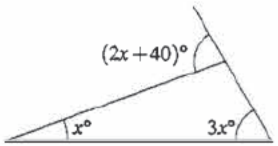
11 Geometry of Rectilinear Figures

11A Angles in intersecting lines and polygons

11A.1 <u>HKCEE MA 1980(1/1*/3) – I – 1</u>

Find the value of x in the figure.



11A.2 <u>HKCEE MA 1980(1*) – I – 15</u>

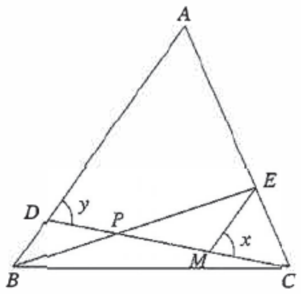
In $\triangle ABC$ (see the figure), $BD = \frac{1}{4}AB$, $CE = \frac{1}{3}AC$, BE intersects CD at P. $x = y$. Prove that

(a) $\triangle EMC$ and $\triangle ADC$ are similar and $EM = \frac{1}{4}AB$,

(b) $\triangle BDP$ and $\triangle EMP$ are congruent,

(c) $PM = CM$,

(d) area of triangle BDP is half the area of triangle PEC.



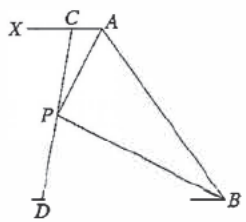
11A.3 <u>HKCEE MA 1981(2) – I – 14</u>

In the figure, $AX//BY$. AP and BP bisect $\angle XAB$ and $\angle YBA$ respectively, and they meet at P. A straight line passing through P meets AX and BY at C and D respectively. Prove that

(a) $\angle APB = 90°$,

(b) $CP = DP$,

(c) $AC + BD = AB$.



11A.4 <u>HKCEE MA 1988 – I – 8(a)</u>

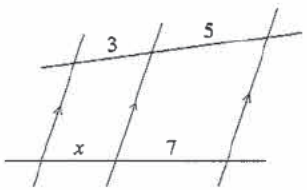
P is a point inside a square $ABCD$ such that PBC is an equilateral triangle. AP is produced to meet CD at Q.

(i) Draw a diagram to represent the above information.

(ii) Calculate $\angle PAB$ and $\angle PQC$.

11A.5 <u>HKCEE MA 1993(I) – I – 1(c)</u>

In the figure, find x.

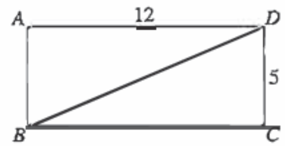


11A.6 <u>HKCEE MA 1995 – I – 1(c)</u>

Find the size of an interior angle of a regular octagon (8-sided polygon).

11A.7 <u>HKCEE MA 1995 – I – 1(d)</u>

In the figure, $ABCD$ is a rectangle. Find BD.



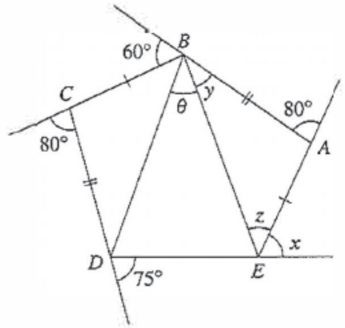
11A.8 <u>HKCEE MA 1996 – I – 10</u>

In the figure, $AB = CD$ and $AE = BC$.

(a) Find x.

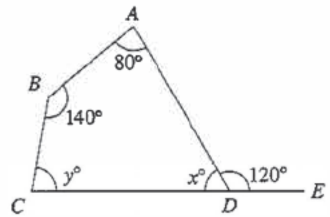
(b) Which two triangles in the figure are congruent?

(c) Find θ, y and z.



11A.9 <u>HKCEE MA 1998 – I – 2</u>

In the figure, CDE is a straight line. Find x and y.

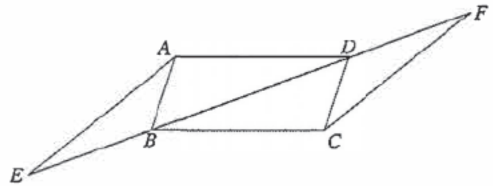


11A.10 <u>HKCEE MA 1999 – I – 14</u>

In the figure, $ABCD$ is a parallelogram. $EBDF$ is a straight line and $EB = DF$.

(a) Prove that $\angle ABE = \angle CDF$.

(b) Prove that $EA//CF$.

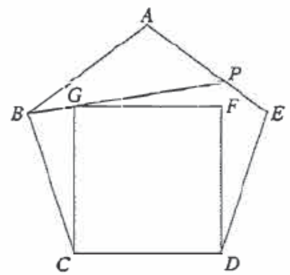


11A.11 <u>HKCEE MA 2000 – I – 13</u>

(To continue as **14A.6**.)

In the figure, $ABCDE$ is a regular pentagon and $CDFG$ is a square. BG produced meets AE at P.

(a) Find $\angle BCG$, $\angle ABP$ and $\angle APB$.

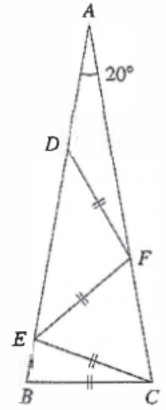


11A.12 HKCEE MA 2002 – I – 10

In the figure, ABC is a triangle in which $\angle BAC = 20°$ and $AB = AC$. D, E are points on AB and F is a point on AC such that $BC = CE = EF = FD$.

(a) Find $\angle CEF$.

(b) Prove that $AD = DF$.



11A.13 HKCEE MA 2004 I – 12

In the figure, AEC, AFB, BCD and DEF are straight lines. $AB = AC$, $CD = CE$ and $\angle CDE = 36°$.

