香港考試局
HONG KONG EXAMINATIONS AUTHORITY

一九九六年香港中學會考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1996

數學 試卷一
MATHEMATICS PAPER I

本評卷參考乃考試局專爲今年本科考試而編寫，供閱卷員參考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員/教師應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員/教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations Authority for markers' reference. The Examinations Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Examinations Authority is counting on the co-operation of markers/teachers in this regard.

考試結束後，各科評卷參考將存放於教師中心，供教師參閱。

After the examinations, marking schemes will be available for reference at the Teachers' Centres.

Hong Kong Certificate of Education Examination
Mathematics Paper I

NOTES FOR MARKERS

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, provided that the method used is sound.

2. In a question consisting of several parts each depending on the previous parts, marks may be awarded to steps or methods correctly deduced from previous erroneous answers. However, marks for the corresponding answers should NOT be awarded. In the marking scheme, marks are classified as:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Others	awarded for correctly completing a proof or arriving at an answer given in a question.

3. Use of notation different from those in the marking scheme should not be penalised.

4. Each mark deducted for *poor presentation* (p.p.) should be denoted by [pp–1] :
   a. At most deduct 1 mark for (p.p.) in each question, up to a maximum of 3 marks for the whole paper.
   b. For similar (p.p.), deduct 1 mark for the first time that it occurs.
   i.e. do not penalise candidates twice in the paper for the same p.p.

5. Each Mark deducted for *wrong/no unit* (u.) should be denoted by [u–1] :
   a. No mark can be deducted for (u.) in Section A.
   b. At most deduct 1 mark for (u.) for the whole paper.

6. Marks entered in the Page Total Box should be the NET total scored on that page.

Solution	Marks	Remarks
1. $r=\frac{h-a}{1+p^2}$	1A	
$r=\frac{8-6}{1+(-4)^2}$	1M	
$=\frac{2}{17}$ (or 0.118)	1A	r.t. 0.118
OR $8=6+r[1+(-4)^2]$	1A	
$2=17r$		
$r=\frac{2}{17}$ (or 0.118)	1A	r.t. 0.118
	(3)	
2. $\frac{a^{\frac{5}{4}}\sqrt[4]{a^3}}{a^{-2}}=\frac{a^{\frac{5}{4}}a^{\frac{3}{4}}}{a^{-2}}$	1A	For $\sqrt[4]{a^3}=a^{\frac{3}{4}}$
$=a^{\frac{5}{4}+\frac{3}{4}-(-2)}$	1M	For applying either $a^m a^n=a^{m+n}$ or $\frac{a^m}{a^n}=a^{m-n}$.
$=a^4$	1A	
	(3)	
3. (a) 4, 1, −2, −5	1A	
(b) $S_{100}=\frac{100}{2}[2(4)+(100-1)(-3)]$	1M+1A	1M for $\frac{100}{2}[2a+(100-1)d]$, a is the 1st term in (a).
$=-14450$	1A	r.t. −14500
OR $T_{100}=4+(100-1)(-3)=-293$	1A	
$S_{100}=\frac{100}{2}[4+(-293)]$	1M	
$=-14450$	1A	
	(4)	
4. Let $f(x)=x^3-x^2-3x-1$, then $f(-1)=(-1)^3-(-1)^2-3(-1)-1=0$.	1A	Accept using long or synthetic division
$\therefore$ $x+1$ is a factor of x^3-x^2-3x-1.		pp-1 for not defining $f(x)$
$x^3-x^2-3x-1=0$		
$(x+1)(x^2-2x-1)=0$	1A	
$x=-1$ or $\frac{2\pm\sqrt{(-2)^2-4(-1)}}{2}$	1A+1M	1A for $x=-1$
		1M for quad. formula
$x=-1$ or $1\pm\sqrt{2}$	1A	For $x=1\pm\sqrt{2}$
	(5)	

