

**香港考試及評核局****HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY****2005年香港中學會考****HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 2005****數學 試卷一****MATHEMATICS PAPER 1**

本評卷參考乃香港考試及評核局專為今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視為標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the teachers' centre.



Hong Kong Certificate of Education Examination  
Mathematics Paper 1

## General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.
4. Use of notation different from those in the marking scheme should not be penalized.
5. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.
6. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
  - a. The symbol  $(u-1)$  should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* in Section A. Do not deduct any marks for *u* in Section B.
  - b. The symbol  $(pp-1)$  should be used to denote 1 mark deducted for *pp*. At most deduct **1 mark** for *pp* in each of Section A and Section B. For similar *pp*, deduct 1 mark for the first time that it occurs. Do not penalize candidates twice in the paper for the same *pp*.
  - c. At most deduct 1 mark in each question. Deduct the mark for *u* first if both marks for *u* and *pp* may be deducted in the same question.
  - d. In any case, do not deduct any marks for *pp* or *u* in those steps where candidates could not score any marks.
7. Marks entered in the Page Total Box should be the NET total scored on that page.
8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’, ‘ft.’ stands for ‘follow through’ and ‘or equivalent’ means ‘accepting equivalent forms of the equation which has been simplified and without uncollected like terms’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
<p>1. <math>P = ab + 2bc + 3ac</math>  <math>ab + 3ac = P - 2bc</math>  <math>a(b + 3c) = P - 2bc</math>  <math>a = \frac{P - 2bc}{b + 3c}</math></p>	<p>1M                      1M                      1A                      -----(3)</p>	<p>for putting <math>a</math> on one side                      for factorization                      or equivalent</p>
<p>2. <math>\frac{(x^3y)^2}{y^5}</math>  <math>= \frac{x^6y^2}{y^5}</math>  <math>= \frac{x^6}{y^{5-2}}</math>  <math>= \frac{x^6}{y^3}</math></p>	<p>1M                      1M                      1A                      -----(3)</p>	<p>for <math>(ab)^n = a^n b^n</math> or <math>(a^m)^n = a^{mn}</math>                      for <math>\frac{b^m}{b^n} = b^{m-n}</math> or <math>\frac{b^m}{b^n} = \frac{1}{b^{n-m}}</math></p>
<p>3. (a) <math>4x^2 - 4xy + y^2</math>  <math>= (2x - y)^2</math></p> <p>(b) <math>4x^2 - 4xy + y^2 - 2x + y</math>  <math>= (2x - y)^2 - 2x + y</math> (by (a))  <math>= (2x - y)^2 - (2x - y)</math>  <math>= (2x - y)(2x - y - 1)</math></p>	<p>1A                      1M                      1A                      -----(3)</p>	<p>or equivalent                      for using the result of (a)                      or equivalent</p>
<p>4. <math>\frac{-3x+1}{4} &gt; x-5</math>  <math>-3x+1 &gt; 4x-20</math>  <math>-7x &gt; -21</math>  <math>7x &lt; 21</math>  <math>x &lt; 3</math></p> <p>For <math>2x+1 \geq 0</math>, we have <math>x \geq \frac{-1}{2}</math>.</p> <p>Therefore, the solution of <math>\frac{-3x+1}{4} &gt; x-5</math> and <math>2x+1 \geq 0</math> is <math>\frac{-1}{2} \leq x &lt; 3</math>.</p> <p>Thus, all integers which satisfy both the inequalities <math>\frac{-3x+1}{4} &gt; x-5</math> and <math>2x+1 \geq 0</math> are 0, 1 and 2.</p>	<p>1M                      1A                      1A                      -----(3)</p>	<p>for putting <math>x</math> on one side</p>

