Marking Scheme

This document was prepared for markers' reference. It should not be regarded as a set of model answers. Candidates and teachers who were not involved in the marking process are advised to interpret its contents with care.

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
   a. The symbol (*u-1*) should be used to denote 1 mark deducted for *u*. At most deduct ***1 mark*** for *u* in each of Section A(1) and Section A(2). Do not deduct any marks for *u* in Section B.
   b. The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deduct ***1 mark*** for *pp* in each of Section A(1) and Section A(2). Do not deduct any marks for *pp* in Section B.
   c. At most deduct 1 mark in each of Section A(1) and Section A(2).
   d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.

7. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Paper 1

Solution	Marks	Remarks
1. $a^{14}\left(\frac{b^3}{a^2}\right)^5$		
$=a^{14}\left(\frac{b^{15}}{a^{10}}\right)$	1M	for $\left(\frac{x}{y}\right)^m=\frac{x^m}{y^m}$
$=a^{14-10}b^{15}$	1M	for $\frac{x^m}{x^n}=x^{m-n}$
$=a^4b^{15}$	1A	
	----------(3)	
2. (a) $\frac{29x-22}{7}\le 3x$		
$29x-22\le 21x$		
$29x-21x\le 22$	1M	for putting x on one side
$8x\le 22$		
$x\le\frac{11}{4}$	1A	$x\le 2.75$
$\frac{29x-22}{7}\le 3x$		
$\frac{29x}{7}-3x\le\frac{22}{7}$	1M	for putting x on one side
$\frac{8x}{7}\le\frac{22}{7}$		
$x\le\frac{11}{4}$	1A	$x\le 2.75$
(b) The required greatest integer is 2 .	1A	
	----------(3)	
3. (a) $m^2+12mn+36n^2$		
$=(m+6n)^2$	1A	or equivalent
(b) $m^2+12mn+36n^2-25k^2$		
$=(m+6n)^2-25k^2$	1M	for using the result of (a)
$=(m+6n+5k)(m+6n-5k)$	1A	or equivalent
	----------(3)	

Solution	Marks	Remarks
4. (a) The 2nd term		
$= \tan \frac{180°}{2+2}$		
$= \tan 45°$	1A	
$= 1$		
(b) The two terms are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$.	1A + 1A	
	----------(3)	
5. (a) $3(2c+5d+4) = 39d$		
$2c+5d+4 = 13d$	1M	for division
$2c = 13d-5d-4$		
$2c = 8d-4$		
$c = 4d-2$	1A	or equivalent
$3(2c+5d+4) = 39d$		
$6c+15d+12 = 39d$	1M	for expanding
$6c = 39d-15d-12$		
$6c = 24d-12$		
$c = 4d-2$	1A	or equivalent
(b) If the value of d is decreased by 1 , the value of c will be decreased by 4 .	1M + 1M	
	----------(4)	
6. Let $\$x$ be the cost of a bottle of milk.		pp−1 for any undefined symbol
$3(2x)+5x = 66$	1A+1M+1A	1A for $y = 2x$ + 1M for $3y+5x$
$11x = 66$		
$x = 6$	1A	
Thus, the cost of a bottle of milk is $\$6$.		u−1 for missing unit
Let $\$x$ be the cost of a bottle of milk and $\$y$ be the cost of a bottle of orange juice. Then, we have		pp−1 for any undefined symbol
$\begin{cases} 3y+5x = 66 \\ y = 2x \end{cases}$	1A + 1A	
So, we have $3(2x)+5x = 66$.	1M	for getting a linear equation in x or y only
Solving, we have $x = 6$.	1A	
Thus, the cost of a bottle of milk is $\$6$.		u−1 for missing unit
The cost of a bottle of milk		
$= \frac{66}{3(2)+5}$	1M + 1A + 1A	1M for fraction + 1A for numerator + 1A for denominator
$= \frac{66}{11}$		
$= \$6$	1A	u−1 for missing unit
	----------(4)	

