Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* in each of Section A(1) and Section A(2). Do not deduct any marks for *u* in Section B.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***1 mark*** for *pp* in each of Section A(1) and Section A(2). Do not deduct any marks for *pp* in Section B.
   c. At most deduct 1 mark in each of Section A(1) and Section A(2).
   d. In any case, do not deduct any marks in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Paper 1

Solution	Marks	Remarks
1. $\frac{mk-t}{k+t}=4$		
[redacted]	1M	
$mk-t=4k+4t$		
$mk-4k=5t$	1M	for putting k on one side
$k(m-4)=5t$		
$k=\frac{5t}{m-4}$	1A	or equivalent
	----------(3)	
2. $\frac{x^{65}}{(x^4y^3)^2}$		
[redacted]	1M	for $(ab)^k=a^kb^k$ or $(a^l)^k=$
[redacted]	1M	for $\frac{c^n}{c^m}=c^{n-m}$ or $\frac{1}{c^{-m}}=c^m$
$=\frac{x^{57}}{y^6}$	1A	
	----------(3)	
3. (a) $81m^2-n^2$		
$=(9m+n)(9m-n)$	1A	or equivalent
(b) $81m^2-n^2+18m-2n$		
[redacted]	1M	for using the result of (a)
$=(9m+n)(9m-n)+2(9m-n)$		
$=(9m-n)(9m+n+2)$	1A	or equivalent
	----------(3)	

Solution	Marks	Remarks
4. (a) 8 090	1A	
(b) 8100	1A	
(c) 8 091.190	1A	
	----------(3)	
5. $\frac{(6)(4)+(9)(7)+(12)(10)+15x}{4+7+10+x}=11$	1M+1A	
$207+15x=11(21+x)$		
$4x=24$		
$x=6$	1A	
	----------(3)	
6. Let x be the original number of girls in the summer camp.		pp−1 for any undefined symbol
Then, the original number of boys in the summer camp is $\frac{7x}{6}$.	1A	can be absorbed
$\frac{7x}{6}-17=x-4$	1M+1A	
$7x-102=6x-24$		
$x=78$	1A	
Thus, the original number of girls in the summer camp is 78 .		
Let x and y be the original numbers of girls and boys in the summer camp respectively.		pp−1 for any undefined symbol
$\begin{cases} 7x=6y \\ y-17=x-4 \end{cases}$	1A+1A	
So, we have $\frac{7x}{6}-17=x-4$.	1M	for getting a linear equation in x or y only
Solving, we have $x=78$.	1A	
Thus, the original number of girls in the summer camp is 78 .		
	----------(4)	