Solution	Marks	Remarks
5. $n - 18 = \frac{2n}{5} + 18$ $\frac{3n}{5} = 36$ $n = 60$	1A + 1M   1A	1A for $\frac{2n}{5} + 1M$ for equating
Suppose that Susan and Teresa have $5k$ marbles and $2k$ marbles respectively. $5k - 18 = 2k + 18$ $3k = 36$ $k = 12$ $n = 5k$ $n = 60$	1A 1M      1A	for $5k$ and $2k$ pp-1 for any undefined symbol for equating
------(3)		
6. (a) Let $\$x$ be the marked price of the calculator. Then, we have $x = 160(1 + 25\%)$ $x = 200$ Thus, the marked price of the calculator is $\$200$ .	1A 1A	u-1 for missing unit
Let $\$x$ be the marked price of the calculator. Then, we have $\left(\frac{x - 160}{160}\right)(100\%) = 25\%$ $x - 160 = 40$ $x = 200$ Thus, the marked price of the calculator is $\$200$ .	1A   1A	accept without 100%   u-1 for missing unit
(b) The selling price of the calculator $= 200(90\%)$ $= \$180$  The percentage profit $= \left(\frac{180 - 160}{160}\right)(100\%)$ $= 12.5\%$	1M     1A	for (a)(90%)
The selling price of the calculator $= 200(90\%)$ $= \$180$  The percentage profit $= \left(\frac{180 - 160}{200 - 160}\right)(25\%)$ $= 12.5\%$	1M   1A	for (a)(90%)
------(4)		
7. The common difference $= 8 - 5 = 3$ $\frac{n}{2}((2)(5) + (n-1)(3)) = 3925$ $3n^2 + 7n - 7850 = 0$ $n = 50$ or $n = \frac{-157}{3}$ (rejected since $n$ is a positive integer) Thus, we have $n = 50$ .	1A 1M 1M 1A   -----(4)	can be absorbed for $\frac{n}{2}((2)(5) + (n-1)(d)) = 3925$ in the form $k_1n^2 + k_2n + k_3 = 0$ where $k_1 \neq 0$

Solution	Marks	Remarks
8. $x = \frac{180(6-2)}{6}$ $= 120$ $y = \frac{180-x}{2}$ $= \frac{180-120}{2}$ $= 30$ $z = 180 - 2y$ $= 180 - (2)(30)$ $= 120$	1A   1M 1A  1M 1A	u-1 for having unit   u-1 for having unit   u-1 for having unit
$z = (180)(5-2) - 2x - 2(x-y)$ $= 540 - 4x + 2y$ $= 540 - (4)(120) + (2)(30)$ $= 120$	1M 1A	u-1 for having unit
------(5)		
9. (a) $2\pi(OA)\left(\frac{100}{360}\right) = 10\pi$ $OA = 18 \text{ cm}$	1M 1A	for $\frac{100}{360}$ u-1 for missing unit
$(OA)\left(\frac{100\pi}{180}\right) = 10\pi$ $OA = 18 \text{ cm}$	1M 1A	for $\frac{100\pi}{180}$ u-1 for missing unit
(b) The area of sector $OABC$ $= \frac{100}{360}\pi(18)^2$ $\approx 282.7433388 \text{ cm}^2$ The area of $\triangle OAC$ $= \frac{1}{2}(18)^2 \sin 100^\circ$ $\approx 159.538856 \text{ cm}^2$ The required area $\approx 282.7433388 - 159.538856$ $\approx 123.2044828$ $\approx 123 \text{ cm}^2$	1M   1M   1A	for $\frac{100}{360}\pi(a)^2$   for $\frac{1}{2}(a)^2 \sin 100^\circ$   u-1 for missing unit r.t. $123 \text{ cm}^2$
The area of sector $OABC$ $= \frac{1}{2}(18)^2\left(\frac{100}{180}\pi\right)$ $\approx 282.7433388 \text{ cm}^2$ The area of $\triangle OAC$ $= \frac{1}{2}(2(18 \sin 50^\circ))(18 \cos 50^\circ)$ $\approx 159.538856 \text{ cm}^2$ The required area $\approx 282.7433388 - 159.538856$ $\approx 123.2044828$ $\approx 123 \text{ cm}^2$	1M   1M   1A	for $\frac{1}{2}(a)^2\left(\frac{100}{180}\pi\right)$   for $\frac{1}{2}(2((a)\sin 50^\circ))((a)\cos 50^\circ)$   u-1 for missing unit r.t. $123 \text{ cm}^2$
------(5)		