Solution	Marks	Remarks
7. (a) The number of badges owned by Tom		
$= 50(1-30\%)$	1M	
$= 35$	1A	
Thus, Tom has 35 badges.		
(b) Note that $50+35=85$ which is an odd number.	1M	
Thus, they will not have the same number of badges.	1A	f.t.
Note that $\frac{50+35}{2}=42.5$ which is not an integer.	1M	
Thus, they will not have the same number of badges.	1A	f.t.
Assume Mary gives x badges to Tom so that they will have the same number of badges.		
Then, we have $50-x=35+x$.	1M	
Solving, we have $x=7.5$.		
Since 7.5 is not an integer, they will not have the same number of badges.	1A	f.t.
If Mary gives 7 of her badges to Tom, then the number of badges owned by Mary is greater than that owned by Tom.	1M	either one
If Mary gives 8 of her badges to Tom, then the number of badges owned by Mary is less than that owned by Tom.		
Thus, they will not have the same number of badges.	1A	f.t.
	----------(4)	
8. (a) The estimated total amount		
$= 16+24+32$	1M + 1A	1M for either correct + 1A for all
$= \$72$	1A	u−1 for missing unit
(b) By (a), the actual total amount they have is greater than \$72 .		
Thus, they have enough money to buy the football.	1A	f.t.
	----------(4)	
9. (a) Note that $\angle ABC+\angle BAC+\angle ACB=180^\circ$ and $\angle CAE+\angle ACD=180^\circ$.	1M	for either one
Then, we have $\angle ABC+\angle BAC+\angle ACB+\angle CAE+\angle ACD=360^\circ$.		
Hence, we have $\angle ABC+(\angle BAC+\angle CAE)+(\angle ACB+\angle ACD)=360^\circ$.		
Therefore, we have $\angle ABC+\angle BAE+\angle BCD=360^\circ$.		
$\angle ABC$		
$= 360^\circ-\angle BAE-\angle BCD$		
$= 360^\circ-108^\circ-126^\circ$		
$= 126^\circ$	1A	u−1 for missing unit
(b) In ΔABC and ΔDCB ,		
$AB=DC$ (given)		
$\angle ABC=126^\circ$ (by (a))		
$\angle DCB=126^\circ$ (given)		
$\angle ABC=\angle DCB$		
$BC=CB$ (common side)		
$\Delta ABC\cong\Delta DCB$ (SAS)		
Marking Scheme:		
Case 1 Any correct proof with correct reasons.	3	
Case 2 Any correct proof without reasons.	2	
Case 3 Incomplete proof with any one correct step and one correct reason.	1	
	----------(5)	

Solution	Marks	Remarks
10. (a) Let $C = ax + bx^2$, where a and b are non-zero constants.	1A	
When $x = 4$, $C = 96$, we have		
$4a + 16b = 96$	1M	for either substitution
$a + 4b = 24$		
When $x = 5$, $C = 145$, we have		
$5a + 25b = 145$		
$a + 5b = 29$		
Solving, we have $b = 5$.	1M	for eliminating one variable
Hence, we have $a = 4$ and $b = 5$.	1A	for both correct
Thus, we have $C = 4x + 5x^2$.		
	----------(4)	
(b) $4x + 5x^2 = 288$	1M	for using (a)
$5x^2 + 4x - 288 = 0$		
$(5x - 36)(x + 8) = 0$	1M	
$x = \frac{36}{5}$ or $x = -8$ (rejected)	1A	7.2
Thus, the perimeter of the tablecloth is $\frac{36}{5}$ metres.		u−1 for missing unit
	----------(3)	

Solution	Marks	Remarks
11. (a) The mean $=\frac{550}{22}$ $=25$	1A	
The median $=26$	1A	
The range $=31-18$ $=13$	1A ----------(3)	
(b) (i) Let x be the mean age of the three new players. $\frac{550-2(31)+3x}{23}=25$	1M + 1A	pp−1 for any undefined symbol
$x=29$ Thus, the mean age of the three new players is 29 .	1A	
The mean age of the three new players $=\frac{2(31)+25}{3}$	1M + 1A	
$=29$	1A	
(ii) Two sets of possible ages of the three new players are $\{25, 31, 31\}$ and $\{26, 30, 31\}$.	1A + 1A ----------(5)	accept 27, 29, 31 and 28, 28, 31