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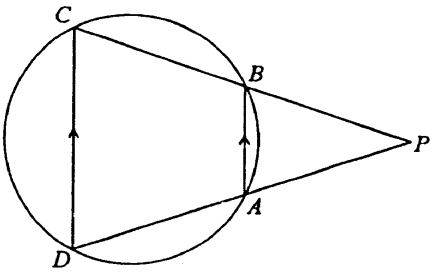
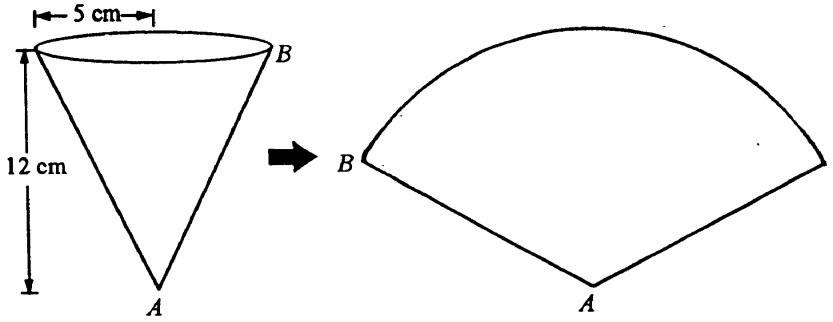
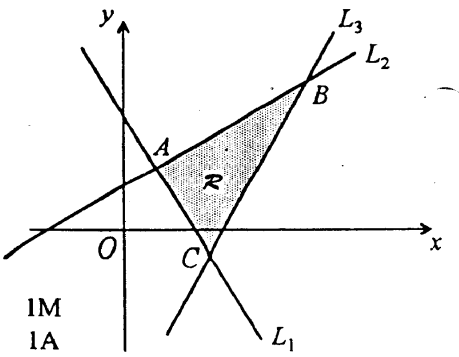
Solution	Marks	Remarks
5.		Accept graphical solutions. Withhold 1 mark for having equal signs in inequalities.
(i) $x > 3$	1A	
(ii) $(x-2)(x-4) < 0$	1A	For factorization, can be omitted
$2 < x < 4$	2A	Accept "$x > 2$ and $x < 4$"
Solution of (i) and (ii): $3 < x < 4$	1A	
	(5)	
6. $\because$ $\angle ABP = \angle DCB$ (corr. $\angle$s, $AB//DC$)		"同位角，$AB//DC$"
$\angle DCB = \angle BAP$ (ext. $\angle$, cyclic quad.)		"圓內接四邊形外角"
$\angle ABP = \angle BAP$		
$\therefore$ $AP = BP$ (sides opp. equal $\angle$s)		Or "base $\angle$s equal", "converse of 'base $\angle$s, iso. Δ' ", "equal $\angle$s, equal sides"
		"等角對邊相等" 或 "等腰三角形底角等的逆定理" 或 "底角相等" 或 "等邊對等角" 或 "等角對等邊"
Marking scheme:		
Case 1 Any correct proof with correct reasons.	5	
Case 2 Any correct proof without reasons.	3	
In addition, any correct argument with correct reason.	1	Maximum 1 mark
Case 3 Any correct argument with correct reason.	1	Maximum 2 marks
	(5)	
7. (a) Area of the shaded region $= (12^2 - 2^2)\pi$ cm^2		
$= 140\pi$ cm^2 (or 440 cm^2)	1A	r.t. 440
(b) (i) The probability that both darts hit the shaded region		
$= \left(\frac{140\pi}{144\pi}\right)^2$ (or 0.972^2)	1M+1M	1M for $\frac{\text{Ans. in (a)}}{\text{Area of the board}}$; 1M for p^2
$= \frac{1225}{1296}$ (or 0.945)	1A	r.t. 0.945
(ii) The probability that only one dart hits the shaded region		
$= 2\left(\frac{140\pi}{144\pi}\right)\left(\frac{4\pi}{144\pi}\right)$	1M+1M	1M for $p(1-p)$; 1M for $2p$ or $p+p$
$= \frac{35}{648}$ (or 0.0540)	1A	r.t. 0.0540
	(7)	pp-1 if no text in (b)