Solution	Marks	Remarks
7. (a) The selling price		
$= 360(1-45\%)$	1M	
$= \$198$	1A	u–1 for missing unit
(b) Let $\$x$ be the cost of the birthday cake.		
$(1+80\%)x = 360$	1M	
$x = 200$		
So, the cost of the birthday cake is $\$200$.		
Note that $200 > 198$.		
Thus, there will be a loss after selling the birthday cake.	1A	f.t.
$\dfrac{\text{The selling price}}{\text{The cost}}$		
$= (1-45\%)(1+80\%)$	1M	
$= 0.99$		
< 1		
Thus, there will be a loss after selling the birthday cake.	1A	f.t.
	----------(4)	
8. (a) The coordinates of B are $(-6, -4)$.	1A	pp–1 for missing '(' or ')'
The coordinates of M		
$= \left(\dfrac{-4+(-6)}{2}, \dfrac{6+(-4)}{2}\right)$	1M	
$= (-5, 1)$	1A	pp–1 for missing '(' or ')'
(b) The slope of OM		
$= \dfrac{1-0}{-5-0}$	1M	either one
$= \dfrac{-1}{5}$		
The slope of AB		
$= \dfrac{6-(-4)}{-4-(-6)}$		
$= 5$		
Since the product of the slope of OM and the slope of AB is -1 , OM is perpendicular to AB .	1A	f.t.
Note that $OA = OB$.	1M	
So, ΔOAB is an isosceles triangle.		
Also note that OM is the median of ΔOAB .		
Thus, OM is perpendicular to AB .	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
9. (a) In ΔABD and ΔACD,		
$\angle ABD = \angle ACD$ (given)		
$\angle BAD = \angle CAD$ (given)		
$AD = AD$ (common side)		
$\Delta ABD \cong \Delta ACD$ (AAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	2	
Case 2 Any correct proof without reasons.	1	
(b) $\angle BAC$		
$= \angle BAD + \angle CAD$		
$= 2(31°)$		
$= 62°$	1A	
Note that ΔABC is an isosceles triangle.		
$\angle ABC$		
$= \frac{1}{2}(180° - \angle BAC)$	1M	
$= \frac{1}{2}(180° - 62°)$		
$= 59°$		
Also note that $\angle ABD = \angle ACD$.		
$\angle CBD$		
$= \angle ABC - \angle ABD$		
$= 59° - 17°$		
$= 42°$	1A	u–1 for missing unit
$\angle CAD$		
$= \angle BAD$		
$= 31°$	1A	
$\angle ADC$		
$= 180° - \angle CAD - \angle ACD$		
$= 180° - 31° - 17°$		
$= 132°$		
Note that $\angle ADB = \angle ADC$.		
$\angle BDC$		
$= 360° - 2(\angle ADC)$	1M	
$= 360° - 2(132°)$		
$= 96°$		
Also note that ΔBCD is an isosceles triangle.		
$\angle CBD$		
$= \frac{1}{2}(180° - \angle BDC)$		
$= \frac{1}{2}(180° - 96°)$		
$= 42°$	1A	u–1 for missing unit
	----------(5)	

Solution	Marks	Remarks
10. (a) The median $= 57$	1A	
The range $= 75 - 23$ $= 52$	1A	
The inter-quartile range $= 65 - 43$ $= 22$	1A ----------(3)	
(b) (i) The inter-quartile range $= 67 - 50$ $= 17$	1M	
The inter-quartile range of the distribution of scores in the second survey is less than the first survey. Thus, the distribution of scores in the second survey is less dispersed than the first survey.	1A	f.t.
The range $= 78 - 44$ $= 34$ The range of the distribution of scores in the second survey is less than the first survey. Thus, the distribution of scores in the second survey is less dispersed than the first survey.	1M 1A	 f.t.
(ii) Note that the least score of the distribution in the second survey is 44 which is greater than the lower quartile (43) of the distribution of scores in the first survey. Thus, the claim is agreed.	1M 1A	 f.t.
Note that in the first survey, there are 9 students whose scores are less than the least score (44) in the second survey. Also note that 25% of these students are 8 students. Thus, the claim is agreed.	1M 1A ----------(4)	 f.t.

Solution	Marks	Remarks
11. (a) Let $f(x)=ax^2+bx$ [redacted].	1A	
Since $f(-2)=28$, we have		
$a(-2)^2+b(-2)=28$	1M	for substitution (either one)
$2a-b=14$		
Since $f(6)=-36$, we have		
$a(6)^2+b(6)=-36$		
$6a+b=-6$		
Solving $2a-b=14$ and $6a+b=-6$, we have $a=1$ and $b=-12$.	1A	for both correct
Thus, we have $f(x)=x^2-12x$.		
	----------(3)	
(b) (i) Note that the vertex of the graph of $y=3(x-6)^2+k$ is $(6,k)$.	1M	
Also note that the graph of $y=3(x-6)^2+k$ and the graph of $y=f(x)$ have the same vertex.		
So, the vertex of the graph of $y=f(x)$ is also $(6,k)$.		
Since $f(6)=-36$, we have $k=-36$.	1A	
(ii) When $x=10$, we have $y=3(10-6)^2-36=12$.	1M	
The coordinates of A are $(10,12)$.		
When $y=12$, we have $3(x-6)^2-36=12$.		
So, we have $x=2$ or $x=10$.		
The coordinates of B are $(2,12)$.		
AB $=10-2$ $=8$	1M	either one
When $x=10$, we have $y=10^2-12(10)=-20$.		
The coordinates of D are $(10,-20)$.		
AD $=12-(-20)$ $=32$		
The area of the rectangle $ABCD$ $=(8)(32)$ $=256$	1A	
	----------(5)	

