Mathematics Compulsory Part Paper 1

Solution	Marks	Remarks
1. $k=\frac{3x-y}{y}$ $yk=3x-y$ $y=\frac{3x}{k+1}$		
2. $\frac{(m^4n^{-1})^3}{(m^{-2})^5}$ $=\frac{m^{12}n^{-3}}{m^{-10}}$ $=\frac{m^{22}}{n^3}$		
3. (a) $x^2-4xy+3y^2=(x-3y)(x-y)$ (b) $x^2-4xy+3y^2+11x-33y$ $=(x-3y)(x-y)+11(x-3y)$ $=(x-3y+11)(x-y)$		
4. Let x, y be the number of regular tickets and concessionary tickets sold respectively $\begin{cases}x=5y\\126x+78y=50976\end{cases}$ $126x+78\left(\frac{x}{5}\right)=50976$ $\begin{cases}x=360\\y=72\end{cases}$ The number of admission tickets sold that day $=432$		
5. (a) $7(x-2)\le\frac{11x+8}{3}$ and $6-x<5$ $21(x-2)\le 11x+8$ and $x>1$ $x\le 5$ and $x>1$ $\therefore 1<x\le 5$ (b) 2, 3, 4, 5 are the only integers satisfying (a) So, there are 4 integers satisfying both inequalities in (a)		

Solution	Marks	Remarks

6. (a) The coordinates of $A' = (-4, -3)$

The coordinates of $B' = (9, 9)$

(b) Slope of AB

$= \frac{-9-4}{9-(-3)}$

$= -\frac{13}{12}$

Slope of A'B'

$= \frac{-3-9}{-4-9}$

$= \frac{12}{13}$

Slope of AB × Slope of A'B'

$= -\frac{13}{12} \times \frac{12}{13}$

$= -1$

$\therefore AB \perp A'B'$

7. (a) $\frac{x}{360°} = \frac{1}{9}$

$x = 40°$

(b) Let N be the number of students in the school

$\frac{180}{N} = \frac{360° - 90° - 40° - 158°}{360°}$

$N = 900$

The number of students in the school

$= 900$

8. (a) $y = \frac{k}{\sqrt{x}}$

When $y = 81$, $x = 144$

$81 = \frac{k}{\sqrt{144}}$

$k = 972$

$y = \frac{972}{\sqrt{x}}$

(b) Change in value of y

$= \frac{972}{\sqrt{324}} - \frac{972}{\sqrt{144}}$

$= -27$

Solution	Marks	Remarks
9. (a) Let x mL be the actual capacity of a standard bottle		
$200-\frac{10}{2}\le x<200+\frac{10}{2}$		
$195\le x<205$		
The least possible capacity is 195 mL		
(b) $23400\le 120x<24600$		
$23.4L\le 120x<24.6L$		
Least total capacity $=23.4\ L$		
No, I don't agree the claim.		
10. (a) $\text{OP}=\text{OR}(given)$		
$PS=RS\,(given)$		
$OS=OS\,(common)$		
$\therefore \Delta OPS\cong\Delta ORS\,(SSS)$		
(b) $\angle POQ=10°$		
$\angle POQ=\angle QOR=10°$		
$\angle POR=20°$		
Area of the sector OPQR		
$=\frac{20°}{360°}\times\pi\times 6^2$		
$=2\pi\text{ cm}^2$		
11. (a) $\frac{895+70+a+80+b}{15}=70$		
$a+b=5$		
$80+b-61=22$		
$b=3$		
$a=2$		
median $=\$69$		
standard deviation		
$=\$7.330302404$		
$\approx\$7.33$ (corr to 3 sig fig)		

Solution	Marks	Remarks
12. (a) Let V_1 cm^3 and V_2 cm^3 be the volume of smaller right pyramid and larger right pyramid respectively $\frac{V_1}{V_2}$ $=\left(\sqrt{\frac{4}{9}}\right)^3$ $=\frac{8}{27}$ V_1+V_2 $=(84)(20)$ $=1680$ V_2 $=1680\times\frac{27}{27+8}$ $=1296$ The volume of the larger pyramid $=1296\text{ cm}^3$ (b) Volume of smaller pyramid $=384$ cm^3 Height of smaller pyramid $=\frac{2}{3}\times 12$ $=8$ $\frac{1}{3}$(Base Area)(8) $=384$ Base Area $=144\text{cm}^2$ Length of square in base $=\sqrt{144}$ $=12$ Total surface area $=\frac{1}{2}(12)\sqrt{\left(\frac{12}{2}\right)^2+8^2}\times 4+144$ $=384\text{ cm}^2$		