(a) Find

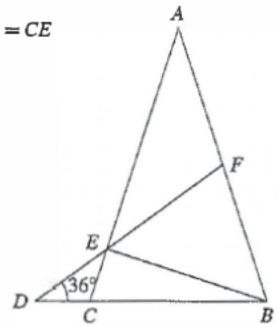
(i) $\angle AEF$,

(ii) $\angle BAC$.

(b) Suppose $AF = FB$.

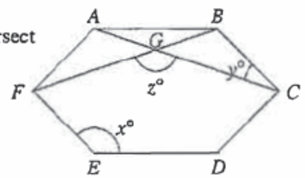
(i) Prove that $\angle AEB$ is a right angle.

(ii) If $AE = 10$ cm, find the area of $\triangle ABC$.



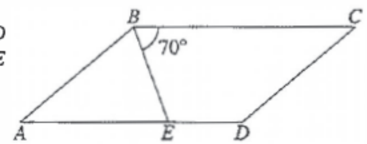
11A.14 HKCEE MA 2005 – I – 8

In the figure, $ABCDEF$ is a regular six-sided polygon. AC and BF intersect at G. Find x, y and z.



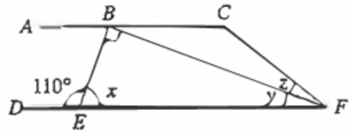
11A.15 HKCEE MA 2006 I 5

In the figure, $ABCD$ is a parallelogram. E is a point lying on AD such that $AE = AB$. It is given that $\angle EBC = 70°$. Find $\angle ABE$ and $\angle BCD$.



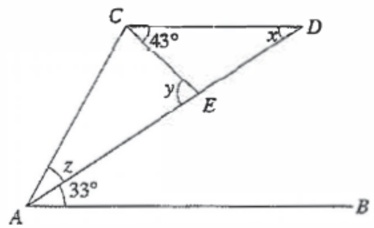
11A.16 HKCEE MA 2007 – I – 8

In the figure, ABC and DEF are straight lines. It is given that $AC // DF$, $BC = CF$, $\angle EBF = 90°$ and $\angle BED = 110°$. Find x, y and z.



11A.17 HKCEE MA 2008 I – 9

In the figure, $AB // CD$. E is a point lying on AD such that $AE = AC$. Find x, y and z.



11A.18 HKDSE MA 2020 – I – 8

In Figure 1, B and D are points lying on AC and AE respectively. BE and CD intersect at the point F. It is given that $AB = BE$, $BD // CE$, $\angle CAE = 30°$ and $\angle ADB = 42°$.

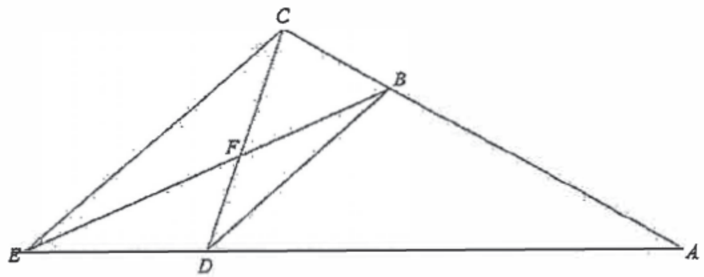


Figure 1

(a) Find $\angle BEC$.

(b) Let $\angle BDC = \theta$. Express $\angle CFE$ in terms of θ.

(5 marks)

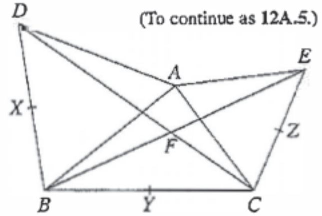
11B Congruent and similar triangles

11B.1 HKCEE MA 1982(2) I – 13

(To continue as **12A.5**.)

In the figure, $\triangle ADB$ and $\triangle ACE$ are equilateral triangles. DC and BE intersect at F.

(a) Prove that $DC = BE$. [Hint: Consider $\triangle ADC$ and $\triangle ABE$.]



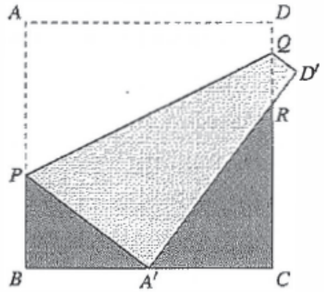
11B.2 HKCEE MA 2001 – I – 11

As shown in the figure, a piece of square paper $ABCD$ of side 12 cm is folded along a line segment PQ so that the vertex A coincides with the mid-point of the side BC. Let the new positions of A and D be A' and D' respectively, and denote by R the intersection of $A'D'$ and CD.

(a) Let the length of AP be x cm. By considering the triangle PBA', find x.

(b) Prove that the triangles PBA' and $A'CR$ are similar.

(c) Find the length of $A'R$.

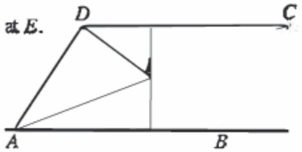


11B.3 HKCEE MA 2003 – I – 8

The figure shows a parallelogram $ABCD$. The diagonals AC and BD cut at E.

(a) Prove that the triangles ABC and CDA are congruent.

(b) Write down all other pairs of congruent triangles.



11B.4 HKCEE MA 2009 – I – 11

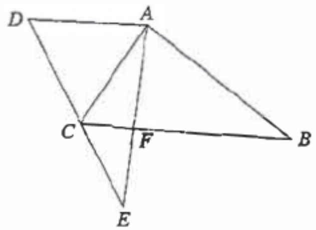
In the figure, C is a point lying on DE. AE and BC intersect at F. It is given that $AC = AD$, $BC = DE$ and $\angle BCE = \angle CAD$.

(a) Prove that $\triangle ABC \cong \triangle AED$.