Solution	Marks	Remarks
<p>10. (a) Let <math>f(x) = ax^3 + bx</math>, where <math>a</math> and <math>b</math> are non-zero constants.</p> <p>Since <math>f(2) = -6</math>, we have  <math>8a + 2b = -6</math>  <math>4a + b = -3</math> ..... (1)</p> <p>Since <math>f(3) = 6</math>, we have  <math>27a + 3b = 6</math>  <math>9a + b = 2</math> ..... (2)</p> <p>Solving (1) and (2), we have</p> $\begin{cases} a = 1 \\ b = -7 \end{cases}$ <p>Thus, we have <math>f(x) = x^3 - 7x</math>.</p> <p>(b) (i) <math>g(x) = f(x) - 6</math>  <math>g(3) = f(3) - 6 = 6 - 6 = 0</math>  Thus, by Factor Theorem, <math>x - 3</math> is a factor of <math>g(x)</math>.</p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p>	<p></p> <p>for substitution (either one)</p> <p>for solving simultaneous equations and can be absorbed</p> <p>for both correct</p> <p>----- (4)</p>
<div style="border: 1px solid black; padding: 5px;"> <math>g(x) = x^3 - 7x - 6</math>  <math>g(3) = 3^3 - 7(3) - 6 = 0</math>  Thus, by Factor Theorem, <math>x - 3</math> is a factor of <math>g(x)</math>. </div>	<p>1A</p>	<p>accept using division correctly</p>
<p>(ii) <math>g(x) = (x-3)(x^2 + 3x + 2)</math>  <math>= (x-3)(x+1)(x+2)</math></p>	<p>1M + 1A</p> <p>1A</p> <p>----- (4)</p>	<p>1M for <math>(x-3)(ax^2 + bx + c)</math></p>

Solution	Marks	Remarks
11. (a) The required probability $= \frac{1}{2}$	1A -----(1)	0.5
(b) The required probability $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{1}{8}$	1M 1A -----(2)	for (a) $p_1 p_2$ , $0 < p_1, p_2 < 1$ 0.125
(c) The required probability $= \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(1 - \frac{1}{2}\right)$ $= \frac{3}{4}$	1M+1M 1A	1M for $(1 - (a))$ + 1M for $(a)p_3$ , $0 < p_3 < 1$ 0.75
The required probability $= 1 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{3}{4}$	1M+1M 1A	1M for $(1 - p_3)$ + 1M for $p_3 = (a)p_4$ , $0 < p_3, p_4 < 1$ 0.75
(d) The required probability $= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$ $= \frac{1}{8}$	-----(3) 1M 1A -----(2)	for (a) $p_4 \left(\frac{1}{2}\right)$ , $0 < p_4 < 1$ 0.125