Solution	Marks	Remarks

13. (a) Let the equation of C be $(x-4)^2+(y+1)^2=r$, where r is a real constant

Since C passes through $(-6,5)$,

So, we have $(-6-2)^2+(5+1)^2=r$

$r=100$

The equation of C: $(x-4)^2+(y+1)^2=100$

(b) Radius of $C=10$

GF

$=\sqrt{(2-(-3))^2+(-1-11)^2}$

$=13$

>10

F lies outside C

(c) (i) F, G, H are collinear

(ii) The equation of straight line which passes through F and H:

$$\frac{y-11}{x+3}=\frac{11-(-1)}{-3-2}$$

$$y=-\frac{12}{5}x+\frac{19}{5}$$

$$12x+5y-19=0$$

Solution	Marks	Remarks
14. (a) $f(x)$ $= (3x+7)(2x^2+ax+4)+bx+c$ $= 6x^3+(3a+14)x^2+(12+7a+b)x+28+c$ $\equiv 6x^3-13x^2-46x+34$ $3a+14=-13$ $a=-9$ (b) (i) Since $g(x)$ is a quadratic polynomial, degree of quotient when $g(x)$ is divided by $2x^2+ax+4$ is 0 $g(x)=A(2x^2-9x+4)+bx+c$, where A is a constant $f(x)-g(x)$ $=(3x+7)(2x^2+ax+4)+bx+c-(A(2x^2-9x+4)+bx+c)$ $=(3x+7-A)(2x^2+ax+4)$ $\therefore$ $f(x)-g(x)$ is divisible by $2x^2+ax+4$ (b) (ii) $f(x)-g(x)=0$ $(3x+7-A)(2x^2-9x+4)=0$ $x=\frac{A-7}{3}$ or $x=4$ or $x=\frac{1}{2}$ Since $\frac{1}{2}$ is not an integer, I don't agree the claim		

Solution	Marks	Remarks

15. $\begin{cases} 0 = a + \log_b 9 \\ 3 = a + \log_b 243 \end{cases}$

$3 = \log_b 243 - \log_b 9$

$3 = \log_b \dfrac{243}{9}$

$b^3 = 27$

$b = 3$

$a = -2$

$y = -2 + \log_3 x$

$y + 2 = \log_3 x$

$x = 3^{y+2}$

16. (a) The total volume of water imported

$= 1.5\times 10^7 + 0.9\times 1.5\times 10^7 + 0.9^2 \times 1.5\times 10^7 + ... + 0.9^{19}\times 1.5\times 10^7$

$= 1.5\times 10^7 \times (1 + 0.9 + 0.9^2 + ... + 0.9^{19})$

$= 1.5\times 10^7 \times \dfrac{1-0.9^{20}}{1-0.9}$

$= 1.31763501\times 10^8$

$\approx 1.32\times 10^8 \text{ m}^3$ (corr to 3 sig fig)

(b) Since the water imported every year is positive,

The total volume of water imported

$< 1.5\times 10^7 + 0.9\times 1.5\times 10^7 + 0.9^2\times 1.5\times 10^7 + ...$

$= 1.5\times 10^7 \times (1 + 0.9 + 0.9^2 + ...)$

$= 1.5\times 10^7 \times \dfrac{1}{1-0.9}$

$= 1.5\times 10^8$

$< 1.6\times 10^8$

No, I don't agree the claim.

Suppose the total water imported can exceed $1.6\times 10^8 \text{ m}^3$ at n^{th} year

$1.5\times 10^7 + 0.9\times 1.5\times 10^7 + 0.9^2\times 1.5\times 10^7 + ... + 0.9^{n-1}\times 1.5\times 10^7 > 1.6\times 10^8$

$1.5\times 10^7 \times (1 + 0.9 + 0.9^2 + ... + 0.9^{n-1}) > 1.6\times 10^8$

$\dfrac{1-0.9^n}{1-0.9} > \dfrac{32}{3}$

$0.9^n < -\dfrac{1}{15}$ which is impossible

No, I don't agree the claim.

Solution	Marks	Remarks
17. (a) Required probability		
$=\dfrac{C_4^4 C_1^{15}}{C_5^{19}}$		
$=\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{2}{17}\right)\left(\dfrac{1}{16}\right)\left(\dfrac{15}{15}\right)C_4^5$		
$=\dfrac{5}{3876}$		r.t. 0.00129
(b) Required probability		
$=\dfrac{C_3^4 C_2^{15}}{C_5^{19}}$		
$=\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{2}{17}\right)\left(\dfrac{15}{16}\right)\left(\dfrac{14}{15}\right)C_3^5$		
$=\dfrac{35}{969}$		r.t. 0.0361
(c) Required probability		
$=1-\dfrac{5}{3876}-\dfrac{35}{969}$		
$=\left(\dfrac{15}{19}\right)\left(\dfrac{14}{18}\right)\left(\dfrac{13}{17}\right)\left(\dfrac{12}{16}\right)\left(\dfrac{11}{15}\right)C_0^5+\left(\dfrac{4}{19}\right)\left(\dfrac{15}{18}\right)\left(\dfrac{14}{17}\right)\left(\dfrac{13}{16}\right)\left(\dfrac{12}{15}\right)C_1^5$ $+\left(\dfrac{4}{19}\right)\left(\dfrac{3}{18}\right)\left(\dfrac{15}{17}\right)\left(\dfrac{14}{16}\right)\left(\dfrac{13}{15}\right)C_2^5$		
$=\dfrac{3731}{3876}$		r.t. 0.963