Solution		Marks	Remarks
12. (a) In ΔABP and ΔPCD,			
$\angle ABP = 90°$	(given)		
$\angle ABP + \angle PCD = 180°$	(int. $\angle$s, $AB // CD$)		
$\angle PCD = 90°$			
Therefore, we have $\angle ABP = \angle PCD$.			
$\angle APD = 90°$	(given)		
$\angle BAP + 90° + \angle APB = 180°$	($\angle$ sum of Δ)		
$\angle BAP = 90° - \angle APB$			
$\angle CPD + 90° + \angle APB = 180°$	(adj. $\angle$s on st. line)		
$\angle CPD = 90° - \angle APB$			
Therefore, we have $\angle BAP = \angle CPD$.			
[redacted]	[redacted]		
	[redacted]		
$\Delta ABP \sim \Delta PCD$	(AAA)		(AA) (equiangular)

Marking Scheme:		
Case 1	Any correct proof with correct reasons.	3
Case 2	Any correct proof without reasons.	2
Case 3	Incomplete proof with any one correct step and one correct reason.	1

----------(3)

Solution	Marks
(b) Since $\Delta ABP \sim \Delta PCD$, we have	
$\frac{AB}{BP} = \frac{PC}{CD}$	1M
$\frac{3}{x} = \frac{11-x}{k}$	
$3k = x(11-x)$	
$x^2 - 11x + 3k = 0$	1
	----------(2)
(c) Note that $x^2 - 11x + 3k = 0$ has real roots.	
$\Delta \geq 0$	1M
$(-11)^2 - 4(1)(3k) \geq 0$	1M+1A
$k \leq \frac{121}{12}$	
Note that k is an integer.	
Thus, the greatest value of k is 10 .	1A
	----------(4)

Solution	Marks	Remarks
13. (a) Let l mm be the length of OX.		
$\pi l^2\left(\frac{288}{360}\right)=2880\pi$	1M	for $\pi l^2\left(\frac{288}{360}\right)$
$l^2=3600$		
$l=60$	1A	
Thus, the length of OX is 60 mm .		u−1 for missing unit
	----------(2)	
(b) Let r mm be the base radius of the container.		
$2\pi r=2\pi(60)\left(\frac{288}{360}\right)$	1M	for $2\pi l\left(\frac{288}{360}\right)$
$r=48$	1A	can be absorbed
The height of the container		
$=\sqrt{60^2-48^2}$	1M	
$=36$ mm	1A	u−1 for missing unit
	----------(4)	
(c) The capacity of the container		
$=\frac{1}{3}\pi(48)^2(36)$	1M	
$\approx 86\,858.75369\text{ mm}^3$		
$\approx 86.85875369\text{ cm}^3$	1M	for $1\text{ cm}^3=1000\text{ mm}^3$
Note that the volume of water is 150 cm^3 .		
So, the volume of water is greater than 86.85875369 cm^3 .		
Thus, the water will overflow.	1A	f.t.
	----------(3)	