Solution	Marks	Remarks
8. (a) Capacity $= \frac{1}{3}\pi(5)^2(12)$ cm^3		
≈ 314 cm^3 (or 100π cm^3)	1A	r.t. 314
(b) (i) $AB = \sqrt{5^2+12^2}$ cm $= 13$ cm	1A	
Area of the sector $= \pi(5)(13)$ cm^2	1M	
≈ 204 cm^2 (or 65π cm^2)	1A	r.t. 204
OR Arc length of the sector $= 10\pi$ cm		
Area of the sector $= \frac{1}{2}\cdot 13\cdot 10\,\pi$ cm^2	1A+1M	1A for 13
≈ 204 cm^2 (or 65π cm^2)	1A	r.t. 204
(ii) Angle of the sector $= \frac{65\pi}{13^2\pi}\times 360°$	1M	
$\approx 138°$	1A	r.t. 138
OR Let the angle of the sector be θ.		
$\frac{1}{2}(13)^2\theta = 65\pi$	1M	
$\theta = \frac{10}{13}\pi$ (or 2.42)	1A	r.t. 2.42
OR Angle of the sector $= \frac{10\pi}{13}$ (or 2.42)	1M+1A	1M for $\frac{\text{Arc length}}{AB}$, r.t. 2.42
	(6)	



9. (a) $3x + 2y - 7 = 0$ .....(1)
$2x - y - 7 = 0$ .....(3)
(3)×2 + (1): $7x - 21 = 0$
$x = 3,\ y = -1$ — 1M 1A
$\therefore\ C = (3, -1)$



(b) $3x + 2y \geq 7$
$3x - 5y \geq -7$
$2x - y \leq 7$ — 1A+1A — 1A for any one being correct; Withhold 1 mark for strict inequalities

(c) Let $P(x, y) = 2x - 2y - 7$, then
$P(1, 2) = -9$, $P(6, 5) = -5$, $P(3, -1) = 1$.
$\therefore$ The maximum value of $2x - 2y - 7$ is 1. — 1M 1A — For testing any one of these pts.; f.t.

(6)

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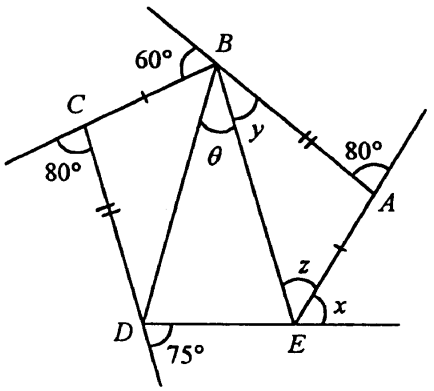
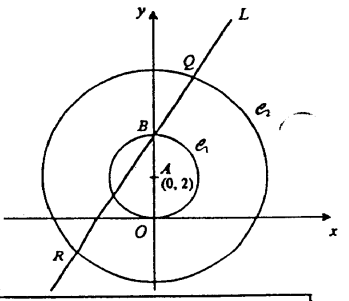
Solution	Marks	Remarks
10. (a) $x = 360° - (80° + 60° + 80° + 75°)$ $= 65°$	1A	
(b) $\Delta ABE \cong \Delta CDB$	1A	
(c) $60° + z + \theta + y = 180$		
$\because y + z = 80°$	1A	
$\therefore \theta = 180° - 60° - 80° = 40°$	1A	
$\angle BDE = \frac{180°-40°}{2} = 70°$	1M	
$y = 180° - 75° - 70° = 35°$	1A	
$z = 80° - 35° = 45°$	1A	
OR $\angle BDE = \angle BED$		
$180° - 75° - y = 180° - 65° - z$	1M	
$z - y = 10°$ .....(1)		
$y + z = 80°$ .....(2)	1A	
Solving (1) and (2), $y = 35°$	1A	
$z = 45°$	1A	
$\theta = 180° - 60° - 35° - 45° = 40°$	1A	
	(7)	