Solution	Marks	Remarks
12. (a) The slope of AB		
$=\frac{24-18}{6-(-2)}$		
$=\frac{3}{4}$		
The equation of the straight line passing through A and B is		
$y-24=\frac{3}{4}(x-6)$	1M	
$3x-4y+78=0$	1A	or equivalent
	----------(2)	
(b) Let $(c, 0)$ be the coordinates of C.		
Note that the slope of AC is $\frac{24-0}{6-c}$.	1M	
Then, we have $\left(\frac{24}{6-c}\right)\left(\frac{3}{4}\right)=-1$.	1M	
Solving, we have $c=24$.	1A	
Thus, the coordinates of C are $(24, 0)$.		pp−1 for missing '(' or ')'
The slope of AC is $\frac{-4}{3}$.	1M	
The equation of AC is $4x+3y-96=0$.		
Putting $y=0$ in $4x+3y-96=0$, we have $x=24$.	1M	for putting $y=0$
Thus, the coordinates of C are $(24, 0)$.	1A	pp−1 for missing '(' or ')'
	----------(3)	
(c) AB		
$=\sqrt{(6-(-2))^2+(24-18)^2}$	1M	either one
$=10$ **units**		
AC		
$=\sqrt{(6-24)^2+(24-0)^2}$		
$=30$ **units**		
The area of ΔABC		
$=\frac{(10)(30)}{2}$		
$=150$ **square units**	1A	
	----------(2)	
(d) The ratio of the area of ΔABD to the area of ΔACD is $r:1$.	1M	
$\frac{90}{150-90}=\frac{r}{1}$		
$r=\frac{3}{2}$	1A	1.5
The ratio of the area of ΔABC to the area of ΔABD is $(r+1):r$.	1M	
$\frac{150}{90}=\frac{r+1}{r}$		
$r=\frac{3}{2}$	1A	1.5
	----------(2)	

Solution	Marks	Remarks
13. (a) Let M be the mid-point of BC. Then, we have $BM = 8$ cm.		
AM $= \sqrt{17^2 - 8^2}$ $= 15$ cm	1M	for Pythagoras' theorem
The area of ΔABC $= \frac{(16)(15)}{2}$ $= 120 \text{ cm}^2$	1A	u−1 for missing unit
	----------(2)	
(b) The volume of the wooden block $ABCDEF$		
$= (120)(20)$	1M	
$= 2400 \text{ cm}^3$	1A	u−1 for missing unit
	----------(2)	
(c) (i) The volume of the wooden block $APQRES$		
$= 2400\left(\frac{4}{16}\right)^2$	1M	
$= 150 \text{ cm}^3$	1A	u−1 for missing unit
The height of the triangular base APQ $= 15\left(\frac{4}{16}\right)$	1M	for using ratio
$= \frac{15}{4}$ cm		
The volume of the wooden block $APQRES$ $= \frac{1}{2}(4)\left(\frac{15}{4}\right)(20)$		
$= 150 \text{ cm}^3$	1A	u−1 for missing unit
(ii) Note that $\frac{\text{the volume of } APQRES}{\text{the volume of } ABCDEF} = \frac{1}{16}$ and $\left(\frac{PQ}{BC}\right)^3 = \frac{1}{64}$.	1M	for finding either ratio
Also note that the two ratios are not equal.	1M	for comparing two ratios
Thus, the two blocks are not similar.	1A	f.t.
Note that $\frac{PQ}{BC} = \frac{1}{4}$ and $\frac{QR}{CD} = 1$.	1M	for finding either ratio
Also note that the two ratios are not equal.	1M	for comparing two ratios
Thus, the two blocks are not similar.	1A	f.t.
Note that $\frac{\text{the area of parallelogram } AQRE}{\text{the area of parallelogram } ACDE} = \frac{1}{4}$ and $\left(\frac{PQ}{BC}\right)^2 = \frac{1}{16}$.	1M	for finding either ratio
Also note that the two ratios are not equal.	1M	for comparing two ratios
Thus, the two blocks are not similar.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
14. (a) (i) The required probability		
$=\left(\frac{8}{10}\right)\left(\frac{7}{9}\right)$	1M	for $\left(\frac{p}{m}\right)\left(\frac{q}{m-1}\right)$, $p<m$ and $q<m-1$
$=\frac{28}{45}$	1A	r.t. 0.622
(ii) The required probability		
$=2\left(\frac{2}{10}\right)\left(\frac{8}{9}\right)$	1M	for $2\left(\frac{s}{n}\right)\left(\frac{t}{n-1}\right)$, $s<n$ and $t<n-1$
$=\frac{16}{45}$	1A	r.t. 0.356
(iii) The required probability		
$=\frac{16}{45}+\frac{28}{45}$	1M	for (a)(i) + (a)(ii)
$=\frac{44}{45}$	1A	r.t. 0.978
The required probability		
$=1-\left(\frac{2}{10}\right)\left(\frac{1}{9}\right)$	1M	
$=\frac{44}{45}$	1A	r.t. 0.978
	----------(6)	
(b) (i) Note that the mean results of Alice and Betty are 275 seconds and 272 seconds respectively	1A	
So, the mean result of Betty is better than that of Alice.	1M	
Thus, Betty is likely to get a better result.	1A	f.t.
Note that the median results of Alice and Betty are 279.5 seconds and 272.5 seconds respectively.	1A	
So, the median result of Betty is better than that of Alice.	1M	
Thus, Betty is likely to get a better result.	1A	f.t.
By comparing each result of Alice with each result of Betty, there are altogether 100 outcomes.	1A	
Alice gets a better result in 38 outcomes out of the 100 outcomes. Betty gets a better result in 61 outcomes out of the 100 outcomes.	1M	
Thus, Betty is likely to get a better result.	1A	f.t.
(ii) Alice gets three results which are better than 267 seconds but Betty gets only one result which is better than 267 seconds.	1M	
Thus, Alice has a greater chance of breaking the record.	1A	f.t.
	----------(5)	