(b) If $AD // BC$,

  (i) prove that $\triangle ABF \sim \triangle DEA$;

  (ii) write down two other triangles which are similar to $\triangle ABF$.

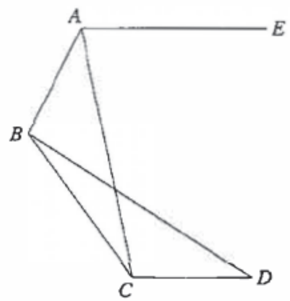


11B.5 HKCEE MA 2010 – I – 9

In the figure, $AB = CD$, $AE // CD$, $\angle BAE = 108°$ and $\angle BCD = 126°$.

(a) Find $\angle ABC$.

(b) Prove that $\triangle ABC \cong \triangle DCB$.

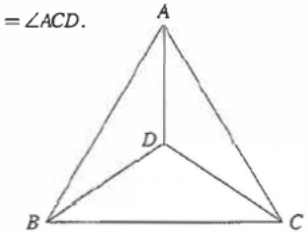


11B.6 HKCEE MA 2011 – I – 9

In the figure, AD is the angle bisector of $\angle BAC$. It is given that $\angle ABD = \angle ACD$.

(a) Prove that $\triangle ABD \cong \triangle ACD$.

(b) If $\angle BAD = 31°$ and $\angle ACD = 17°$, find $\angle CBD$.



11B.7 HKDSE MA 2013 – I – 7

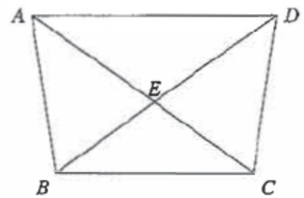
In the figure, $ABCD$ is a quadrilateral. The diagonals AC and BD intersect at E. It is given that $BE = CE$ and $\angle BAC = \angle BDC$.

(a) Prove that $\triangle ABC \cong \triangle DCB$.

(b) Consider the triangles in the figure.

  (i) How many pairs of congruent triangles are there?

  (ii) How many pairs of similar triangles are there?

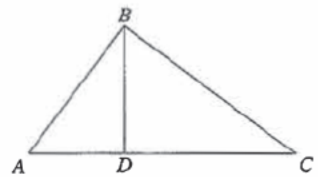


11B.8 HKDSE MA 2014 – I – 9

In the figure, D is a point lying on AC such that $\angle BAC = \angle CBD$.

(a) Prove that $\triangle ABC \sim \triangle BDC$.

(b) Suppose that $AC = 25$ cm, $BC = 20$ cm and $BD = 12$ cm. Is $\triangle BCD$ a right angled triangle? Explain your answer.



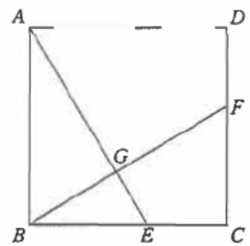
11B.9 HKDSE MA 2015 – I – 13

In the figure, $ABCD$ is a square. E and F are points lying on BC and CD respectively such that $AE = BF$. AE and BF intersect at G.

(a) Prove that $\triangle ABE \cong \triangle BCF$.

(b) Is $\triangle BGE$ a right-angled triangle? Explain your answer.

(c) If $CF = 15$ cm and $EG = 9$ cm, find BG.



11B.10 HKDSE MA 2016 I 13

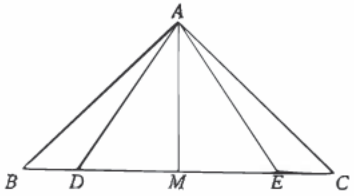
In the figure, ABC is a triangle. D, E and M are points lying on BC such that $BD = CE$, $\angle ADC = \angle AEB$ and $DM = EM$.

(a) Prove that $\triangle ACD \cong \triangle ABE$.

(b) Suppose that $AD = 15$ cm, $BD = 7$ cm and $DE = 18$ cm.

  (i) Find AM.

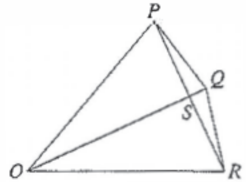
  (ii) Is $\triangle ABE$ a right-angled triangle? Explain your answer.



11B.11 HKDSE MA 2017 – I – 10 (To continue as **12A.31**.)

In the figure, $OPQR$ is a quadrilateral such that $OP = OQ = OR$. OQ and PR intersect at the point S. S is the mid-point of PR.

(a) Prove that $\triangle OPS \cong \triangle ORS$.



11B.12 HKDSE MA 2018 I 13

In the figure, $ABCD$ is a trapezium with $\angle ABC = 90°$ and $AB // DC$. E is a point lying on BC such that $\angle AED = 90°$.

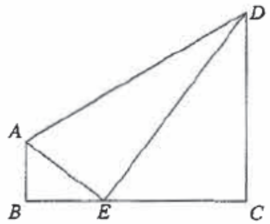
(a) Prove that $\triangle ABE \sim \triangle ECD$.

(b) It is given that $AB = 15$ cm, $AE = 25$ cm and $CE = 36$ cm.

  (i) Find the length of CD.

  (ii) Find the area of $\triangle ADE$.

  (iii) Is there a point F lying on AD such that the distance between E and F is less than 23 cm? Explain your answer.



11B.13 HKDSE MA 2019 I 14

In the figure, $ABCD$ is a square. It is given that E is a point lying on AD. BD and CE intersect at the point F. Let G be a point such that $BG // EC$ and $CG // DB$.

(a) Prove that

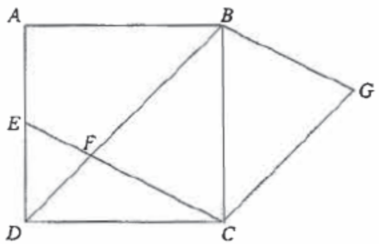
  (i) $\triangle BCG \cong \triangle CBF$,

  (ii) $\triangle BCF \sim \triangle DEF$.

