the horizontal ground. P is 9km due east of R and Q is due south of R. The distance between P and Q is 14km. Find the bearing of Q from P.



13 Basic Trigonometry

13A Trigonometric functions

13A.1 HKCEE MA 1980(1/1*/3) – I – 4

$\sin\theta = \cos 120° = -\frac{1}{2} \Rightarrow \theta = 210° \text{ or } 330°$

13A.2 HKCEE MA 1981(1/2/3) – I – 4

$0° \le \theta \le 180° \Rightarrow 200° \le 200° + \theta \le 380°$

$\therefore \cos(200° + \theta) = \sin 120° = \frac{\sqrt{3}}{2} \Rightarrow 200° + \theta = 330°$

$\theta = 130°$

13A.3 HKCEE MA 1982(1/2/3) – I – 5

$2\sin^2\theta + 5\sin\theta - 3 = 0$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$

$\sin\theta = \frac{1}{2} \text{ or } -3 \text{ (rej.)} \Rightarrow \theta = 30° \text{ or } 150°$

13A.4 HKCEE MA 1983(A/B) – I – 7

$2\cos^2\theta + 5\sin\theta + 1 = 0$

$2(1 - \sin^2\theta) + 5\sin\theta + 1 = 0$

$2\sin^2\theta - 5\sin\theta - 3 = 0$

$(2\sin\theta + 1)(\sin\theta - 3) = 0$

$\sin\theta = -\frac{1}{2} \text{ or } 3 \text{ (rej.)} \Rightarrow \theta = 210° \text{ or } 330°$

13A.5 HKCEE MA 1984(A/B) – I – 7

(a) $\frac{\sin\theta}{\cos\theta} = \frac{1 + \cos\theta}{\sin\theta}$

$\sin^2\theta = \cos\theta + \cos^2\theta$

$0 = \cos\theta + \cos^2\theta - (1 - \cos^2\theta)$

$2\cos^2\theta + \cos\theta - 1 = 0$

(b) $(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{2} \text{ or } -1 \text{ (rej.)} \Rightarrow \theta = 60°$

13A.6 HKCEE MA 1985(A/B) – I – 6

$2\tan^2\theta = 1 - \tan\theta$

$2\tan^2\theta + \tan\theta - 1 = 0$

$(2\tan\theta - 1)(\tan\theta + 1) = 0$

$\tan\theta = \frac{1}{2} \text{ or } -1$

$\theta = 27°, 180° + 27° \text{ or } 135°, 180° + 135°$

$= 27°, 207° \text{ (nearest deg)}, 135° \text{ or } 315°$

13A.7 HKCEE MA 1986(A/B) – I – 4

$\sin^2\theta + 7\sin\theta = 5\cos^2\theta = 5(1 - \sin^2\theta)$

$6\sin^2\theta + 7\sin\theta - 5 = 0$

$(2\sin\theta - 1)(3\sin\theta + 5) = 0$

$\sin\theta = \frac{1}{2} \text{ or } -\frac{5}{3} \text{ (rejected)}$

$\theta = 30° \text{ or } 180° - 30° = 150°$

13A.8 HKCEE MA 1987(A/B) – I – 4

$2\sin^2\theta = 3\cos\theta$

$2(1 - \cos^2\theta) = 3\cos\theta$

$2\cos^2\theta + 3\cos\theta - 2 = 0$

$(2\cos\theta - 1)(\cos\theta + 2) = 0$

$\cos\theta = \frac{1}{2} \text{ or } -2 \text{ (rejected)}$

$\theta = 60° \text{ or } 360° - 60° = 300°$

13A.9 (HKCEE MA 1988 – I – 2)

(a) $\frac{\sin(180° - \theta)}{\sin(90° + \theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

(b) $\sin^2(180° - \phi) + \sin^2(270° + \phi) = \sin^2\phi + (-\cos\phi)^2 = 1$

13A.10 HKCEE MA 1989 – I – 7

$\frac{3\sin\theta}{\cos\theta} = 2\cos\theta$

$3\sin\theta = 2\cos^2\theta = 2(1 - \sin^2\theta)$

$2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(\sin\theta + 2) = 0$

$\sin\theta = \frac{1}{2} \text{ or } -2 \text{ (rejected)}$

$\theta = 30° \text{ or } 180° - 30° = 150°$

13A.11 HKCEE MA 1990 – I – 3

$\frac{1 - \cos^2\theta}{\cos\theta} = -\frac{3}{2}$

$2 - 2\cos^2\theta = -3\cos\theta$

$2\cos^2\theta - 3\cos\theta - 2 = 0$

$(2\cos\theta + 1)(\cos\theta - 2) = 0$

$\cos\theta = \frac{-1}{2} \text{ or } 2 \text{ (rejected)}$

$\theta = 120° \text{ or } 360 - 120° = 240°$

13A.12 HKCEE MA 1991 – I – 5

$\sin^2\theta - 3\cos\theta - 1 = 0$

$(1 - \cos^2\theta) - 3\cos\theta - 1 = 0$

$\cos^2\theta + 3\cos\theta = 0$

$\cos\theta(\cos\theta + 3) = 0$

$\cos\theta = 0 \text{ or } -3 \text{ (rejected)}$

$\theta = 90° \text{ or } 270°$

13A.13 HKCEE MA 1992 – I – 1(b)