Solution	Marks	Remarks
11. (a) (i) Equation of $\mathscr{C}_1$: $x^2+(y-2)^2=4$ (or $x^2+y^2-4y=0$)	1A	
(ii) $B=(0,4)$	1A	Can be omitted
Equation of L : $y=2x+4$	1A	Or equivalent
(b) $\begin{cases} x^2+(y-2)^2=25 \\ y=2x+4 \end{cases}$		
$x^2+(2x+4-2)^2=25$ (or $(y-4)^2+4(y-2)^2=100$)	1M	Eliminate either x or y
$5x^2+8x-21=0$ (or $5y^2-24y-68=0$)	1A	
$(5x-7)(x+3)=0$		
$x=\frac{7}{5}$ or -3. ($y=\frac{34}{5}$ or -2)	1A	Accepted $x=1.4$ or -3, $y=6.8$ or -2.
$Q=(\frac{7}{5},\frac{34}{5})$ [or (1.4, 6.8)], $R=(-3,-2)$.	1A	Must indicate Q and R
(c) (i) The mid-point of QR is the point on L which is nearest to A.	1M	Attempt to find mid-pt. of QR
Mid-point of $QR=\left(\frac{\frac{7}{5}+(-3)}{2},\frac{\frac{34}{5}+(-2)}{2}\right)$		
$=(-\frac{4}{5},\frac{12}{5})$ [or (–0.8, 2.4)]	1A	
OR Let L' be the line through A perpendicular to L.		
Equation of L': $\frac{y-2}{x}=-\frac{1}{2}$ or $x+2y-4=0$	1M	For attempting to find the eqtn. of L'
The required point is the intersection of L and $L'=(-\frac{4}{5},\frac{12}{5})$.	1A	
(ii) The intersection of QA and $\mathscr{C}_1$ is the pt. on $\mathscr{C}_1$ which is nearest to Q.	1M	
The required point is $\left(\frac{3(0)+2(\frac{7}{5})}{5},\frac{3(2)+2(\frac{34}{5})}{5}\right)$	1M	
$=(\frac{14}{25},\frac{98}{25})$ [or (0.56, 3.92)]	1A	
OR Equation of QA : $24x-7y+14=0$	1M	For finding eqtn. of QA
Solving the equations of QA and $\mathscr{C}_1$, we have $625y^2-2500y+196=0$		
$y=\frac{2}{25}$ (rej.) or $\frac{98}{25}$	1M	For solving & giving 1 soln. only
The required pt. is $(\frac{14}{25},\frac{98}{25})$ [or (0.56, 3.92)]	1A	



Solution	Marks	Remarks

12. (a) (i)

Table 1 (Mr. Chan)

Month	Loan Interest ($)	Loan Repaid ($)	Outstanding Balance ($)
1	750.00	8 250.00	41 750.00
2	626.25	8 373.75	33 376.25
3	500.64	8 499.36	24 876.89
4	(*a*) 373.15	(*b*) 8 626.85	(*c*) 16 250.04 (05)
5	243.75	8 756.25	7 493.79 (80)
6	112.41	7 493.79 (80)	0.00

1A+1M — 1A for (*a*), 1M for $a + b = 9000$ or $b + c = 24876.89$

1A

(ii) Amount of last payment = $ 7 606.20 (21) — 1A — Accept 7606.2

(iii) Total interest earned by the bank = $ 2 606.20 — 1A — Accept 2606.2

(b)

Table 2 (Mrs. Lee)

Month	Instalment ($)	Loan Interest ($)	Loan Repaid ($)	Outstanding Balance ($)
1	9 000.00	750.00	8 250.00	41 750.00
2	10 800.00	626.25	10 173.75	31 576.25
3	12 960.00	473.64	12 486.36	19 089.89
4	15 552.00	286.35	15 265.65	3 824.24
5	3 881.60	57.36	3 824.24	0.00

1A+1A+1A — 1A for 10800.00, 1A for 626.25, 1A for the whole row

1A

(c) Mr. Cheung saves $ 1 2000 each month.