Solution	Marks	Remarks
15. (a) By sine formula		
$\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \angle BAC}$	1M	
$\frac{AB}{\sin(180° - 73° - 59°)} = \frac{24}{\sin 73°}$		
$AB = \frac{24 \sin 48°}{\sin 73°}$		
$AB \approx 18.65041003$ cm		
$AB \approx 18.7$ cm	1A	r.t. 18.7 cm
	----------(2)	
(b) (i) By cosine formula		
$BD^2 = AB^2 + AD^2 - 2(AB)(AD) \cos \angle BAD$	1M	accept $BD = 2AB \sin \frac{\angle BAD}{2}$
$BD^2 \approx (18.65041003)^2 + (18.65041003)^2 - 2(18.65041003)^2 \cos 92°$		
$BD \approx 26.83196445$ cm		
$BD \approx 26.8$ cm	1A	r.t. 26.8 cm
(ii) Let Q be the foot of perpendicular from B to AC.		
$\sin \angle BAC = \frac{BQ}{AB}$	1M	
BQ		
$\approx 18.65041003 \sin 73°$		
≈ 17.83547581 cm		
The angle between the plane ABC and the plane ACD is $\angle BQD$.	1M	for identifying the required angle
$\sin \frac{\angle BQD}{2} = \frac{\frac{BD}{2}}{BQ}$	1M	
$\sin \frac{\angle BQD}{2} \approx \frac{13.41598223}{17.83547581}$		
$\angle BQD \approx 97.56395848°$		
$\angle BQD \approx 97.6°$	1A	r.t. 97.6°
Thus, the required angle is 97.6°.		
$\cos \angle BQD = \frac{BQ^2 + DQ^2 - BD^2}{2(BQ)(DQ)}$	1M	for using cosine formula
$\cos \angle BQD \approx \frac{(17.83547581)^2 + (17.83547581)^2 - (26.83196445)^2}{2(17.83547581)(17.83547581)}$		
$\angle BQD \approx 97.56395848°$		
$\angle BQD \approx 97.6°$	1A	r.t. 97.6°
Thus, the required angle is 97.6°.		
(iii) Note that $\sin \frac{\angle BPD}{2} = \frac{BD}{2BP}$ and BD is a constant.		
The length of BP is the shortest when P is at Q.	1M	
Note also that the length of BP varies inversely as $\sin \frac{\angle BPD}{2}$.	1M	
Thus, $\angle BPD$ increases from $\angle BAD$ (92°) to $\angle BQD$ (97.6°) and then decreases to $\angle BCD$ (68.0°).	1A	
	----------(9)	