$\sin x = \frac{1}{2} \Rightarrow x = 180° - 30° = 150°$

13A.14 HKCEE MA 1992 – I – 1(c)

$\frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A} = \cos A$

13A.15 HKCEE MA 1993 – I – 3

$\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{3}{2}$

$2\sin\theta + 2\cos\theta = 3\sin\theta - 3\cos\theta$

$-\sin\theta = 5\cos\theta$

$\tan\theta = -5$

$\theta = 78.7° \text{ or } 180° + 78.7° = 259° \text{ (3 s.f.)}$

13A.16 HKCEE MA 1994 – I – 2(b)

$\sin x° = \sin 36° \Rightarrow x = 180 - 36 = 144$

13A.17 HKCEE MA 1994 – I – 2(c)

$\cos y° = -\cos 36° = \cos(180° + 36°) \Rightarrow y = 216$

13A.18 HKCEE MA 1995 – I – 6

$2\sin^2\theta + 5\sin\theta - 3 = 0$

$(2\sin\theta - 1)(\sin\theta + 3) = 0$

$\sin\theta = \frac{1}{2} \text{ or } -3 \text{ (rejected)}$

$\theta = 30° \text{ or } 180° - 30° = 150°$

13A.19 HKCEE MA 2010 – I – 4

(a) 2nd term $= \tan\frac{180°}{(2)+2} = \tan 45° = 1$

(b) (Note that if the product of two different numbers is 1, one of them is > 1 and the other < 1. Besides, the sequence is decreasing when n increases. Hence, the larger term must come before the 2nd term.)

$\tan\frac{180°}{(1)+2} = \tan 60° = \sqrt{3}$

$\Rightarrow \frac{1}{\sqrt{3}} = \tan 30° = \tan\frac{180°}{6} = \frac{180°}{(5)+1}$

$\therefore$ Required terms are the 1st one, $\sqrt{3}$, and 5th one, $\frac{1}{\sqrt{3}}$.

13B Trigonometric ratios in right angled triangles

13B.1 HKCEE MA 1980(1/1*/3) – I – 5

Let ℓ mm be the length of rod. Then

$\frac{\sqrt{3}}{2} = \cos 30° = \frac{\ell - 50}{\ell}$

$\sqrt{3}\ell = 2(\ell - 50)$

$100 = (2 - \sqrt{3})\ell \Rightarrow \ell = 373 \text{ (3 s.f.)}$

Hence, the rod is 373 mm long.

13B.2 HKCEE MA 1993 – I – 1(b)

$h = 100\cos 40° = 76.6 \text{ (3 s.f.)}$

13B.3 HKCEE MA 1994 – I – 5

(a) $BE = \sqrt{1^2 + 2^2} = \sqrt{5}$ $(= 2.24)$

(b) $\tan x° = \frac{1}{2} \Rightarrow x = 26.5651 = 26.6 \text{ (3 s.f.)}$

$\tan\angle EBC = 2 \Rightarrow \angle EBC = 63.4349$

$\Rightarrow y = 63.4349 - x = 36.9 \text{ (3 s.f.)}$

13B.4 HKCEE MA 1995 – I – 1(e)

$\frac{1}{3} = \cos A = \frac{2}{AC} \Rightarrow AC = 6$

13B.5 HKCEE MA 1997 – I – 6

(a) $\angle LAB = 20° + (180° - 140°) = 60°$

$\angle ALB = 110° - 20° = 90°$

$\therefore$ Distance $= LB = 20\sin 60° = 10\sqrt{3} = 17.3$ (km, 3 s.f.)

(b) $\angle ABL = 180° - 90° - 60° = 30°$

$\therefore$ Bearing $= 180° + 140° - 30° = 290°$

13B.6 HKCEE MA 1998 – I – 3

$\tan x° = \frac{7}{5} \Rightarrow x = 54.5$

$\Rightarrow y = 180 - 90 - 54.5 = 35.5$

13B.7 HKCEE MA 2000 – I – 4

$a = \sqrt{10^2 - 7^2} = \sqrt{51} = 7.14$

$\cos x° = \frac{7}{10} \Rightarrow x = 45.6$

13B.8 HKCEE MA 2008 – I – 4

$\sin\angle RQP = \frac{9}{14} \Rightarrow \angle RQP = 40.01°$

$\therefore$ Bearing = S40.0°W or $(180° + 40.0°) = 220°$