Month	Savings ($)	Instalment ($)	Balance ($)
1	12 000.00	9 000.00	3 000.00
2	15 000.00	10 800.00	4 200.00
3	16 200.00	12 960.00	3 240.00
4	15 240.00	15 552.00	−312.00

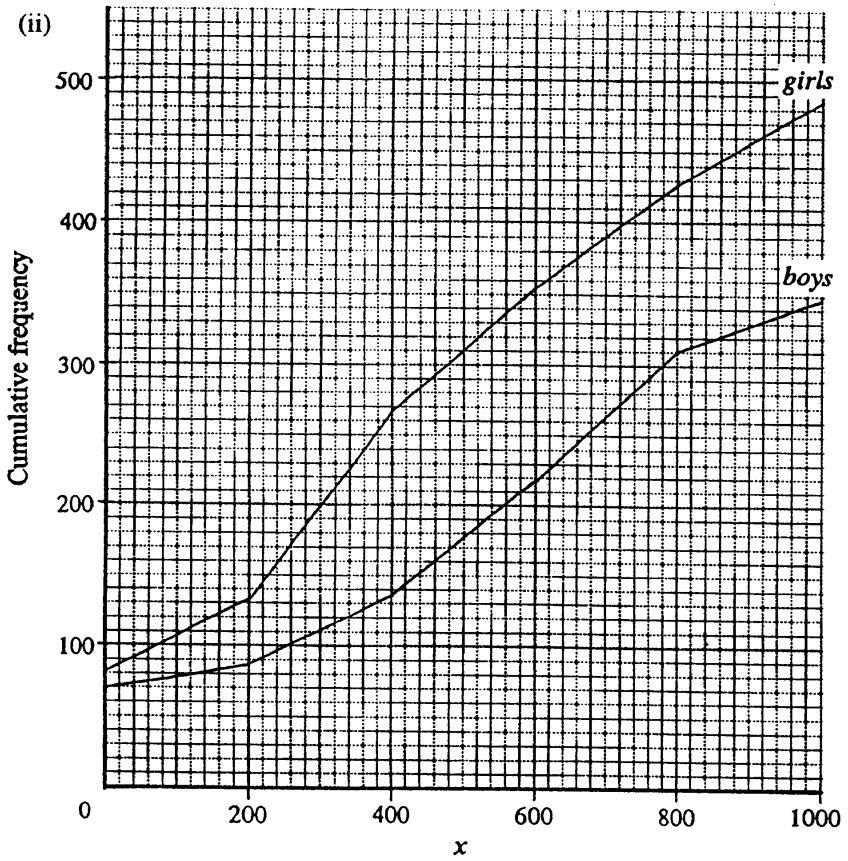
Mr. Cheung will not have enough savings to pay the 4th instalment.