(b) Suppose that $\angle BCF = \angle BGC$.

  (i) Let $BC = \ell$. Express DF in terms of ℓ.

  (ii) Someone claims that $AE > DF$. Do you agree? Explain your answer.



11B.14 HKDSE MA 2020 I 18

In Figure 2, U, V and W are points lying on a circle. Denote the circle by C. TU is the tangent to C at U such that TVW is a straight line.

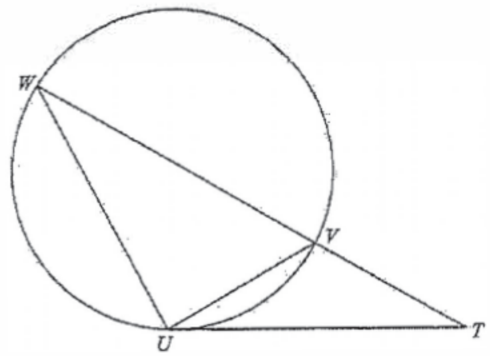


Figure 2

(a) Prove that $\Delta UTV \sim \Delta WTU$. (2 marks)

(b) It is given that VW is a diameter of C. Suppose that $TU = 780$ cm and $TV = 325$ cm.

  (i) Express the circumference of C in terms of π.

  (ii) Someone claims that the perimeter of ΔUVW exceeds 35 m. Do you agree? Explain your answer.

(5 marks)

11 Geometry of Rectilinear Figures

11.1 HKCEE MA 1980(1/1*/3) – I – 1

$x^\circ + 3x^\circ = (2x+40)^\circ$ (ext. $\angle$ of $\triangle$)
$x = 20$

11.2 HKCEE MA 1980(1*) – I – 15

(a) In $\triangle EMC$ and $\triangle ADC$,
$x = y$ (given)
$\angle ECM = \angle ACD$ (common)
$\angle MEC = \angle DAC$ ($\angle$ sum of $\triangle$)
$\therefore \triangle EMC \sim \triangle ADC$ (AAA)
Hence, $\dfrac{EM}{AD} = \dfrac{EC}{AC} = \dfrac{1}{3}$ (corr. sides, $\sim \triangle$s)
$EM = \dfrac{1}{3}AD$
$= \dfrac{1}{3}\left(\dfrac{3}{4}AB\right) = \dfrac{1}{4}AB$

(b) $\because x = y$ (given)
$\therefore AB // EM$ (corr. $\angle$s equal)
In $\triangle BDP$ and $\triangle EMP$,
$\angle BPD = \angle EPM$ (vert. opp. $\angle$s)
$\angle PBD = \angle PEM$ (alt. $\angle$s, $AB//EM$)
$BD = EM = \dfrac{1}{4}AB$ (proved)
$\therefore \triangle BDP \cong \triangle EMP$ (AAS)

(c) $PD = PM$ (corr. sides, $\cong \triangle$s)
$\dfrac{CM}{CD} = \dfrac{EC}{AC} = \dfrac{1}{3}$ (corr. sides, $\sim\triangle$s)
$\Rightarrow DM = \dfrac{2}{3}CD = 2CM$
$\therefore PM = CM$ $(= PD)$

(d) $\because PM = CM$ (proved)
$\therefore$ Area of $\triangle EMP$ = Area of $\triangle EMC$
$\because \triangle BDP \cong \triangle EMP$ (proved)
$\therefore$ Area of $\triangle BDP$ = Area of $\triangle EMP$
Hence, Area of $\triangle BDP = \dfrac{1}{2}$Area of $\triangle PEC$

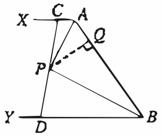
11.3 HKCEE MA 1981(2) – I – 14

(a) $\angle XAB + \angle YBA = 180^\circ$ (int. $\angle$s, $XA//YB$)
$2\angle PAB + 2\angle PBA = 180^\circ$ (given)
$\angle PAB + \angle PBA = 90^\circ$
$\therefore$ In $\triangle ABP$,
$\angle APB = 180^\circ - (\angle PAB + \angle PBA)$ ($\angle$ sum of $\triangle$)
$= 180^\circ - 90^\circ$ (proved)
$= 90^\circ$

(b) Let Q be on AB such that $\angle APQ = \angle APC$.
In $\triangle APC$ and $\triangle APQ$,
$AP = AP$ (common)
$\angle CAP = \angle QAP$ (given)
$\angle APC = \angle APQ$ (by construction)
$\therefore \triangle APC \cong \triangle APQ$ (AAS)
$\therefore CP = PQ$ (corr. sides, $\cong \triangle$s)
Besides,
$\angle QPB = 90^\circ - \angle APQ = 90^\circ \quad \angle APC$ (corr. $\angle$s, $\cong \triangle$s)
$\Rightarrow \angle DPB = 180^\circ - 90^\circ \quad \angle APC$ (adj. $\angle$s on st. line)
$= 90^\circ - \angle APC$
$= \angle QPB$

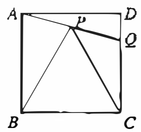
$\therefore$ In $\triangle BPD$ and $\triangle BPQ$,
$PB = PB$ (common)
$\angle PBD = \angle QBP$ (given)
$\angle DPB = \angle QPB$ (proved)
$\therefore \triangle BPD \cong \triangle BPQ$ (AAS)
$PD \quad PQ$ (corr. sides, $\cong \triangle$s)
$\therefore CP = DP$ $(= PQ)$

(c) $\because AC = AQ$ (corr. sides, $\cong \triangle$s)
$BD = BQ$ (corr. sides, $\cong \triangle$s)
$\therefore AC + BD = AQ + BQ = AB$