Solution	Marks	Remarks

18. (a) $\begin{cases} y = 19 \\ y = 2x^2 - 2kx + 2x - 3k + 8 \end{cases}$

$19 = 2x^2 - 2kx + 2x - 3k + 8$

$2x^2 - 2kx + 2x - 3k - 11 = 0$

Δ

$= (-2k+2)^2 - 4(2)(-3k-11)$

$= 4k^2 + 16k + 92$

$= 4(k+2)^2 + 76$

≥ 76 , for all real values of k

$\therefore \Delta > 0$, for all real values of k

L and Γ intersect at two distinct points.

(b) (i) a, b are the roots of $2x^2 - 2kx + 2x - 3k - 11 = 0$

ab

$= \dfrac{-3k-11}{2}$

$= -\dfrac{3k+11}{2}$

$a + b$

$= -\dfrac{-2k+2}{2}$

$= k - 1$

$(a-b)^2$

$= a^2 + b^2 - 2ab$

$= (a+b)^2 - 4ab$

$= (k-1)^2 - 4(-\dfrac{3k+11}{2})$

$= k^2 - 2k + 1 + 6k + 22$

$= k^2 + 4k + 23$

(ii) AB

$= \sqrt{(a-b)^2}$

$= \sqrt{k^2 + 4k + 23}$

$= \sqrt{(k+2)^2 + 19}$

$\geq \sqrt{19} = 4.358898944$, for all real values of k

No, it is impossible.

Solution	Marks	Remarks
19. (a) $\dfrac{AC}{\sin(180° - 30° - 42°)} = \dfrac{24}{\sin 30°}$ $AC = 45.65071278$ $AC \approx 45.7\text{ cm}$ (corr to 3 sig fig) The length of AC is 45.7 cm		
(b) (i) $\dfrac{CF}{CF + AC} = \dfrac{2}{10}$ $CF = \dfrac{1}{4}AC$ $CF = \dfrac{1}{4} \times 45.65071278$ $CF = 11.4126782$ $CF \approx 11.4\text{ cm}$ (corr to 3 sig fig)		
(ii) Area of $\Delta ABF = \dfrac{1}{2}(AB)(AF)\sin 30°$ $AF = CF + AC = 57.06339098$ $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos 42°$ $AB^2 = (45.65071278)^2 + 24^2 - 2(24)(45.65071278)\cos 42°$ $AB = 32.11826911$ Area of ΔABF $= \dfrac{1}{2}(32.11826911)(57.06339098)\sin 30°$ $= 458.1943369\text{ cm}^2$ $\approx 458\text{ cm}^2$ (corr to 3 sig fig)		

Solution	Marks	Remarks
(iii) $BF^2 = AF^2 + AB^2 - 2(AF)(AB)\cos 30°$ $BF^2 = (57.06339098)^2 + (32.11826911)^2 - 2(57.06339098)(32.11826911)\cos 30°$ $BF = 33.36690449$ Let h be the height of ΔABF with base BF $\frac{1}{2}(h)(BF) = \text{Area of } \Delta ABF$ $\frac{1}{2}(h)(BF) = 458.1943369$ $\frac{1}{2}(h)(33.36690449) = 458.1943369$ $h = 27.46400026$ Let the inclination of the thin metal sheet ABC to horizontal ground be θ $\sin\theta = \frac{10}{h}$ $\sin\theta = \frac{10}{27.46400026}$ $\theta = 21.35300646°$ The inclination of the thin metal sheet ABC to horizontal ground = $21.4°$		
(iv) $BF = 33.36690449$ $BD = \sqrt{AB^2 - 10^2}$ $BD = 30.52184808$ $DF = \sqrt{CF^2 - 10^2}$ $DF = 56.18033989$ Let $s = \frac{BF + BD + DF}{2} = 60.03454623$ Area of ΔBDF $= \sqrt{s(s-BF)(s-BD)(s-DF)}$ $= 426.741482$ < 460 No, I don't agree the claim.		