Solution	Marks	Remarks
14. (a) The required probability		
$=\left(\frac{9}{12}\right)\left(1-\frac{1}{6}\right)+\left(\frac{3}{12}\right)\left(1-\frac{1}{3}\right)$	1M+1M	1M for $p_1p_2+p_3p_4$ 1M for $p_1+p_3=1$
$=\frac{19}{24}$	1A	r.t. 0.792
	----------(3)	
(b) (i) The required probability		
$=\left(1-\frac{1}{6}\right)\left(1-\frac{1}{3}\right)$	1M	for $\left(\frac{m}{6}\right)\left(\frac{n}{3}\right)$, $1\le m<6$, $1\le n<3$
$=\left(\frac{5}{6}\right)\left(\frac{2}{3}\right)$		
$=\frac{5}{9}$	1A	r.t. 0.556
(ii) The required probability		
$=\left(\frac{3}{12}\right)\left(1-\frac{1}{3}\right)\left(\frac{2}{11}\right)\left(1-\frac{1}{3}\right)$	1M+1M	1M for $p_5p_6(p_7)^2$ 1M for $p_5=\frac{r}{12}$, $p_6=\frac{r-1}{11}$, $1<r$
$=\left(\frac{3}{12}\right)\left(\frac{2}{11}\right)\left(\frac{2}{3}\right)^2$		
$=\frac{2}{99}$	1A	r.t. 0.0202
(iii) The probability of not making complaints by the two selected customers		
$=\left(\frac{9}{12}\right)\left(\frac{5}{6}\right)\left(\frac{8}{11}\right)\left(\frac{5}{6}\right)+2\left(\frac{3}{12}\right)\left(\frac{9}{11}\right)\left(\frac{5}{9}\right)+\frac{2}{99}$	1M	for p_8+2p_9((b)(i))+(b)(ii)
$=\frac{62}{99}$	1A	r.t. 0.626
$>\frac{1}{2}$		
Thus, the probability of not making complaints by the two selected customers is greater than the probability of making complaints by both of them.	1A	f.t.
The probability of not making complaints by the two selected customers		
$=1-\left(\frac{9}{12}\right)\left(\frac{1}{6}\right)-\left(\frac{3}{12}\right)\left(\frac{1}{3}\right)-\left(\frac{9}{12}\right)\left(\frac{5}{6}\right)\left(\left(\frac{8}{11}\right)\left(\frac{1}{6}\right)+\left(\frac{3}{11}\right)\left(\frac{1}{3}\right)\right)$ $-\left(\frac{3}{12}\right)\left(\frac{2}{3}\right)\left(\left(\frac{9}{11}\right)\left(\frac{1}{6}\right)+\left(\frac{2}{11}\right)\left(\frac{1}{3}\right)\right)$	1M	for $1-p_8-p_9-p_{10}-p_{11}-p_{12}-p$
$=\frac{62}{99}$	1A	r.t. 0.626
$>\frac{1}{2}$		
Thus, the probability of not making complaints by the two selected customers is greater than the probability of making complaints by both of them.	1A	f.t.
	----------(8)	

Solution	Marks	Remarks
15. (a) <table><tr><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>7</td><td>8</td><td>9</td><td>10</td></tr></table>	1M 1A ----------(2)	for 16 integers
(b) The sum of all integers in the 1st row of the 99th table $=\frac{99}{2}(2(99)+(99-1)(1))$ $=14\,652$	 1M 1A ----------(2)	
(c) For the 99th table, there are 99 rows. The sum of all integers in the 1st row is 14 652 . Note that the sums of all integers in the rows of the table form an arithmetic sequence. In this arithmetic sequence, the 1st term is 14 652 , the common difference is 99 and there are 99 terms. The sum of all integers in the 99th table $=\frac{99}{2}(2(14\,652)+(99-1)(99))$ $=1\,930\,797$	 1M+1A 1A	 1M for using (b) as the 1st term
For the 99th table, there are 99 rows. The sum of all integers in the 1st row is 14 652 . Note that the sums of all integers in the rows of the table form an arithmetic sequence. In this arithmetic sequence, there are 99 terms, the 1st term is 14 652 and the last term is 24 354 . The sum of all integers in the 99th table $=\frac{99}{2}(14\,652+24\,354)$ $=1\,930\,797$	 1M+1A 1A ----------(3)	 1M for using (b) as the 1st term
(d) For the kth table, there are k rows. The sum of all integers in the 1st row is $\left(\frac{k}{2}\right)(2k+(k-1)(1))$. Note that the sums of all integers in the rows of the table form an arithmetic sequence. In this arithmetic sequence, the 1st term is $\left(\frac{k}{2}\right)(3k-1)$, the common difference is k and there are k terms. The sum of all integers in the kth table $=\left(\frac{k}{2}\right)\left(2\left(\frac{k}{2}\right)(3k-1)+(k-1)(k)\right)$ $=k^2(2k-1)$ Also note that $k^2(2k-1)$ is a product of odd numbers. Therefore, $k^2(2k-1)$ is an odd number. Thus, there is no odd number k satisfying the required condition.	 1A 1M 1A 1A ----------(4)	 f.t.