11.4 HKCEE MA 1988 – I – 8(a)

(i)



(ii) $\angle ABC = 90^\circ$ (property of square)
$\angle PBC = 60^\circ$ (property of equil $\triangle$)
$\Rightarrow \angle ABP = 90^\circ - 60^\circ = 30^\circ$
$AB = BC$ (property of square)
$= BP$ (property of equi $\triangle$)
$\Rightarrow \angle PAB = \angle APB$ (base $\angle$s, isos. $\triangle$)
$= (180^\circ - 30^\circ) \div 2 = 75^\circ$ ($\angle$ sum of $\triangle$)
$\angle PQC = 180^\circ - \angle PAB = 105^\circ$ (int. $\angle$s, $AB//DC$)

11.5 HKCEE MA 1993(I) – I – 1(c)

$\dfrac{x}{7} = \dfrac{3}{5}$ (intercept thm) $\Rightarrow$ $x = \dfrac{21}{5}$

11.6 HKCEE MA 1995 – I – 1(c)

Required $\angle = (8-2)180^\circ \div 8 = 135^\circ$ ($\angle$ sum of polygon)

11.7 HKCEE MA 1995 – I – 1(d)

$AB = DC = 5$ and $\angle A = 90^\circ$ (property of rectangle)
$\therefore BD = \sqrt{AB^2 + AD^2} = 13$ (Pyth. thm)

11.8 HKCEE MA 1996 – I – 10

(a) $x = 360^\circ - 80^\circ \quad 60^\circ - 80^\circ - 75^\circ = 65^\circ$
(sum of ext. $\angle$s of polygon)

(b) $\triangle ABE$ and $\triangle CDB$ (SAS)

(c) In $\triangle ABE$, $y + z = 80^\circ$ (ext. $\angle$ of $\triangle$)
$\because \triangle ABE \cong \triangle CDB$
$\therefore \angle CDB = y$ (corr. $\angle$s, $\cong \triangle$s)
$BD = BE$ (corr. sides, $\cong \triangle$s)
$\angle BDE = \angle BED$ (base $\angle$s, isos. $\triangle$)
$= 180^\circ - z \quad (65^\circ)$ (adj. $\angle$s on st. line)
$= 115^\circ - z$
$\therefore \angle CDB + \angle BDE + 75^\circ = 180^\circ$ (adj. $\angle$s on st. line)
$y + (115^\circ - z) + 75^\circ = 180^\circ$
$z \quad y = 10^\circ$
Hence, $\begin{cases} z - y = 10^\circ \\ y + z = 80^\circ \end{cases} \Rightarrow \begin{cases} y = 35^\circ \\ z = 45^\circ \end{cases}$
$\therefore$ In $\triangle BDE$, $\theta = 180^\circ \quad 2\angle BED$ ($\angle$ sum of $\triangle$)
$= 180^\circ \quad 2(115^\circ \quad z) = 40^\circ$

11.9 HKCEE MA 1998 – I – 2

$x = 180 \quad 120 = 60$ (adj. $\angle$s on st. line)
$y = (4-2)180 - 80 - 140 - x$ ($\angle$ sum of polygon)
$= 80$

11.10 HKCEE MA 1999 – I – 14

(a) $\angle ABE \; 180^\circ - \angle ABD$ (adj. $\angle$s on st. line)
$180^\circ - \angle CDB$ (alt. $\angle$s, $AB//DC$)
$\angle CDF$ (adj. $\angle$s on st. line)

(b) In $\triangle ABE$ and $\triangle CDF$,
$AB = CD$ (property of //gram)
$EB = FC$ (given)
$\angle ABE = \angle CDF$ (proved)
$\therefore \triangle ABE \cong \triangle CDF$ (SAS)
$\Rightarrow \angle E = \angle F$ (corr. $\angle$s, $\cong \triangle$s)
$\Rightarrow EA//CF$ (alt. $\angle$s equal)

11.11 HKCEE MA 2000 – I – 13

(a) $\angle A = \angle ABC = \angle BCD$ (given)
$= (5 \quad 2)180^\circ \div 5$ ($\angle$ sum of polygon)
$= 108^\circ$
$\angle GCD = 90^\circ$ (property of square)
$\Rightarrow \angle BCG = 108^\circ - 90^\circ = 18^\circ$
$BC = CD = CG$ (given)
$\angle GBC = \angle BGC$ (base $\angle$s, isos. $\triangle$)
In $\triangle BCG$, $\angle GBC = (180^\circ - \angle BCG) \div 2$ ($\angle$ sum of $\triangle$)
$= 81^\circ$
$\angle ABP = 108^\circ \quad 81^\circ = 27^\circ$
$\angle APB = 180^\circ - \angle A - \angle ABP = 45^\circ$ ($\angle$ sum of $\triangle$)

11.12 HKCEE MA 2002 – I – 10

(a) In $\triangle ABC$, $\angle B = \angle C$ (base $\angle$s, isos. $\triangle$)
$= (180^\circ - 20^\circ) \div 2$ ($\angle$ sum of $\triangle$)
$= 80^\circ$
In $\triangle CBE$, $\angle E = \angle B = 80^\circ$ (base $\angle$s, isos. $\triangle$)
$\therefore \angle ECB = 180^\circ - 2(80^\circ)$ ($\angle$ sum of $\triangle$)
$= 20^\circ$
$\therefore \angle ECF = 80^\circ \quad 20^\circ = 60^\circ$
Thus, $\triangle CEF$ is equilateral. $\Rightarrow \angle CEF = 60^\circ$