Solution	Marks	Remarks
<p>12. (a) Since the volume of the right circular cone is equal to the volume of the hemisphere, we have</p> $\frac{1}{3}\pi(h-4)^2h = \frac{2}{3}\pi(h-4)^3$ $h = 2(h-4) \quad (\because h \neq 4)$ $h = 8$	<p>1M+1M</p> <p>1A</p> <p>------(3)</p>	<p>1M for <math>\frac{\pi}{3}r^2h</math> + 1M for <math>\frac{2}{3}\pi r^3</math></p> <p>u-1 for having unit</p>
<p>(b) The length of the slant edge of the right circular cone</p> $= \sqrt{8^2 + 4^2}$ $= \sqrt{80}$ $= 4\sqrt{5} \text{ cm}$ <p>The total surface area of the solid</p> <p>= the curved surface area of the cone + the surface area of the hemisphere</p> $= \pi(4)(4\sqrt{5}) + 2\pi(4^2)$ $= 16(\sqrt{5} + 2)\pi$ $\approx 212.9280006$ $\approx 213 \text{ cm}^2$	<p>1M</p> <p>1A</p> <p>------(3)</p>	<p>for <math>\sqrt{(a)^2 + ((a) - 4)^2}</math></p> <p>for <math>\pi((a) - 4)l + 2\pi((a) - 4)^2</math></p> <p>u-1 for missing unit</p>
<p>(c) The increase in the total surface area</p> $= 2\left(\frac{(8)(8)}{2} + \frac{\pi(4^2)}{2}\right)$ $\approx 114.2654825$ $\approx 114 \text{ cm}^2$	<p>1M</p> <p>1A</p> <p>------(2)</p>	<p>for <math>\left(\frac{(a)(2(a) - 8)}{2} + \frac{\pi((a) - 4)^2}{2}\right)</math></p> <p>u-1 for missing unit</p>



Solution	Marks	Remarks
<p>13. (a) Putting <math>y=0</math> in <math>2x-y+4=0</math>, we have <math>x=-2</math>. Thus, the coordinates of <math>A</math> are <math>(-2, 0)</math>. Putting <math>x=0</math> in <math>2x-y+4=0</math>, we have <math>y=4</math>. Thus, the coordinates of <math>B</math> are <math>(0, 4)</math>.</p> <p>(b) <math>\therefore</math> the slope of <math>L_1</math> is 2. <math>\therefore</math> the slope of <math>L_2</math> is <math>\frac{-1}{2}</math>. Thus, the equation of <math>L_2</math> is <math display="block">y = \frac{-x}{2} + 4</math><math display="block">x + 2y - 8 = 0</math></p> <p>(c) Putting <math>y=0</math> in <math>x+2y-8=0</math>, we have <math>x=8</math>. So, the coordinates of <math>C</math> are <math>(8, 0)</math>. Therefore, we have <math>OC:AC=4:5</math>. The required ratio <math>= 4^2:(5^2-4^2)</math> <math>= 16:9</math></p>	<p>1A</p> <p>1A</p> <p>------(2)</p> <p>1M</p> <p>1M+1A</p> <p>------(3)</p> <p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>pp-1 for missing '(' or ')'</p> <p>pp-1 for missing '(' or ')'</p> <p>can be absorbed</p> <p>1M for slope-intercept form or point-slope form + 1A or equivalent</p> <p>for finding the coordinates of <math>C</math></p> <p>can be absorbed</p> <p>for <math>OC^2:(AC^2-OC^2)</math> accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
<p>Putting <math>y=0</math> in <math>x+2y-8=0</math>, we have <math>x=8</math>. So, the coordinates of <math>C</math> are <math>(8, 0)</math>. Let the coordinates of <math>D</math> be <math>(a, b)</math>. Then, we have <math>b=2a</math> and <math>a+2b-8=0</math>. Solving, the coordinates of <math>D</math> are <math>(1.6, 3.2)</math>. The area of <math>\triangle ODC</math> <math display="block">= \frac{(8)(3.2)}{2}</math><math display="block">= 12.8</math> The area of <math>\triangle ABC</math> <math display="block">= \frac{(10)(4)}{2}</math><math display="block">= 20</math> The required ratio <math>= 12.8:(20-12.8)</math> <math>= 12.8:7.2</math> <math>= 16:9</math></p>	<p>1M</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>for finding the coordinates of <math>C</math></p> <p>can be absorbed</p> <p>accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
<p>Let the coordinates of <math>D</math> be <math>(a, b)</math>. Then, we have <math>b=2a</math> and <math>a+2b-8=0</math>. Solving, the coordinates of <math>D</math> are <math>(1.6, 3.2)</math>. <math>OD^2 = (1.6)^2 + (3.2)^2 = 12.8</math> and <math>AB^2 = 2^2 + 4^2 = 20</math> The required ratio <math>= 12.8:(20-12.8)</math> <math>= 16:9</math></p>	<p>1A</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>can be absorbed for either one</p> <p>accept 1:s and t:1 with s r.t. 0.563 and t r.t. 1.78</p>
	<p>------(4)</p>	