Solution	Marks	Remarks
16. (a) (i) $f(x)$		
$=\frac{1}{2}x-\frac{1}{144}x^2-6$		
$=\frac{-1}{144}(x^2-72x)-6$	1M	
$=\frac{-1}{144}(x^2-72x+36^2-36^2)-6$	1M	
$=\frac{-1}{144}(x-36)^2+3$		
Thus, the coordinates of the vertex are $(36,3)$.	1A	
(ii) $g(x)$		
$=f(x+4)+5$	1A	
$=\frac{-1}{144}((x+4)-36)^2+3+5$		
$=\frac{-1}{144}(x-32)^2+8$	1A	accept $\frac{-1}{144}x^2+\frac{4}{9}x+\frac{8}{9}$
(iii) $h(x)$		
$=2^{f(x+4)}+5$	1A	
$=2^{\frac{-1}{144}(x-32)^2+3}+5$	1A ----------(7)	accept $2^{\frac{-1}{144}x^2+\frac{4}{9}x-\frac{37}{9}}+5$
(b) (i) $2^{f(s)}=8$		
$2^{f(s)}=2^3$		
$f(s)=3$	1M	
$\frac{-1}{144}(s-36)^2+3=3$ (by (a)(i))		
$s=36$	1A	
Thus, the required temperature is $36°C$.		
$2^{f(s)}=8$		
$2^{f(s)}=2^3$		
$f(s)=3$	1M	
$\frac{1}{2}s-\frac{1}{144}s^2-6=3$		
$s^2-72s+1296=0$		
$(s-36)^2=0$		
$s=36$	1A	
Thus, the required temperature is $36°C$.		
(ii) v		
$=h(t)$	1A	accept $v=2^{f(t+4)}+5$
$=2^{\frac{-1}{144}(t-32)^2+3}+5$	1M ----------(4)	for using (a)(iii)

Solution	Marks	Remarks
17. (a) (i) By rotating B anticlockwise through $90°$ with respect to A, the coordinates of D are $(-6, 8)$.	1A	
The coordinates of the centre of the circle $ABCD$ = the coordinates of the mid-point of BD $=\left(\frac{8+(-6)}{2}, \frac{6+8}{2}\right)$	1M	
$=(1, 7)$	1A	
(ii) The radius of the circle $ABCD$ $=\sqrt{(1-0)^2+(7-0)^2}$	1M	
$=5\sqrt{2}$ units	1A	r.t. 7.07 units
$AB=\sqrt{6^2+8^2}=10$ units $BD=\sqrt{AB^2+AD^2}=\sqrt{10^2+10^2}=10\sqrt{2}$ units	1M	either one
The radius of the circle $ABCD$ $=\frac{BD}{2}=5\sqrt{2}$ units	1A	r.t. 7.07 units
	----------(5)	
(b) (i) The radius of the circle $A_1B_1C_1D_1$ $=\frac{2(5\sqrt{2})\sin 45°}{2}$	1M	
$=5$ units		
The required ratio $=5^2:(5\sqrt{2})^2$	1M	
$=1:2$	1A	accept $0.5:1$
(ii) The areas of the shaded regions form a geometric sequence. The total area of all the shaded regions $=\left(10^2-\pi\left(\frac{10}{2}\right)^2\right)+\frac{1}{2}\left(10^2-\pi\left(\frac{10}{2}\right)^2\right)+\cdots+\left(\frac{1}{2}\right)^9\left(10^2-\pi\left(\frac{10}{2}\right)^2\right)$ $=\left(10^2-\pi\left(\frac{10}{2}\right)^2\right)\left(1+\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^2+\cdots+\left(\frac{1}{2}\right)^9\right)$		
$=\left(10^2-\pi\left(\frac{10}{2}\right)^2\right)\left(\frac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}}\right)$	1M	for sum of geometric sequence
≈ 42.8784529 square units		
p $\approx\frac{42.8784529}{\pi(5\sqrt{2})^2}$	1M	
≈ 0.272972709 Thus, the design of the logo is good.	1A	f.t.
	----------(6)	