(b) $\angle EDF = \angle DEF$ (base $\angle$s, isos. $\triangle$)
$= 180^\circ - \angle CEF - \angle BEC$ (adj. $\angle$s on st. line)
$= 40^\circ$
$\angle DFA = 40^\circ - \angle A = 20^\circ$ (ext. $\angle$ of $\triangle$)
$\because \angle DFA = \angle DAF = 20^\circ$ (proved)
$\therefore AD = DF$ (sides opp. equal $\angle$s)

11.13 HKCEE MA 2004 – I – 12

(a) (i) $\angle AEF = \angle CED$ (vert. opp. $\angle$s)
$= \angle CDE$ (base $\angle$s, isos. $\triangle$)
$= 36^\circ$

(ii) $\angle ABC = \angle ACB$ (base $\angle$s, isos. $\triangle$)
$= \angle CDE + \angle CED$ (ext. $\angle$ of $\triangle$)
$= 72^\circ$
$\therefore \angle BAC = 180^\circ \quad 2(72^\circ) = 36^\circ$ ($\angle$ sum of $\triangle$)

(b) (i) $\because \angle FAE = \angle AEF = 36^\circ$ (proved)
$\therefore AF = FE$ (sides opp. equal $\angle$s)
$\because AF = FB,\ FE = FB$ (given)
$\therefore \angle EFB = \angle A + \angle AEF = 72^\circ$ (ext. $\angle$ of $\triangle$)
$\angle FEB = \angle FBE$ (base $\angle$s, isos. $\triangle$)
$= (180^\circ - \angle EFB) \div 2 = 54^\circ$
Hence, $\angle AEB = \angle AEF + \angle FEB = 36^\circ + 54^\circ = 90^\circ$

(ii) $AC = AB = \dfrac{AE}{\cos\angle A} = \dfrac{10}{\cos 36^\circ}$
$BE = AE\tan\angle A = 10\tan 36^\circ$
$\therefore$ Area of $\triangle ABC = \dfrac{1}{2}AC \cdot BE = 44.9$ (cm², 3.s.f.)

11.14 HKCEE MA 2005 – I – 8

$x = (6-2)180 \div 6 = 120$ ($\angle$ sum of polygon)
In $\triangle ABC$, $\angle B = 120^\circ$
$\because AB = BC$ (given)
$\therefore y^\circ = \angle BAC$ (base $\angle$s, isos.$\triangle$)
$y = (180 - \angle B) \div 2$ ($\angle$ sum of $\triangle$)
$= 30$
$\angle ABG = \angle BAG = 30^\circ$
$z^\circ = \angle AGB$ (vert. opp. $\angle$s)
$z = 180 - 30 \quad 30 = 120$ ($\angle$ sum of $\triangle$)

11.15 HKCEE MA 2006 – I – 5

$\angle ABE = \angle AEB$ (base $\angle$s, isos. $\triangle$)
$= \angle CBE = 70^\circ$ (alt. $\angle$s, $BC//AD$)
$\angle BCD = 180^\circ - \angle ABC$ (int. $\angle$s, $AB//DC$)
$= 180^\circ - (70^\circ - 70^\circ) = 40^\circ$

11.16 HKCEE MA 2007 – I – 8

$x = 180^\circ - 110^\circ = 70^\circ$ (adj. $\angle$s on st. line)
$\angle CBF = z$ (base $\angle$s, isos. $\triangle$)
$\angle EBC = 110^\circ$ (alt. $\angle$s, $AC//DF$)
$z = 110^\circ \quad 90^\circ = 20^\circ$
$y = 180^\circ - 90^\circ - x = 20^\circ$ ($\angle$ sum of $\triangle$)

11.17 HKCEE MA 2008 – I – 9

$x = 33^\circ$ (alt. $\angle$s, $CD//AB$)
$y = 43^\circ + x = 76^\circ$ (ext. $\angle$ of $\triangle$)
$\angle ACE = y = 76^\circ$ (base $\angle$s, isos. $\triangle$)
$z = 180^\circ - \angle ACE - y = 28^\circ$ ($\angle$ sum of $\triangle$)

11.18 HKDSE MA 2020 – I – 8

8a
$AB = BE$ (given)
$\angle AEB = \angle BAE$ (base $\angle$s, isos. Δ)
$\angle AEB = 30^\circ$
$\angle ADB = \angle BED + \angle DBE$ (ext. $\angle$ of Δ)
$42^\circ = 30^\circ + \angle DBE$
$\angle DBE = 12^\circ$
$\angle BEC = \angle DBE$ (alt. $\angle$s, $BD // CE$)
$= 12^\circ$

b
$\angle DCE = \angle BDC$ (alt. $\angle$s, $BD // CE$)
$= \theta$
$\angle CEF + \angle CFE + \angle ECF = 180^\circ$ ($\angle$ sum of Δ)
$12^\circ + \angle CFE + \theta = 180^\circ$
$\theta = 168^\circ - \theta$

11B Congruent and similar triangles

11B.1 HKCEE MA 1982(2) – I – 13

(a) $\angle DAB = \angle EAC = 60°$ (property of equil. $\triangle$)
$\angle DAB + \angle BAC = \angle EAC + \angle BAC$
$\angle DAC = \angle BAE$
In $\triangle ADC$ and $\triangle ABE$,
$DA = BA$ (property of equil. $\triangle$)
$\angle DAC = \angle BAE$ (proved)
$AC = AE$ (property of equil. $\triangle$)
$\therefore \triangle ADC \cong \triangle ABE$ (SAS)
$\therefore DC = BE$ (corr. sides, $\cong \triangle$s)

11B.2 HKCEE MA 2001 – I – 11

(a) $PA' = PA = x$ cm
In $\triangle PBA'$, $x^2 = PB^2 + BA'^2$ (Pyth. thm)
$x^2 = (12-x)^2 + (12 \div 2)^2$
$x^2 = 144 - 24x + x^2 + 36 \Rightarrow x = 7.5$