Marking Scheme:		
Mr. Cheung cannot afford to use the repayment scheme as described in (b).	1A	
Putting the balance $3000 into his savings after paying the 1st instalment.	1M	Can be omitted
Showing (the 4th month's instalment) > (the 4th month's savings) i.e. 15552.00 > 15240.00 OR (the 1st 4 months' instalments) > (the 1st 4 months' savings) i.e. 48312 > 48000	1A	

Solution	Marks	Remarks
13. (a) Let $F = k_1xt + k_2t^2$ for some constants k_1 and k_2.	1A	
Then $Q = 20000 + k_1xt + k_2t^2$.		
Hence $\begin{cases} 30600 = 20000 + k_1(85)(40) + k_2(40)^2 \\ 28100 = 20000 + k_1(75)(60) + k_2(60)^2 \end{cases}$	1M+1A	1M for substitution
$\begin{cases} 53 = 17k_1 + 8k_2 \\ 9 = 5k_1 + 4k_2 \end{cases}$	1A	Or equivalent forms
$7k_1 = 35$ or $28k_2 = -112$		
$\begin{cases} k_1 = 5 \\ k_2 = -4 \end{cases}$	1	
$\therefore \quad Q = 20000 + 5xt - 4t^2$		
(b) (i) When $x = 82$ and $t = 45$, then		
$Q = 20000 + 5(82)(45) - 4(45)^2$		
$= 30350$	1A	r.t. 30400
(ii) When $Q = 30350$ and $x = 78$, then		
$30350 = 20000 + 5(78)t - 4t^2$	1M	Use ans. in (b)(i)
$4t^2 - 390t + 10350 = 0$		
$2t^2 - 195t + 5175 = 0$		
$\Delta = 195^2 - 4(2)(5175)$		
$= -3375$		
$\therefore$ There is no real solution for t.		
Thus it is not possible to achieve the same value of Q in (i) by varying t.	1M+1A	f.t.
(c) When $x = 80$,		
$Q = 20000 + 5(80)t - 4t^2$		
$= 20000 - 4(t^2 - 100t)$		
$= 30000 - 4(t - 50)^2$	1M+1A	1M for $a+b(t-50)^2$ or $a+b(2t-100)^2$
$\therefore$ Q is maximum when $t = 50$.		
Thus the amount of time required is 50 seconds.	1A	

Solution	Marks	Remarks
14. (a) The unclearly printed number in Table 3 is 23.7.	1A	pp-1 for 23.7%
(b) There are rounding off errors.	1A	
(c) (i) The c.f. table of the distribution of x ($x \le 1000$) for boys		

x ($\le$)	c.f.
0	70
200	87
400	135
600	218
800	310
1000	346

	1A	For 87, pp-1 for writing intervals in the left column
	1A	
(ii) 	1M+1A	
(iii) The median of x for boys = 490	1A	Accept 480–500
The median of x for girls = 410	1A	Accept 400–420
(iv) From the cumulative frequency polygons, there are 265(±5) boys and 390(±5) girls spending up to \$700	1M	For either one correct or a vertical line through x=700
$\therefore$ The total number = 265+390 = 655	1A	Accept 645–665
(d) From Tables 3 and 4, 20.0% of boys and 15.0% of girls did not spend any money on buying clothes for Christmas.		
$\therefore$ In this survey, more boys did not spend any money on buying clothes for Christmas.	1	The exact figures need not be quoted
We have to consider the percentages instead of the frequencies because the number of boys and the number of girls in this survey are not equal.	1	