Solution	Marks	Remarks
14. (a) $\sin 30^\circ = \frac{BE}{120}$ $BE = 60$ cm $\cos 30^\circ = \frac{CE}{120}$ $CE = 60\sqrt{3}$ cm	1A  1A -----(2)	r.t. 104 cm $CE \approx 103.9230485$ cm
(b) By sine formula, we have $\frac{AB}{\sin 40^\circ} = \frac{120}{\sin 60^\circ}$ $AB \approx 89.06726388$ $AB \approx 89.1$ cm                     and $\frac{AC}{\sin 80^\circ} = \frac{120}{\sin 60^\circ}$ $AC \approx 136.4589651$ $AC \approx 136$ cm	1M  1A+1A -----(3)	for either one  $AB$ r.t. 89.1 cm $AC$ r.t. 136 cm
(c) $CD = \sqrt{AC^2 - AD^2}$ $CD \approx \sqrt{136.4589651^2 - 100^2}$ $CD \approx 92.84960504$ cm  $DE = \sqrt{AB^2 - (AD - BE)^2}$ $DE \approx \sqrt{89.06726388^2 - (100 - 60)^2}$ (by (a)) $DE \approx 79.58000688$ cm	1M  1M	for $\sqrt{AB^2 - (100 - BE)^2}$
By cosine formula, we have $\cos \angle CDE = \frac{DE^2 + CD^2 - CE^2}{2DE \cdot CD}$	1M	
$\cos \angle CDE \approx \frac{79.58000688^2 + 92.84960504^2 - (60\sqrt{3})^2}{2(79.58000688)(92.84960504)}$ (by (a)) $\angle CDE \approx 73.67434913^\circ$ $\angle CDE \approx 73.7^\circ$	1M  1A	r.t. 73.7°
The required distance $= CD \sin \angle CDE$ $\approx 92.84960504 \sin 73.67434913^\circ$ $\approx 89.10586658$ $\approx 89.1$ cm	1M  1A	r.t. 89.1 cm
Let $x$ cm be the shortest distance from $C$ to $DE$ . Then, we have $\frac{1}{2}(x)(DE) = \frac{1}{2}(CD)(DE) \sin \angle CDE$ $x = CD \sin \angle CDE$ $x \approx 92.84960504 \sin 73.67434913^\circ$ $x \approx 89.10586658$ $x \approx 89.1$ Thus, the required distance is 89.1 cm.	1M  1A	r.t. 89.1
	-----(6)	

Solution	Marks	Remarks
<p>15. (a) The mean = 122 marks</p> <p>The mean deviation  <math display="block">\frac{38+36+32+29+22+19+(3)(2)+1+(2)(12)+14+15+22+(3)(24)+36}{20}</math>                     = 18.3 marks</p> <p>The standard deviation = 22 marks</p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>------(4)</p>	
<p>(b) The total number of the top 20% students in the music test = (20)(20%) = 4</p> <p>The least score for the top 20% students in the music test = 146 marks</p> <p>The score obtained by Mary = 122 + (22)(1) = 144 &lt; 146</p> <p>Thus, Mary is not one of the top 20% students in the music test.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>can be absorbed</p> <p>must show reasons</p>
<p>The total number of the top 20% students in the music test = (20)(20%) = 4</p> <p>The least score for the top 20% students in the music test = 146 marks</p> <p>The standard score of the least score for the top 20% students in the music test  <math display="block">= \frac{146 - 122}{22}</math> <math display="block">= \frac{12}{11}</math>                     &gt; 1</p> <p>Thus, Mary is not one of the top 20% students in the music test.</p>	<p>1A</p> <p>1M</p> <p>1A</p>	<p>can be absorbed</p> <p>must show reasons</p>
<p>(c) (i) The required probability = <math>\frac{1}{20}</math></p> <p>(ii) The required probability = <math>2\left(\left(\frac{1}{20}\right)\left(\frac{1}{19}\right) + \left(\frac{1}{20}\right)\left(\frac{1}{19}\right)\right)</math> = <math>\frac{1}{95}</math></p>	<p>1A</p> <p>1M+1M</p> <p>1A</p> <p>------(4)</p>	<p>0.05</p> <p>1M for <math>\left(\frac{1}{n}\right)\left(\frac{1}{n-1}\right)</math> where <math>n \geq 10</math> + 1M for the four cases r.t. 0.0105</p>