Solution	Marks	Remarks
16 (a) The coordinates of R are $(64,-48)$.	1A	
So, the coordinates of the mid-point of PR are $(40,16)$.		
The slope of PR $=\frac{80+48}{16-64}$ $=\frac{-8}{3}$		
Therefore, the slope of the perpendicular bisector of PR is $\frac{3}{8}$.	1M	
The equation of the perpendicular bisector of PR is		
$y-16=\frac{3}{8}(x-40)$	1M	
$3x-8y+8=0$	1A	or equivalent
	----------(4)	
(b) Note that the circumcentre of ΔPQR is the point of intersection of PS and the perpendicular bisector of PR .		
Let $(16,h)$ be the coordinates of the circumcentre of ΔPQR .	1M	for x-coordinate
Since $(16,h)$ lies on $3x-8y+8=0$, we have $3(16)-8h+8=0$.	1M	for substitution
So, we have $h=7$.	1A	
Thus, the coordinates of the circumcentre of ΔPQR are $(16,7)$.		
	----------(3)	
(c) (i) The coordinates of the centre of C $=(16,7)$		
The radius of C $=80-7$ $=73$	1M	
Thus, the equation of C is $(x-16)^2+(y-7)^2=73^2$.	1A	$x^2+y^2-32x-14y-5024=0$
(ii) Let N be the circumcentre of ΔPQR .		
By (b), we have $N=(16,7)$.		
If the centre of C and the in-centre of ΔPQR are the same point, then the circumcentre and the in-centre of ΔPQR are the same point.		
Let M be the mid-point of PR .		
By (a), we have $M=(40,16)$.		
Also, we have $S=(16,-48)$.		
NS $=7-(-48)$ $=55$	1M	either one
NM $=\sqrt{(40-16)^2+(16-7)^2}$ $=\sqrt{657}$		
Since $NS\neq NM$, N is not the in-centre of ΔPQR .		
Thus, the centre of C and the in-centre of ΔPQR are not the same point.	1A	f.t.
	----------(4)	

Solution	Marks	Remarks
17. (a) (i) By cosine formula, we have		
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\angle BAC$	1M	
$BC^2 = 20^2 + 30^2 - 2(20)(30)\cos 56°$		
[redacted]		
Thus, the length of BC is 25.1 cm .	1A	r.t. 25.1 cm
(ii) By sine formula, we have		
$\frac{\sin\angle ACB}{AB} = \frac{\sin\angle BAC}{BC}$	1M	
$\frac{\sin\angle ACB}{20} \approx \frac{\sin 56°}{25.07924472}$		
[redacted]		
$\angle ACB \approx 41.4°$	1A	r.t. 41.4°
(iii) Let h cm be the perpendicular distance from A to BC .		
h		
$= AC\sin\angle ACB$	1M	
$\approx 30\sin 41.38644619°$		
[redacted]		
The required distance		
$\approx 19.83403205 - 4$		
[redacted]		
Thus, the perpendicular distance from A to DE is 15.8 cm .	1A	r.t. 15.8 cm
(iv) Note that $\Delta ADE \sim \Delta ABC$.		
$\frac{DE}{BC} = \frac{h-4}{h}$	1M	
$\frac{DE}{25.07924472} \approx \frac{15.83403205}{19.83403205}$		
[redacted]		
Thus, the length of DE is 20.0 cm .	1A	r.t. 20.0 cm
	----------(8)	
(b) Let θ be the angle between the metal sheet ADE and the horizontal ground.		
(i) $\cos\theta = \frac{\text{The perpendicular distance from } P \text{ to } DE}{h-4}$	1M	
$\cos\theta \approx \frac{\frac{2(120)}{20.02142397}}{15.83403205}$		
[redacted]		
Thus, the angle between the metal sheet ADE and the horizontal ground is 40.8° .	1A	r.t. 40.8°
(ii) $\sin\theta = \frac{AP}{h-4}$		
$AP \approx 15.83403205\sin 40.79515196°$		
[redacted]		
Thus, the shortest distance from A to the horizontal ground is 10.3 cm .	1A	r.t. 10.3 cm
	----------(3)	