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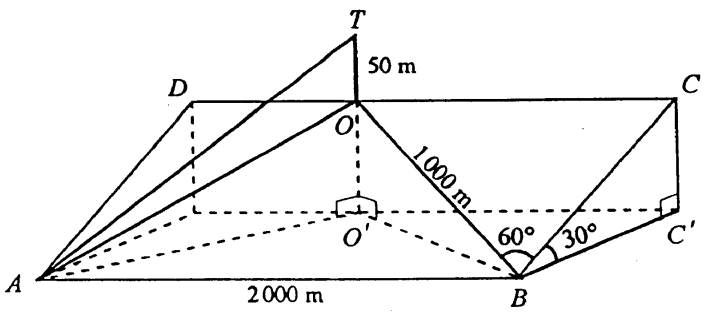
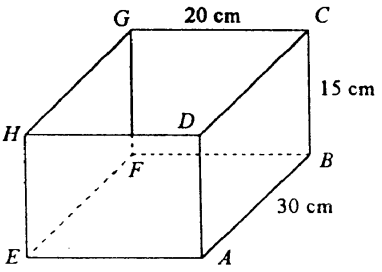
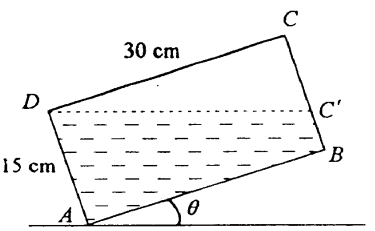
Solution	Marks	Remarks
15. (a) $BC = 1000\cos 60° = 500$ (m)	1A	
$CC' = 500\sin 30° = 250$ (m)	1A	
(b) Let the inclination of BO with the horizontal be α.		
$\sin\alpha = \frac{250}{1000}$	1M	
$\alpha = 14.5°$ (or 14°29')	1A	r.t. 14.5
(c) $AO = \sqrt{1000^2 + 2000^2 - 2(1000)(2000)\cos 30°}$	2A	
≈ 1239.31		
≈ 1240 (or $1000\sqrt{5-2\sqrt{3}}$)		
OR $AD = BC = 1000\cos 60°$		
$DO = DC - OC = (2000 - 1000\sin 60°)$	1A	For DO
$AO = \sqrt{(1000\cos 60°)^2 + (2000 - 1000\sin 60°)^2}$	1A	
≈ 1240		
$AO' = \sqrt{(AO)^2 - (OO')^2}$		
$\approx \sqrt{(1239.31)^2 - (250)^2}$	1M	
≈ 1213.83		
$AT = \sqrt{(AO')^2 + (TO')^2}$		
$= \sqrt{(1239.31)^2 - (250)^2 + (300)^2}$	1M	
≈ 1250.36		
≈ 1250 (m)	1A	r.t. 1250
(d) Route I takes $\left(\frac{1000}{0.3} + 60\right)$ s ≈ 3393 s (or 56 min. 33 sec.)	1M+1A	1M for applying $t = \frac{d}{s}$
Route II takes $\left(\frac{2000}{0.8} + \frac{1250.36}{3.2}\right)$ s ≈ 2891 s (or 48 min. 11 sec.)		1A for either, r.t. 3390, 2890 resp.
Hence route II takes a shorter time.	1	



Solution	Marks	Remarks
16. (a) (i) Area $= [20\times30+2(20\times15+30\times15)]$	1M	For any two of 20×30, 20×15 and 30×15
$= 2100\ (cm^2)$	1A	
(ii) Capacity $= 20\times30\times15$		
$= 9000\ (cm^3)$	1A	
(b) (i) $\tan\theta = \frac{15}{30}$	1M	For $\frac{CC'}{30}$ with substitution
$= \frac{1}{2}$		
$\theta = 26.6°$ (or 26°34′)	1A	r.t. 26.6
(ii) If $\tan\theta = \frac{1}{3}$, then		
$V = \frac{1}{2}(30)(30\tan\theta)(20)$	1A	
$= \frac{1}{2}(30)(30)(\frac{1}{3})(20)$		
$= 3000$	1A	
(iii) If $V = 6750$, then		
$\frac{1}{2}(15)(\frac{15}{\tan\theta})(20) = 9000 - 6750$	1M+1A	1M for $9000 - 6750$ or showing that the water level is below B.
$\tan\theta = 1$		
$\theta = 45°$	1A	
(c) Suppose water is poured out by tilting the edge AB and the inclination of AE with the horizontal is ϕ.		
Then $\tan\phi = \frac{15}{20} = \frac{3}{4}$	1A	
$\therefore$ $\tan\phi > \tan\theta$, ϕ is larger than the value of θ in (b)(i).	1	
OR Since $AB > AE$ and AD is common, ϕ is larger than the value of θ in (b)(i).	1A 1	





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