Solution	Marks	Remarks
<p>16. (a) (i) The required interest  <math>= (200\ 000)\left(\frac{6\%}{12}\right)</math>  <math>= \\$1\ 000</math></p> <p>(ii) The required amount  <math>= 200\ 000 + 1\ 000 - x</math>  <math>= \\$ (201\ 000 - x)</math></p> <p>(iii) The required amount  <math>= 200\ 000\left(1 + \frac{6\%}{12}\right)^n - x\left(1 + \frac{6\%}{12}\right)^{n-1} - x\left(1 + \frac{6\%}{12}\right)^{n-2} - \dots - x</math>  <math>= 200\ 000(1.005)^n - x[(1.005)^{n-1} + (1.005)^{n-2} + \dots + 1]</math>  <math>= 200\ 000(1.005)^n - x\left[\frac{(1.005)^n - 1}{1.005 - 1}\right]</math>  <math>= \\$ \{200\ 000(1.005)^n - 200x[(1.005)^n - 1]\}</math></p>	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1</p> <p>-----(6)</p>	<p></p> <p>pp-1 for missing '(' or ')'</p> <p>for sum of GP</p> <p>pp-1 for missing '(', ')', '[', ']', '{' or '}'</p>
<p>(b) (i) Assume that Peter has not yet fully repaid the loan after paying the <math>n</math>th instalment but the loan is fully repaid after Peter has paid the <math>(n+1)</math>th instalment. Then, by (a)(iii),</p> $0 < 200\ 000(1.005)^n - (200)(1\ 800)[(1.005)^n - 1] \leq \frac{1\ 800}{1 + \frac{6\%}{12}}$ $360\ 000 \leq 160\ 000(1.005)^{n+1} < 360\ 000(1.005)$ $2.25 \leq (1.005)^{n+1} < 2.26125$ $\frac{\log(2.25)}{\log(1.005)} \leq n+1 < \frac{\log(2.26125)}{\log(1.005)}$ $162.5911713 \leq n+1 < 163.5911713$ <p>Thus, the required number of the months is 163 .</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>accept either inequality</p> <p>for taking log</p>
<p>Let the required time be <math>n</math> months. By (a)(iii), we have</p> $200\ 000(1.005)^n - (200)(1\ 800)[(1.005)^n - 1] \leq 0$ $360\ 000 \leq 160\ 000(1.005)^n$ $(1.005)^n \geq \frac{9}{4}$ $n \geq \frac{\log(2.25)}{\log(1.005)}$ $n \geq 162.5911713$ <p>Thus, the required number of the months is 163 .</p>	<p>1M</p> <p>1M</p> <p>1A</p>	<p>accept using (a)(iii) = 0</p> <p>for taking log</p>
<p>(ii) By(a)(i), the monthly instalment of \$ 900 is less than the loan interest of \$1 000 for the 1st month.          Therefore, the loan can never be fully repaid.          Thus, the bank refuses his request.</p>	<p>1M</p> <p>1A</p>	<p>for comparing the result of (a)(i)</p>
<p>If the loan can be fully repaid in <math>m</math> months, then by (a)(iii),</p> $200\ 000(1.005)^m - (200)(900)[(1.005)^m - 1] \leq 0$ $180\ 000 \leq -20\ 000(1.005)^m$ <p>which has no solution.          Therefore, the loan can never be fully repaid.          Thus, the bank refuses his request.</p>	<p>1M</p> <p>1A</p>	<p>for mentioning no solution</p>
	<p>----- (5)</p>	