(b) In $\triangle PBA'$ and $\triangle A'CR$,
$\angle B = \angle C = 90°$ (given)
$\angle BPA' = 180° - \angle B - \angle PA'B$ ($\angle$ sum of $\triangle$)
$= 90° - \angle PA'B$
$\angle CA'R = 180° - \angle PA'R - \angle PA'B$ (adj. $\angle$s on st. line)
$= 90° - \angle PA'B$
$\Rightarrow \angle BA'P = \angle CA'R$
$\angle BA'P = \angle CRA'$ ($\angle$ sum of $\triangle$)
$\therefore \triangle PBA' \sim \triangle A'CR$ (AAA)

(c) $\dfrac{PA'}{PB} = \dfrac{A'R}{A'C}$ (corr. sides, $\sim \triangle$s)
$\dfrac{7.5}{12-7.5} = \dfrac{A'R}{6} \Rightarrow A'R = 10$ (cm)

11B.3 HKCEE MA 2003 – I – 8

(a) In $\triangle ABC$ and $\triangle CDA$,
$AB = CD$ (property of //gram)
$BC = DA$ (property of //gram)
$AC = CA$ (common)
$\therefore \triangle ABC \cong \triangle CDA$ (SSS)

(b) $\triangle ABD \cong \triangle CDB$, $\triangle ABE \cong \triangle CDE$, $\triangle ADE \cong \triangle CBE$

11B.4 HKCEE MA 2009 – I – 11

(a) $\angle ADC = \angle ACE - \angle CAD$ (ext. $\angle$ of $\triangle$)
$= \angle ACE - \angle BCE$ (given)
$= \angle ACB$
In $\triangle ABC$ and $\triangle AED$,
$AC = AD$ (given)
$BC = ED$ (given)
$\angle ACB = \angle ADE$ (proved)
$\therefore \triangle ABC \cong \triangle AED$ (SAS)

(b) (i) In $\triangle ABF$ and $\triangle DEA$,
$\angle AFB = \angle DAE$ (alt. $\angle$s, $AD // BC$)
$\angle ABF = \angle DEA$ (corr. $\angle$s, $\cong \triangle$s)
$\angle BAF = \angle EDA$ ($\angle$ sum of $\triangle$)
$\therefore \triangle ABF \sim \triangle DEA$ (AAA)

(ii) $\triangle CEF$, $\triangle CBA$

11B.5 HKCEE MA 2010 – I – 9

(a) $\angle EAC + \angle ACD = 180°$ (int. $\angle$s, $AE // CD$)
In $\triangle ABC$, $\angle ABC + \angle BAC + \angle BCA = 180°$ ($\angle$ sum of $\triangle$)
$\angle ABC + (108° - \angle EAC) + (126° - \angle ACD) = 180°$
$\angle ABC + 234° - (180°) = 180°$ (proved)
$\angle ABC = 126°$

(b) In $\triangle ABC$ and $\triangle DCB$,
$AB = DC$ (given)
$\angle ABC = \angle DCB = 126°$ (proved)
$BC = CB$ (common)
$\therefore \triangle ABC \cong \triangle DCB$ (SAS)

11B.6 HKCEE MA 2011 – I – 9

(a) In $\triangle ABD$ and $\triangle ACD$,
$\angle BAD \quad \angle CAD$ (given)
$AD = AD$ (common)
$\angle ABD = \angle ACD$ (given)
$\therefore \triangle ABD \cong \triangle ACD$ (ASA)

(b) $\angle CAD = \angle BAD = 31°$ (given)
In $\triangle ACD$,
$\angle ADC = 180° - 31° - 17° = 132°$ ($\angle$ sum of $\triangle$)
$\angle ADB = \angle ADC = 132°$ (corr. $\angle$s, $\cong \triangle$s)
$DB = DC$ (corr. sides, $\cong \triangle$s)
$\angle BDC = 360° - 132° - 132° = 96°$ ($\angle$s at a pt)
$\angle CBD = \angle BCD$ (base $\angle$s, isos. $\triangle$)
$= (180° - 96°) \div 2 = 42°$ ($\angle$ sum of $\triangle$)

11B.7 HKDSE MA 2013 – I – 7

(a) $\because BE = CE$ (given)
$\therefore \angle BCE = \angle CBE$ (base $\angle$s, isos. $\triangle$)
In $\triangle ABC$ and $\triangle DCB$,
$\angle BAC = \angle BDC$ (given)
$\angle ACB = \angle DCB$ (proved)
$BC = CB$ (common)
$\therefore \triangle ABC \cong \triangle DCB$ (AAS)

(b) (i) 3 ($\triangle ABC \cong \triangle DCB$, $\triangle ABE \cong \triangle DCE$, $\triangle ABD \cong \triangle DCA$)
(ii) 4 (the 3 in (i) and $\triangle ADE \sim \triangle CBE$)

11B.8 HKDSE MA 2014 – I – 9

(a) In $\triangle ABC$ and $\triangle BDC$,
$\angle C = \angle C$ (common)
$\angle BAC = \angle DBC$ (given)
$\angle ABC = \angle BDC$ ($\angle$ sum of $\triangle$)
$\therefore \triangle ABC \sim \triangle BDC$ (AAA)

(b) $\dfrac{AC}{BC} = \dfrac{BC}{DC}$ (corr. sides, $\sim \triangle$s)
$\dfrac{25}{20} = \dfrac{20}{DC}$
$DC = 16$
$BC^2 = 20^2 = 400$
$BD^2 + CD^2 = 12^2 + 16^2 = 400 = BC^2$
$\therefore \triangle BCD$ is a right-$\angle$ed $\triangle$. (converse of Pyth. thm)

11B.9 HKDSE MA 2015 – I – 13

(a) In $\triangle ABE$ and $\triangle BCF$,
$AB = BC$ (property of square)
$\angle B = \angle C = 90°$ (property of square)
$AE = BF$ (given)
$\therefore \triangle ABE \cong \triangle BCF$ (RHS)

(b) $\angle AEB = \angle BFC$ (corr. sides, $\cong \triangle$s)
In $\triangle BEG$,
$\angle BGE = 180° \quad \angle GBE \quad \angle GEB$ ($\angle$ sum of $\triangle$)
$= 180° - \angle GBE - \angle BFC$ (proved)
$= \angle BCF = 90°$ ($\angle$ sum of $\triangle$)
$\therefore$ YES.