Solution	Marks	Remarks						
<p>17. (a) (i) Note that <math>\angle QRP = 90^\circ</math> (<math>\angle</math> in semi-circle)</p> <p>In <math>\triangle OQR</math> and <math>\triangle ORP</math>,  <math>\therefore \angle QRO = 90^\circ - \angle PRO</math>  <math>\angle RPO = 90^\circ - \angle PRO</math> (<math>\angle</math> sum of <math>\triangle</math>)  <math>\therefore \angle QRO = \angle RPO</math>  <math>\angle QOR = 90^\circ = \angle ROP</math> (given)  <math>\angle OQR = \angle ORP</math> (<math>\angle</math> sum of <math>\triangle</math>)                      Therefore, <math>\triangle OQR \sim \triangle ORP</math> (AAA)</p> <p>So, we have <math>\frac{OR}{OQ} = \frac{OP}{OR}</math>.</p> <p>Thus, we can conclude that <math>OR^2 = OP \cdot OQ</math>.</p>		<p>[半圓上的圓周角]</p> <p>[<math>\triangle</math>內角和]</p> <p>[已知]</p> <p>[<math>\triangle</math>內角和]</p> <p>[等角] (AA) (equiangular)</p>						
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<p>(ii) In <math>\triangle MON</math> and <math>\triangle POR</math>,  <math>\angle MNO = \angle PRO</math> (<math>\angle</math>s in the same segment)</p> <p><math>\therefore \angle MON = 90^\circ</math> (<math>\angle</math> in semi-circle)  <math>\angle POR = 90^\circ</math> (given)  <math>\therefore \angle MON = \angle POR</math>  <math>\angle OMN = \angle OPR</math> (<math>\angle</math> sum of <math>\triangle</math>)                      Therefore, <math>\triangle MON \sim \triangle POR</math> (AAA)</p>		<p>[同弓形內的圓周角]</p> <p>[對同弧的圓周角]</p> <p>[半圓上的圓周角]</p> <p>[已知]</p> <p>[<math>\triangle</math>內角和]</p> <p>[等角] (AA) (equiangular)</p>						
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<p>(b) (i) By (a)(i), we have <math>OR^2 = OP \cdot OQ</math>  <math>OR^2 = (4)(9)</math>  <math>OR = 6</math>                      Thus, the coordinates of <math>R</math> are <math>(0, 6)</math>.</p>	1A	pp-1 for missing '(' or ')'						
<p>(ii) Note that <math>PR = \sqrt{4^2 + 6^2} = \sqrt{52} = 2\sqrt{13}</math>.</p> <p>By (a)(ii), we have <math>\frac{MN}{2\sqrt{13}} = \frac{3\sqrt{13}}{6}</math>.</p> <p>Therefore, we have <math>MN = \frac{13}{2}</math>.</p> <p>Hence, the radius of the circle <math>MONR</math> is <math>\frac{13}{4}</math>.</p> <p>Let the coordinates of the centre of the circle <math>MONR</math> be <math>(a, b)</math>.                      Then, we have  <math>b = \frac{OR}{2} = \frac{6}{2} = 3</math> and  <math>a = -\sqrt{\left(\frac{13}{4}\right)^2 - 3^2} = \frac{-5}{4}</math>.</p> <p>So, the coordinates of the centre of the circle <math>MONR</math> are <math>\left(\frac{-5}{4}, 3\right)</math>.</p>	1M 1M 1A	3.25 -1.25 pp-1 for missing '(' or ')'						
<p>------(5)</p> <p>------(6)</p>								