(c) $BE = CF = 15$ cm (corr. sides, $\cong \triangle$s)
$BG = \sqrt{BE^2 - EG^2} = 12$ cm (Pyth. thm)

11B.10 HKDSE MA 2016 – I – 13

(a) $DE = ED$ (common)
$BD + DE = CE + ED$ (given)
$BE = CD$
In $\triangle ACD$ and $\triangle ABE$,
$BE = CD$ (proved)
$\angle AEB = \angle ADC$ (given)
$AE = AD$ (sides opp. equal $\angle$s)
$\therefore \triangle ACD \cong \triangle ABE$ (SAS)

(b) (i) $\because DM = EM$ (given)
$\therefore AM \perp DE$ (property of isos. $\triangle$)
$AM = \sqrt{AD^2 - (DE \div 2)^2} = 12$ (cm) (Pyth. thm)
(ii) $AB = \sqrt{AM^2 + BM^2} = 20$ (cm) (Pyth. thm)
$BE^2 = 25^2 = 625$
$AB^2 + AE^2 = AB^2 + AD^2$ (corr. sides, $\cong \triangle$s)
$= 20^2 + 15^2 = 625 = BE^2$
$\therefore$ YES. (converse of Pyth. thm)

11B.11 HKDSE MA 2017 – I – 10

(a) $\because OP = OR$ and $PS = RS$ (given)
$\therefore OS \perp PR$ (property of isos. $\triangle$)
In $\triangle OPS$ and $\triangle ORS$,
$OP = OR$ (given)
$OS = OS$ (common)
$\angle OSP = \angle OSR$ (proved)
$\therefore \triangle OPS \cong \triangle ORS$ (RHS)

11B.12 HKDSE MA 2018 – I – 13

(a) $\angle C = 180° \quad \angle B = 90°$ (int. $\angle$s, $AB // DC$)
$\angle BAE = 180° - \angle ABE - \angle AEB$quad ($\angle$ sum of $\triangle$)
$= 90° \quad \angle AEB$
$\angle CED = 180° - \angle AED - \angle AEB$ (adj. $\angle$s on st. line)
$= 90° \quad \angle AEB$
$\therefore BAE = \angle CED$
In $\triangle ABE$ and $\triangle ECD$,
$\angle B = \angle C = 90°$ (proved)
$\angle BAE = \angle CED$ (proved)
$\angle BEA = \angle CDE$ ($\angle$ sum of $\triangle$)
$\therefore \triangle ABE \sim \triangle ECD$ (AAA)

(b) (i) $BE = \sqrt{AE^2 - AB^2} = 20$ cm (Pyth. thm)
$\dfrac{AB}{BE} = \dfrac{EC}{CD}$ (corr. sides, $\sim \triangle$s)
$\dfrac{15}{20} = \dfrac{36}{CD}$
$CD = 48$ cm

(ii) $DE = \sqrt{CD^2 + CE^2} = 60$ cm (Pyth. thm)
Area of $\triangle ADE = \dfrac{1}{2}(25)(60) = 750$ (cm²)
(iii) $AD = \sqrt{25^2 + 60^2} = 65$ (cm) (Pyth. thm)
Let ℓ cm be the shortest distance from E to AD.
$\dfrac{AD \cdot \ell}{2} =$ Area of $\triangle ADE$
$\ell = 2 \times 750 \div 65$
$= 23.077 > 23$
$\therefore$ NO.

11B.13 HKDSE MA 2020 – I – 18

18a
$\angle TUV = \angle TWU$ ($\angle$ in alt. segment)
$\angle UTV = \angle WTU$ (common $\angle$)
$\angle UVT = \angle WUT$ (3rd $\angle$ of $\triangle$)
$\triangle UTV \sim \triangle WTU$ (A.A.A.)

bi
$\triangle UTV \sim \triangle WTU$ (from (a))
$\dfrac{TU}{TW} = \dfrac{TV}{TU}$ (corr. sides, ~$\triangle$s)
$\dfrac{TU}{TV + VW} = \dfrac{TV}{TU}$
$\dfrac{780}{325 + VW} = \dfrac{325}{780}$
$VW = 1547$ cm
The circumference of $C = \pi(1547)$
$= 1547\pi$ cm

ii
$\triangle UTV \sim \triangle WTU$ (from (a))
$\dfrac{UV}{WU} \quad \dfrac{TV}{TU}$ (corr. sides, ~$\triangle$s)
$\dfrac{UV}{UW} = \dfrac{325}{780}$
$UV = \dfrac{5}{12}UW$
$\angle VUW = 90°$ ($\angle$ in semi-circle)
$UV^2 + UW^2 = VW^2$ (Pyth. Thm.)
$\left(\dfrac{5}{12}UW\right)^2 + UW^2 \quad 1547^2$
$UW = 1428$ cm
The perimeter of $\triangle UVW = UV + UW + VW$
$= \dfrac{5}{12}UW + UW + VW$
$= \dfrac{5}{12}(1428) + 1428 + 1547$
$= 3570$ cm
$= 35.7$ m
> 35 m
Therefore, the perimeter of $\triangle UVW$ exceeds 35 m.
The claim is agreed with.